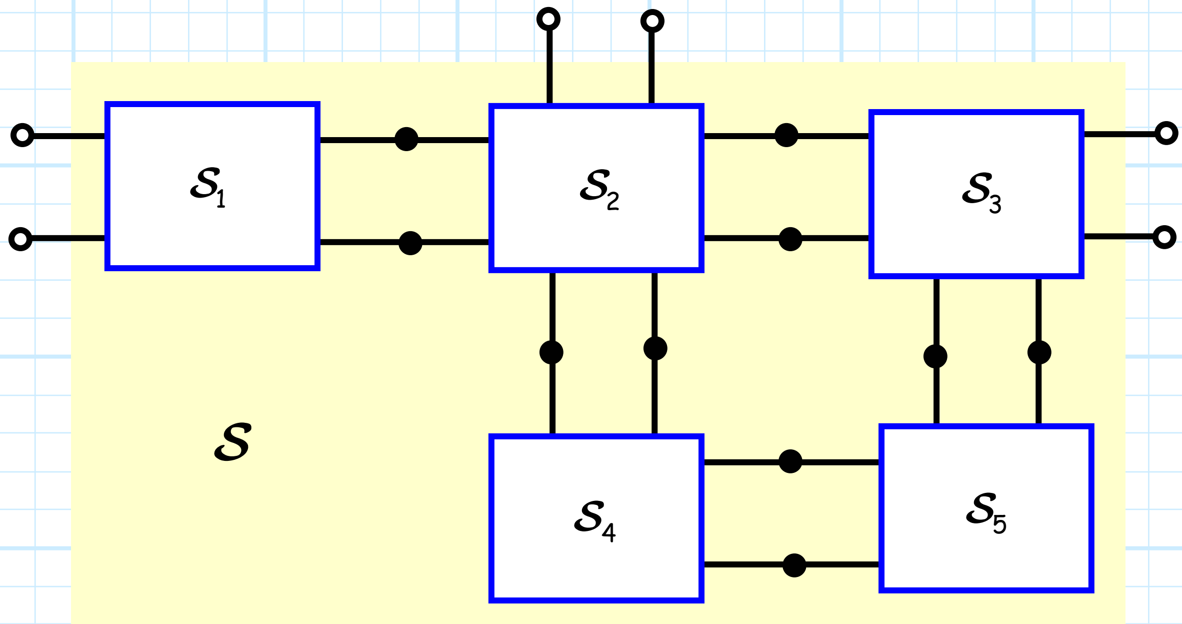
Signal Flow Graphs

Consider a complex **3-port** microwave network, constructed of **5** simpler microwave devices:



where $\mathcal{S}_n$ is the **scattering matrix** of each device, and $\mathcal{S}$ is the **overall** scattering matrix of the **entire** 3-port network.

Q: *Is there any way to determine this **overall** network scattering matrix $\mathcal{S}$ from the **individual** device scattering matrices $\mathcal{S}_n$?*

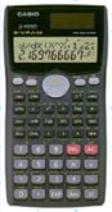
A: **Definitely**! Note the wave **exiting** one port of a device is a wave **entering** (i.e., incident on) another (and vice versa). This is a **boundary condition** at the port connection between devices.

Add to this the scattering parameter equations from each individual device, and we have a **sufficient** amount of math to determine the relationship between the incident and exiting waves of the remaining three ports—in other words, the scattering matrix of the **3-port network**!

Q: *Yikes! Wouldn't that require a lot of **tedious** algebra!*

A: It sure would! We might use a **computer** to assist us, or we might use a tool employed since the early days of microwave engineering—the **signal flow graph**.

Signal flow graphs are helpful in (count em') **three ways**!



Way 1 - Signal flow graphs provide us with a **graphical** means of **solving** large systems of simultaneous equations.

Way 2 – We'll see the a signal flow graph can provide us with a **road map** of the wave **propagation paths** throughout a microwave device or network. If we're paying attention, we can glean great **physical insight** as to the inner working of the microwave device represented by the graph.

Way 3 - Signal flow graphs provide us with a quick and accurate method for **approximating** a network or device. We will find that we can often replace a rather complex graph with a much **simpler** one that is **almost** equivalent.

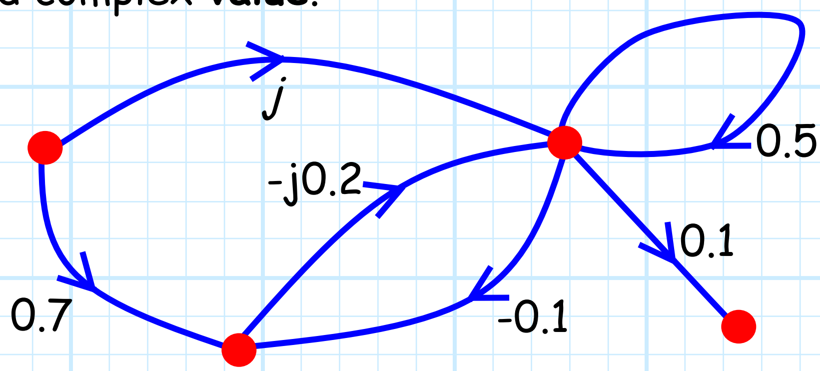

bunny, 64 spheres

We find this to be very helpful when **designing** microwave components. From the analysis of these approximate graphs, we can often determine **design rules** or equations that are tractable, and allow us to design components with (near) optimal performance.

Q: *But what* ***is*** *a signal flow graph?*

A: First, some **definitions**!

Every signal flow graph consists of a set of **nodes**. These nodes are connected by **branches**, which are simply contours with a specified **direction**. Each branch likewise has an associated complex **value**.



Q: *What could this possibly have to do with* ***microwave engineering****?*

A: Each **port** of a microwave device is represented by **two nodes**—the "*a*" node and the "*b*" node. The "*a*" node simply represents the value of the **normalized amplitude** of the wave incident on that port, evaluated **at** the plane of that port:

$$a_n \doteq \frac{V_n^+(z_n = z_{nP})}{\sqrt{Z_{0n}}}$$

Likewise, the "b" node simply represents the **normalized amplitude** of the wave **exiting** that port, evaluated **at** the plane of that port:

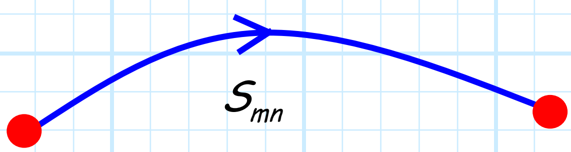
$$b_n \doteq \frac{V_n^-(z_n = z_{nP})}{\sqrt{Z_{0n}}}$$

Note then that the **total voltage** at a port is simply:

$$V_n(z_n = z_{nP}) = (a_n + b_n)\sqrt{Z_{0n}}$$

The value of the **branch** connecting two nodes is simply the value of the **scattering parameter** relating these two voltage values:

$$a_n \doteq \frac{V_n^+(z_n = z_{nP})}{\sqrt{Z_{0n}}}$$

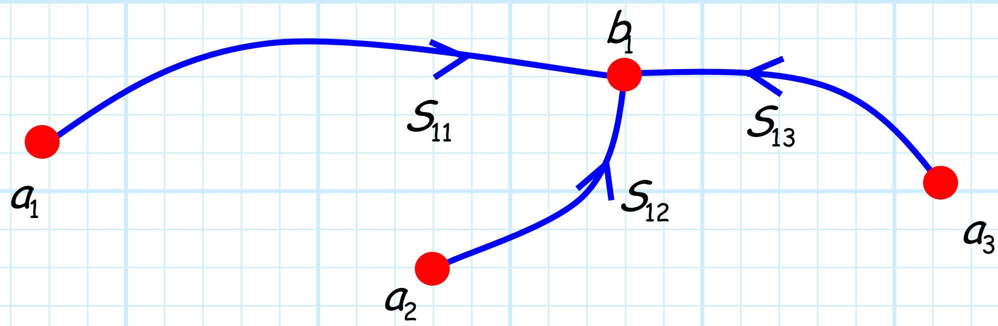


$$b_m \doteq \frac{V_m^-(z_m = z_{mP})}{\sqrt{Z_{0m}}}$$

The signal flow graph above is simply a **graphical** representation of the equation:

$$b_m = S_{mn}\, a_n$$

Moreover, if **multiple** branches enter a node, then the voltage represented by that node is the **sum** of the values from each branch. For example, the signal flow graph:



is a **graphical** representation of the equation:

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2 + S_{13}\, a_3$$

Now, consider a **two-port device** with a scattering matrix $\mathcal{S}$:

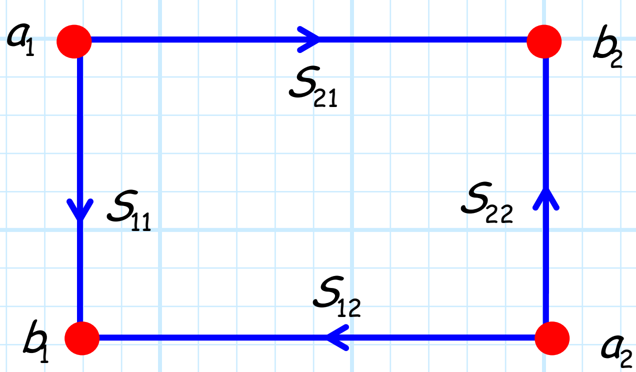
$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

So that:

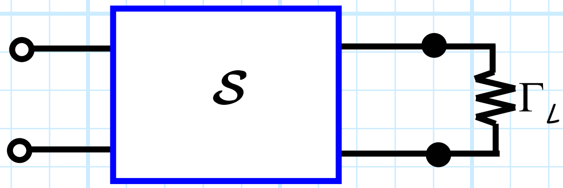
$$b_1 = S_{11}\, a_1 + S_{12}\, a_2$$

$$b_2 = S_{21}\, a_1 + S_{22}\, a_2$$

We can thus **graphically** represent a **two-port device** as:



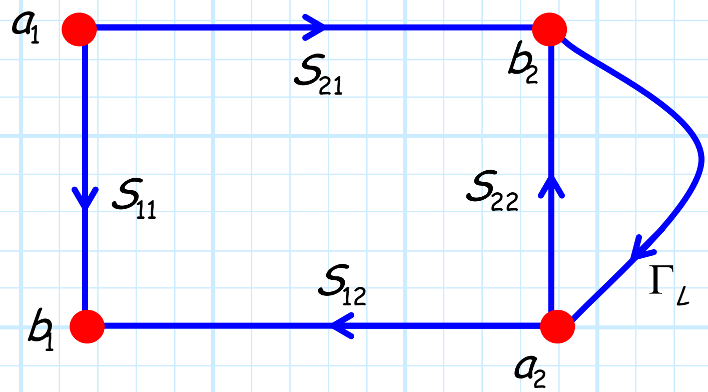
Now, consider a case where the second port is **terminated** by some load Γ_L:



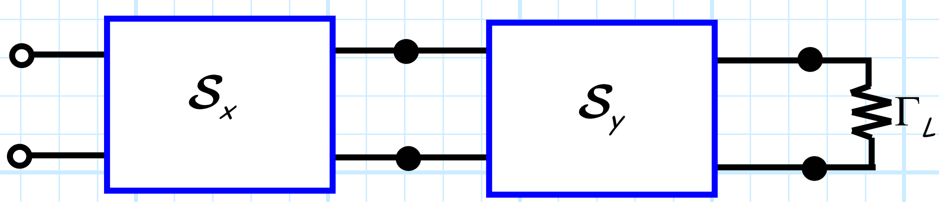
We now have yet **another** equation:

$$V_2^+(z_2 = z_{2P}) = \Gamma_L \, V_2^-(z_2 = z_{2P})$$
$$a_2 = \Gamma_L \, b_2$$

Therefore, the signal flow graph of this **terminated** network is:



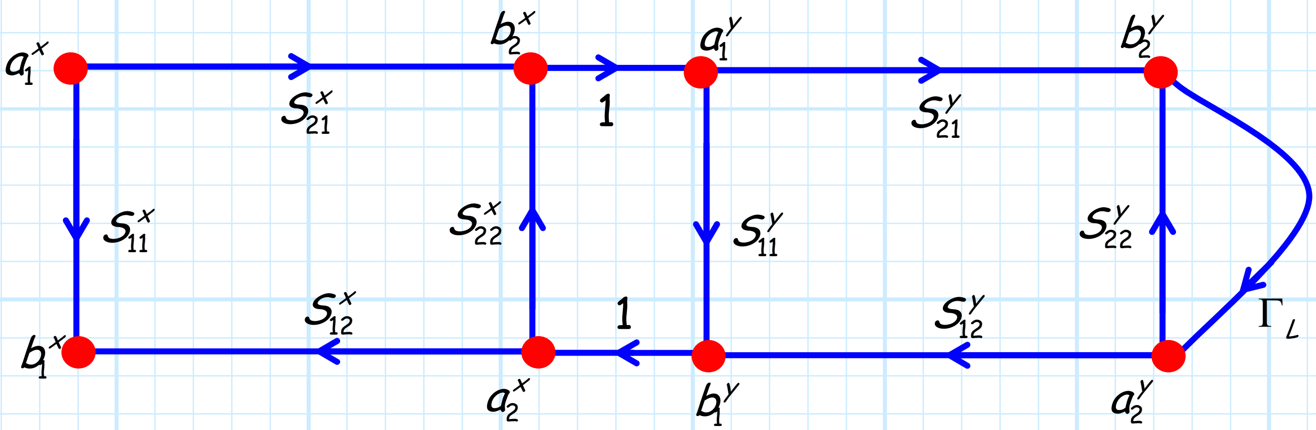
Now let's cascade **two different** two-port networks



Here, the output port of the first device is **directly** connected to the input port of the second device. We describe this mathematically as:

$$a_1^y = b_2^x \qquad \text{and} \qquad b_1^y = a_2^x$$

Thus, the signal flow graph of this network is:

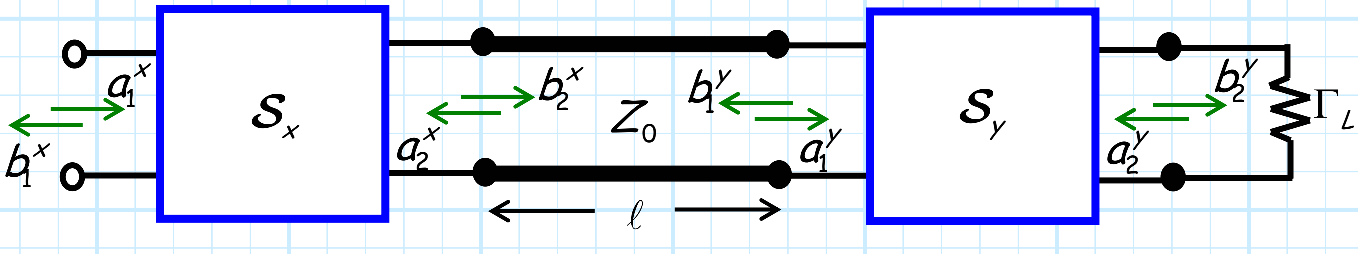


Q: *But what happens if the networks are connected with **transmission lines**?*

A: Recall that a length ℓ of transmission line with characteristic impedance Z_0 is likewise a **two-port** device. Its scattering matrix is:

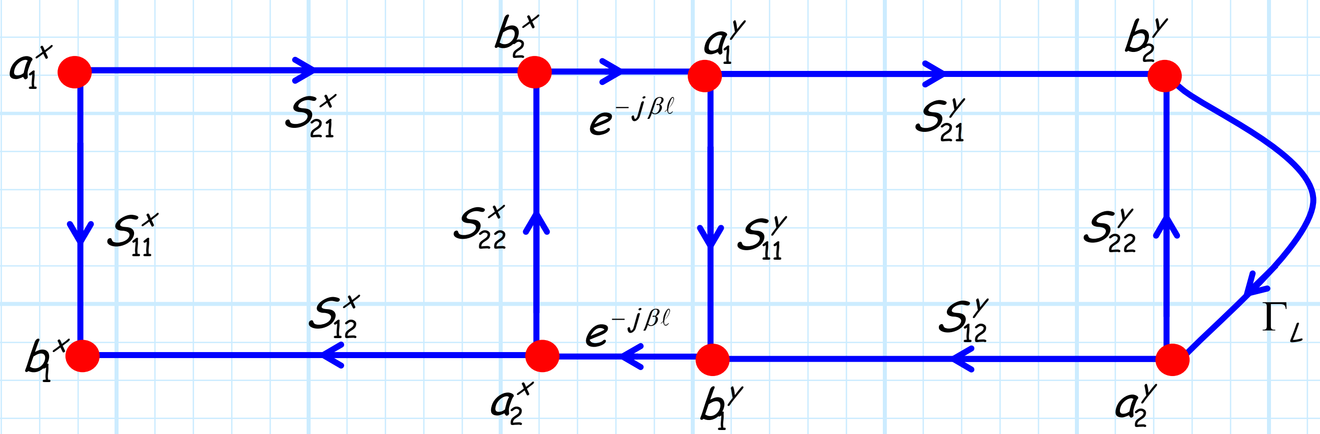
$$\mathcal{S} = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

Thus, if the two devices are connected by a length of **transmission line**:



$$a_1^y = e^{-j\beta\ell}\, b_2^x \qquad\qquad a_2^x = e^{-j\beta\ell}\, b_1^y$$

so the signal flow graph is:



Note that there is **one** (and only one) **independent variable** in this representation.

This **independent** variable is node a_1^x.

This is the only node of the *sfg* that does **not** have any **incoming** branches. As a result, its value depends on **no other** node values in the *sfg*.

→ From the standpoint of a *sfg*, independent nodes are essentially **sources**!

Of course, this likewise makes sense physically (do **you** see why?). The node value a_1^x represents the complex amplitude of the wave **incident** on the one-port network. If this value is **zero**, then **no power** is incident on the network—the rest of the nodes (i.e., wave amplitudes) will likewise be **zero**!

Now, say we wish to determine, for example:

1. The **reflection coefficient** Γ_{in} of the one-port device.

2. The **total current** at port 1 of second network (i.e., network y).

3. The **power absorbed** by the load at port 2 of the second (y) network.

In the first case, we need to determine the value of dependent node b_1^x:

$$\Gamma_{in} = \frac{b_1^x}{a_1^x}$$

For the second case, we must determine the value of wave amplitudes a_1^y and b_1^y:

$$I_1^y = \frac{a_1^y - b_1^y}{\sqrt{Z_0}}$$

And for the third and final case, the values of nodes a_2^y and b_2^y are required:

$$P_{abs} = \frac{\left|b_2^y\right|^2 - \left|a_2^y\right|^2}{2}$$

Q: *But just how the heck do we* ***determine*** *the values of these wave amplitude "nodes"?*

A: One way, of course, is to solve the **simultaneous equations** that describe this network.

From network x and network y:

$$b_1^x = S_{11}^x\, a_1^x + S_{12}^x\, a_2^x \qquad\qquad b_1^y = S_{11}^y\, a_1^y + S_{12}^y\, a_2^y$$

$$b_2^x = S_{21}^x\, a_1^x + S_{22}^x\, a_2^x \qquad\qquad b_2^y = S_{21}^y\, a_1^y + S_{22}^y\, a_2^y$$

From the transmission line:

$$a_1^y = e^{-j\beta\ell}\, b_2^x \qquad\qquad a_2^x = e^{-j\beta\ell}\, b_1^y$$

And finally from the load:

$$a_2 = \Gamma_L\, b_2$$

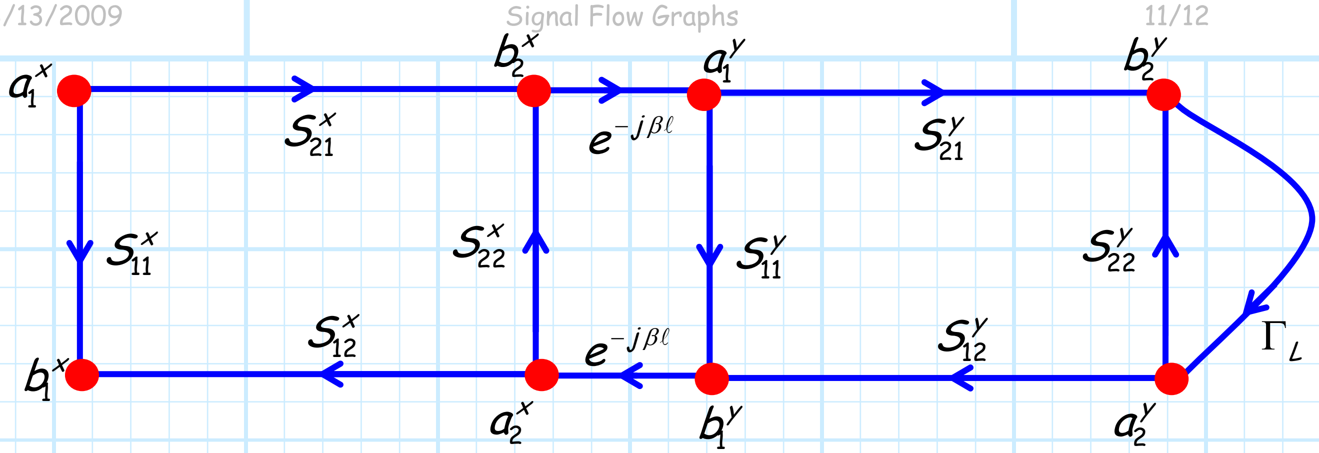
But another, **EVEN BETTER** way to determine these values is to **decompose (reduce)** the signal flow graph!

Q: *Huh?*

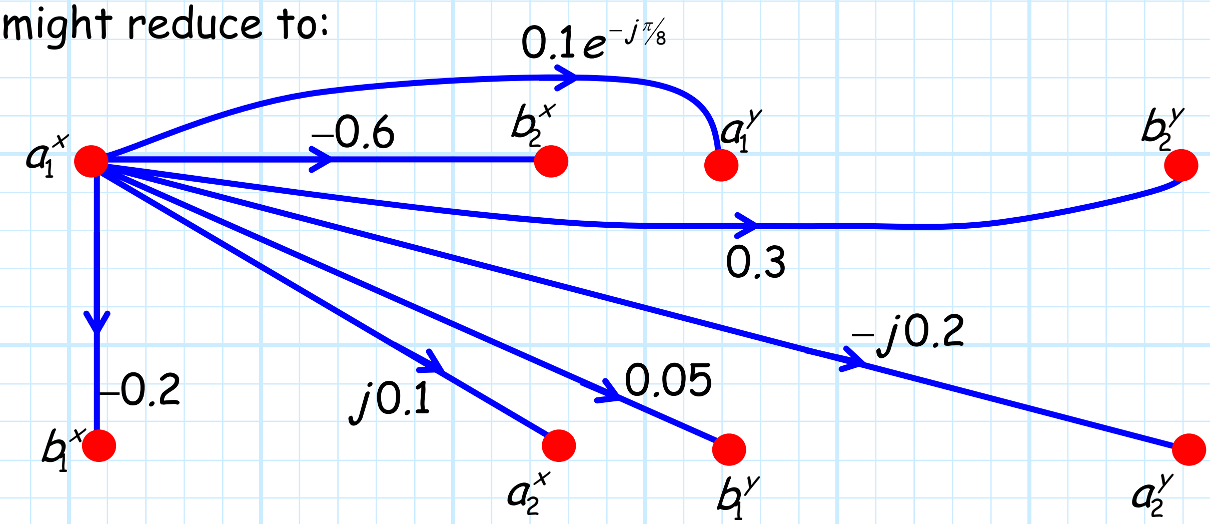
A: Signal flow graph **reduction** is a method for **simplifying** the **complex** paths of that signal flow graph into a more **direct** (but equivalent!) form.

Reduction is really just a **graphical** method of **decoupling** the simultaneous equations that are **described** by the *sfg*.

For instance, in the example we are considering, the *sfg*:



might reduce to:



From **this** graph, we can **directly** determine the value of each node (i.e., the value of each wave amplitude), in terms of the one independent variable a_1^x.

$$b_1^x = -0.2\, a_1^x$$

$$b_2^x = -0.6\, a_1^x \qquad a_2^x = j\,0.1\, a_1^x$$

$$b_1^y = 0.05\, a_1^x \qquad a_1^y = 0.1\, e^{-j\pi/8} a_1^x$$

$$b_2^y = 0.3\, a_1^x \qquad a_2^y = -0.2\, a_1^x$$

And of course, we can then determine values like:

1.

$$\Gamma_{in} = \frac{b_1^x}{a_1^x} = \frac{-0.2\, a_1^x}{a_1^x} = -0.2$$

2. $$I_1^y = \frac{a_1^y - b_1^y}{\sqrt{Z_0}} = \frac{0.1\, e^{-j\pi/8} - 0.05}{\sqrt{Z_0}}\, a_1^x$$

3. $$P_{abs} = \frac{\left|b_2^y\right|^2 - \left|a_2^y\right|^2}{2} = \frac{(0.3)^2 - (0.2)^2}{2}\left|a_1^x\right|^2$$

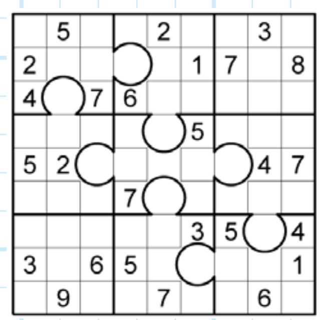
Q: *But **how** do we reduce the* sfg *to its simplified state? Just what is the **procedure**?*

A: Signal flow graphs can be reduced by sequentially applying one of **four simple rules**.

Q: *Can these rules be applied in **any order**?*

A: **No!** The rules can only be applied when/where the structure of the *sfg* allows. You must **search** the *sfg* for structures that allow a rule to be applied, and the *sfg* will then be (a little bit) reduced. You then search for the **next** valid structure where a rule can be applied. Eventually, the *sfg* will be **completely reduced**!

Q: *????*



A: It's a bit like solving a **puzzle**. Every *sfg* is different, and so each will require a different reduction procedure. It requires a little **thought**, but with a little practice, the reduction procedure is **easily** mastered.

You may even find that it's kind of **fun**!