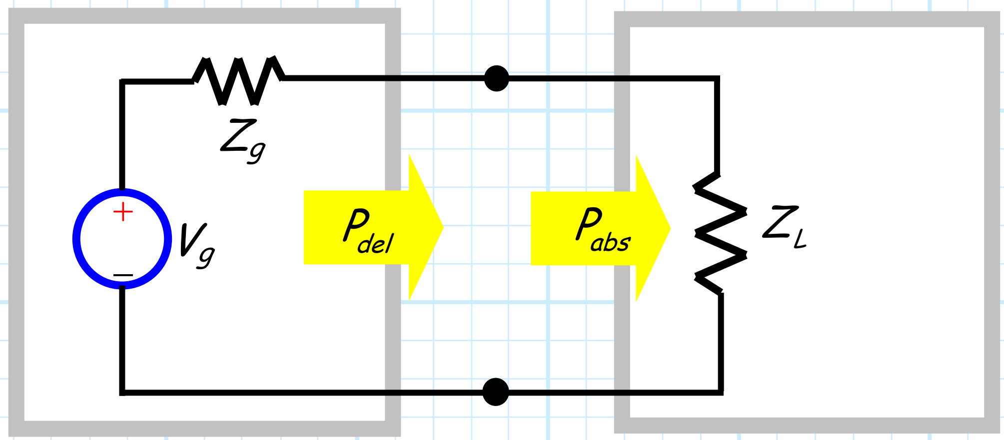
Special Cases of Source and Load Impedance

Consider again the power **absorbed** by the load (delivered by the source):

$$P_{del} = P_{abs} = \frac{|V_g|^2}{2} \frac{R_L}{|Z_g + Z_L|^2}$$



It is evident that this power transfer is dependent on **each** and **every** element of the equivalent circuit—the **source parameters** V_g and Z_g, as well as the **load impedance** Z_L.

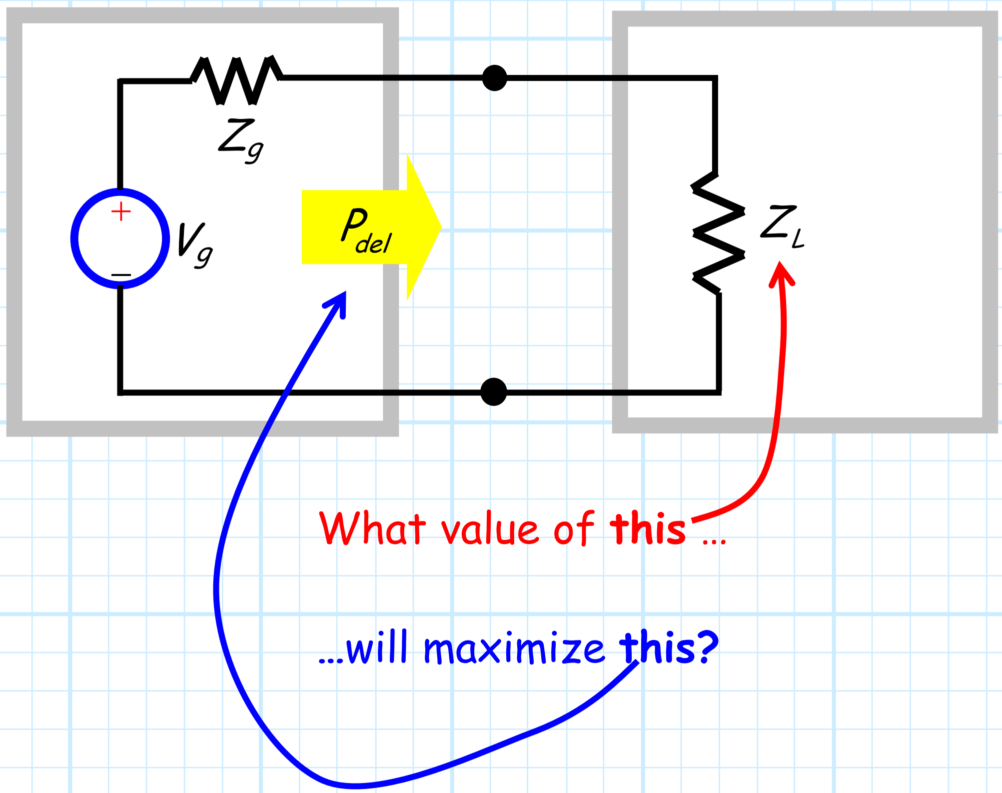
Q: *I assume that we want to* ***maximize*** *this power transfer.* ***How*** *can we maximize P_{abs}??*

A: The answer to that question is among the best known in electrical engineering. Unfortunately, it is also frequently misunderstood and misapplied—**so pay attention**!

Match the Load to the Source

First, let's ask **this** question:

Q1: *What **load** impedance Z_L will maximize the power delivered by the **source** (i.e., maximize P_{del})?*



A1: The **load** impedance $Z_L = Z_g^*$ will maximize the power delivered by the **source**.

The Available Power of the Source

We can likewise determine what the **value** of this maximum power is. For $Z_L = Z_g^*$, we find:

$$P_{del}\big|_{Z_L=Z_g^*} = \frac{|V_g|^2}{2}\frac{R_g}{|Z_g + Z_g^*|^2} = \frac{|V_g|^2}{2}\frac{R_g}{|2R_g|^2} = \frac{|V_g|^2}{8R_g}$$

This maximum delivered power is **very important** and is dubbed the **available power** P_{avl} of the **source**:

$$P_{avl} = \frac{|V_g|^2}{8R_g}$$

Note the available power is dependent **just on source parameters** (i.e., V_g and R_g), and so P_{avl} is a parameter of the **source only**.

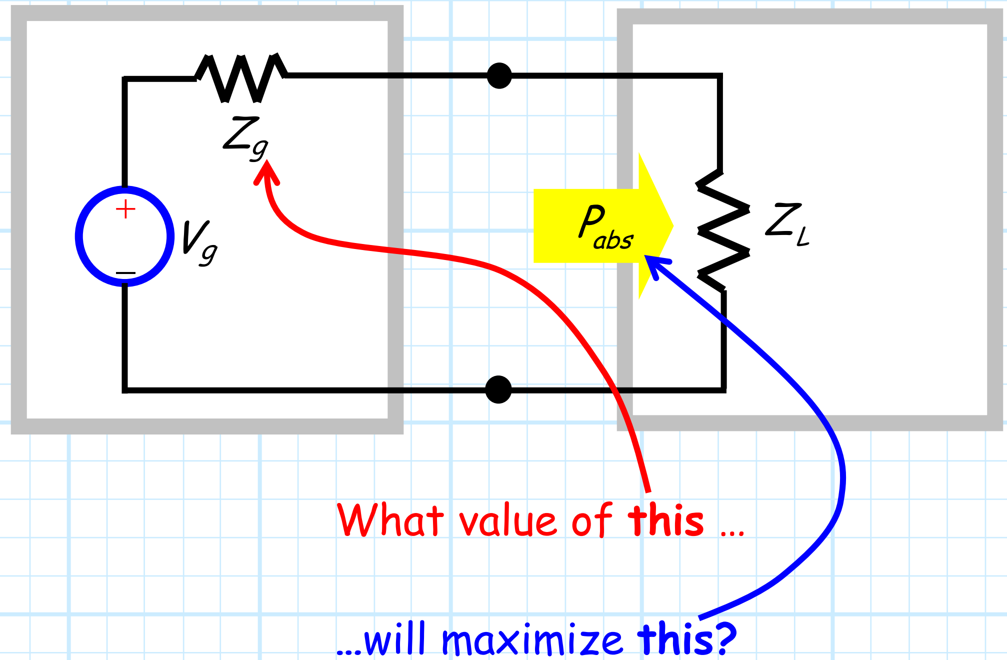
This available power is the **most** that can be **delivered** by the source(i.e., $P_{del} \leq P_{avl}$), and this available source power can **only** be delivered if a load $Z_L = Z_g^*$ is connected:

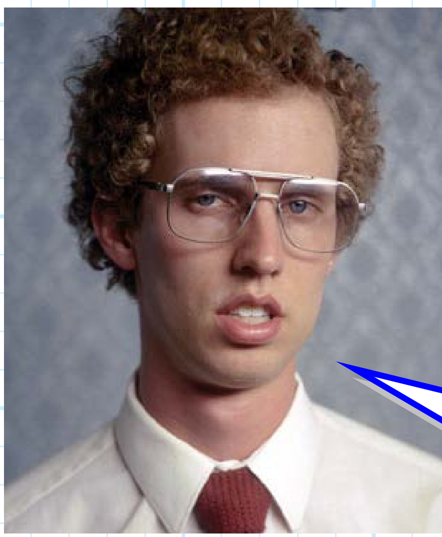
$$P_{del} = P_{avl} = \frac{|V_g|^2}{8R_g} = \qquad \text{iff} \qquad Z_L = Z_g^*$$

A Completely Different Question!

Now, let's ask **completely different** question:

Q2: *What **source** impedance Z_g will maximize the power absorbed by the **load** (i.e., maximize P_{abs})?*





*Like, we already know the answer. Absorbed power is maximized by a **conjugate match**:*

$$Z_g = Z_L^* . \quad \textit{Gosh!}$$

NOT!

Not so fast! It can be shown that the value of Z_g that **maximizes** the absorbed power:

$$P_{abs} = \frac{\left|V_g\right|^2}{2} \frac{R_L}{\left|Z_g + Z_L\right|^2}$$

is:

$$\boxed{Z_g = -jX_L}$$

where X_L is the imaginary (i.e., reactive portion of the load ($X_L = \text{Im}\{Z_L\}$).

Thus, we conclude:

Q2: *What **source** impedance Z_g will maximize the power absorbed by the **load** (i.e., maximize P_{abs})?*

A2: The **source** impedance $Z_g = -jX_L$ will maximize the power absorbed by the **load** (i.e., maximize P_{abs}).

Don't Make This Mistake!

Although it is **very** common for electrical engineers to incorrectly assume the answer to question **Q2** is the answer to **Q1** (i.e., $Z_g = Z_L^*$), and this is **far** from the correct answer!

Using the **correct** solution $Z_g = -jX_L$, we find:

$$P_{abs}\Big|_{Z_g=-jX_L} = \frac{|V_g|^2}{2} \frac{R_L}{|Z_g + Z_L|^2} = \frac{|V_g|^2}{2\,R_L}$$

Whereas if we enforce a **"conjugate match"** $Z_g = Z_L^*$ the load instead absorbs:

$$P_{abs}\Big|_{Z_g=Z_L^*} = \frac{|V_g|^2}{2} \frac{R_L}{|Z_L^* + Z_L|^2} = \frac{1}{4}\left(\frac{|V_g|^2}{2\,R_L}\right)$$

The power absorbed by the load when $Z_g = Z_L^*$ is just **25%** of the power absorbed if $Z_g = -jX_L$!

Dazed and Confused

Q: *But if Z_L is **not** equal to Z_g^* ($Z_L \neq Z_g^*$) isn't the absorbed power **less** than the available power ??*

A: You bet!

If $Z_g = -jX_L$, the absorbed power is **far less** than the **available power**.

Q: *I'm so **confused!***

I thought you said that setting $Z_g = -jX_L$ maximized the absorbed power??

A: See the next page!

ZL cannot alter available power—
but Zg sure the heck can!

A: Here's the deal; altering the value of **load** impedance Z_L changes the delivered power P_{del} but does not alter the available power P_{avl} of the source.

The best we can do is set Z_L such that **all available power** is delivered to the load (i.e., set $Z_L = Z_g^*$).

Contrast this with altering the value of **source** impedance Z_g. Changing Z_g **will** alter the **available power** P_{avl} of the source!

Recall:

$$P_{avl} = \frac{|V_g|^2}{8\,R_g}$$

The ideal source impedance ($Z_g = -jX_L$) is purely reactive, so $R_g = 0$—the available power is therefore **infinite**!

Of course, achieving infinite available power is **not practical**—available power P_{avl} of any **realizable** source is finite.

Don't ever do this!

Still, engineers attempting to **maximize** the power **absorbed by a load** should:

1. Attempt to select/design/alter the **source** such that its **available power** P_{avl} **is maximized**.

2. Attach a **load** that is conjugate matched ($Z_L = Z_g^*$) to this source, such that **all available power is delivered** to the load.

A problem that often arises is a source with a **large available power** has a very **low source impedance**, such that it is **difficult/impractical** to provide a load where $Z_L = Z_g^*$.

Engineers sometimes alter/design/select **another source** that it **easier** to "match", but usually this results in a dramatic **decrease in available power**!

See what I mean?

For **example**, consider two cases:

Source	Available Power	Delivered Power
1	500 mW	200mW
2	100 mW	100 mW

For **which** source is "power transfer maximized"?

For source 2, **100%** of the **available power** is delivered to the load—clearly the **load is matched** to the source impedance.

For source 1, **only 40%** of the available power is delivered to the load—the load is most definitely **not matched** to source impedance.

Yet, the **mismatched load** absorbs **twice** the power of the mismatched case.

It does so because the available power of source 1 is **five times** larger than that of source 2.

→ It's better to have **most** of **alot** than **all** of very **little**!!

Be careful!

Hence, we need to be **careful** when considering a conjugate match (e.g., what does "maximum power transfer" **really** mean?).

→ Selecting/altering the **load to match a source** is a **good** idea, but selecting/altering the **source to match a load** is typically **not**.

This question has—and continues to—spark many **arguments** among electrical engineers!!