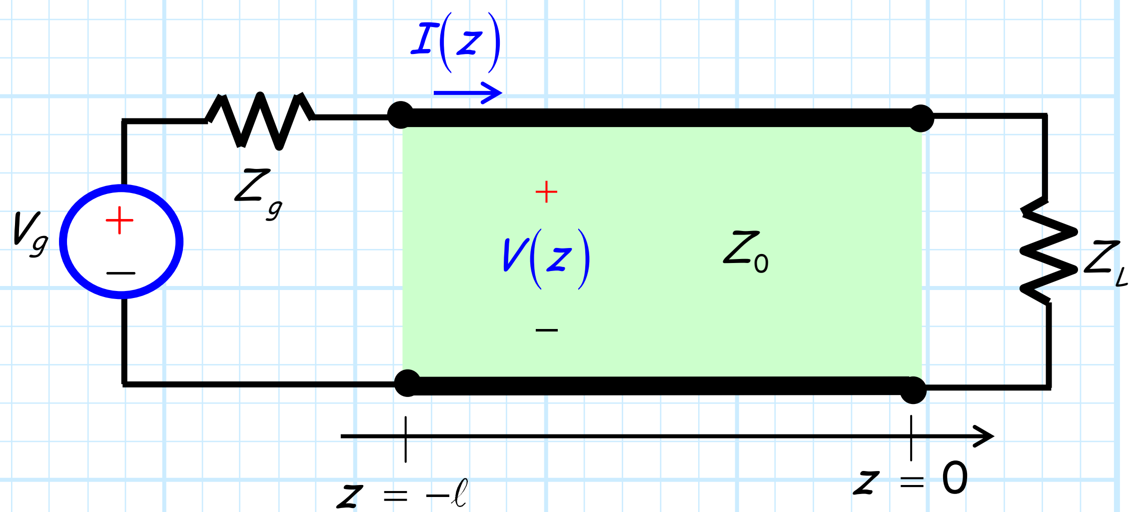
Special Cases of Source and Load Impedance

Let's look at **specific cases** of:

1. Z_g
2. and Z_L,

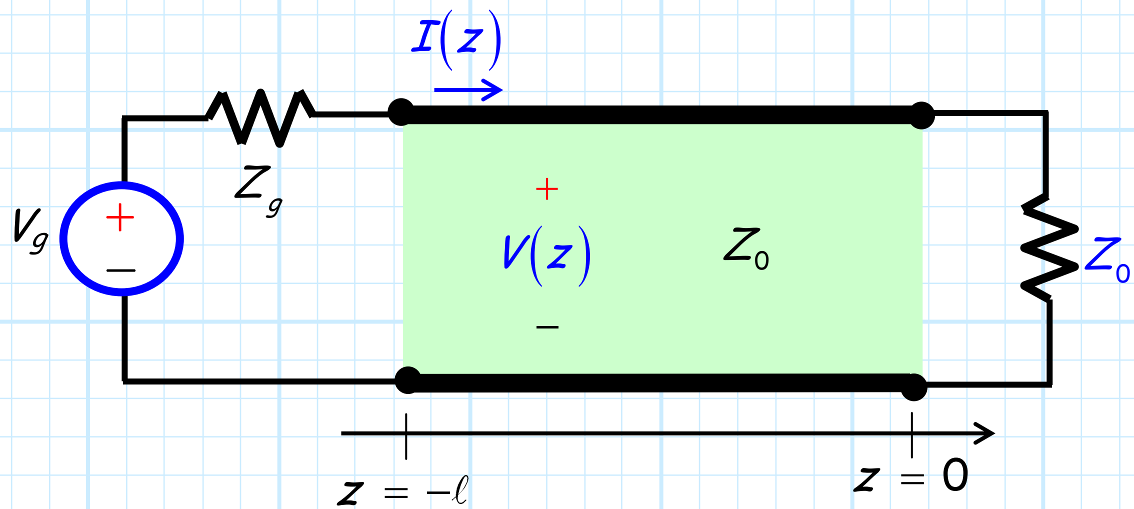


and then determine how they **affect**:

1. V_0^+
2. and P_{abs}.

The first special case: the matched load

$$Z_L = Z_0 + j0$$



Of course for this case—where the load is "**matched**" to the transmission line—we find that $\Gamma_L = 0$ and thus $\Gamma_{in} = 0$.

As a result, the complex amplitude of the plus-wave **simplifies** to:

$$V_0^+ = V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right) + Z_g\left(1-\Gamma_{in}\right)}$$

$$= V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0 + Z_g}$$

The absorbed power is just the incident power

Likewise, the absorbed/delivered power is simply that of the **incident** wave:

$$P_{abs} = \frac{|V_0^+|^2}{2\,Z_0}\left(1-|\Gamma_L|^2\right) = \frac{|V_0^+|^2}{2\,Z_0}$$

as the **matched** condition causes the reflected power to be **zero** ($P^- = 0$)!

Now, recognizing that the **input impedance** of the transmission line for this case will likewise be $Z_{in} = Z_0$, we can **alternatively** express the **absorbed power** (without determining V_0^+!) :

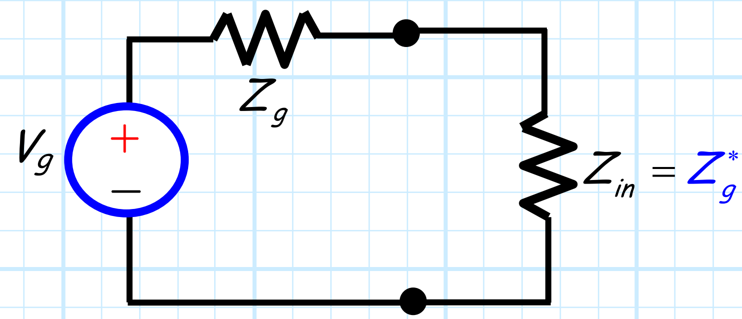
$$P_{abs} = \frac{1}{2}\frac{|V_g|^2}{|Z_g + Z_{in}|^2}\,\text{Re}\{Z_{in}\} = \frac{1}{2}\frac{|V_g|^2}{|Z_g + Z_0|^2}\,Z_0 = \frac{|V_g|^2}{2}\frac{Z_0}{|Z_g + Z_0|^2}$$

The 2nd special case: the conjugate match

$$Z_{in} = Z_g^*$$

For this case, we find the load Z_L takes on **whatever value** required to make $Z_{in} = Z_g^*$.

→ The input impedance is a **conjugate match** to the source impedance.



This is of course is a **very** important case!

First, using the fact that:

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_g^* - Z_0}{Z_g^* + Z_0}$$

We **can** show that (trust me!) that the plus-wave amplitude **simplifies** to:

$$V_0^+ = V_g\, e^{-j\beta\ell}\, \frac{Z_g^* + Z_0}{4\,\text{Re}\{Z_g\}}$$

Yawn: the load absorbs the available power

Not a particularly interesting result, but now let's look at the **absorbed power**.

$$\begin{aligned} P_{abs} &= \frac{1}{2}\frac{|V_g|^2}{|Z_g + Z_{in}|^2}\text{Re}\{Z_{in}\} \\ &= \frac{1}{2}\frac{|V_g|^2}{|Z_g + Z_g^*|^2}\text{Re}\{Z_g^*\} \\ &= \frac{1}{2}\frac{|V_g|^2}{|2\text{Re}\{Z_g^*\}|^2}\text{Re}\{Z_g^*\} \\ &= \frac{1}{2}|V_g|^2\frac{1}{4\text{Re}\{Z_g^*\}} \\ &= \frac{|V_g|^2}{8\,R_g} = P_{avl} \end{aligned}$$

The absorbed power is **of course** the **available power** of the source.

Since the input impedance is a **conjugate match** to the source, the source is delivering energy at the **highest rate** it possibly **can**!

But, the load still reflects power!

Q: *But if the power **delivered** is at a maximum, so too is the power **absorbed** by the load. I.E.:*

$$P_{abs} = P_{del} = P_{avl}$$

*If the power absorbed from the load is at a **maximum**, then the power **reflected** from the load must be at a **minimum**.*

*The load must be absorbing **all** the incident power; **right**?*

A: You might think so—and many engineers in fact **do** think so.

→ But those engineers are **incorrect**!

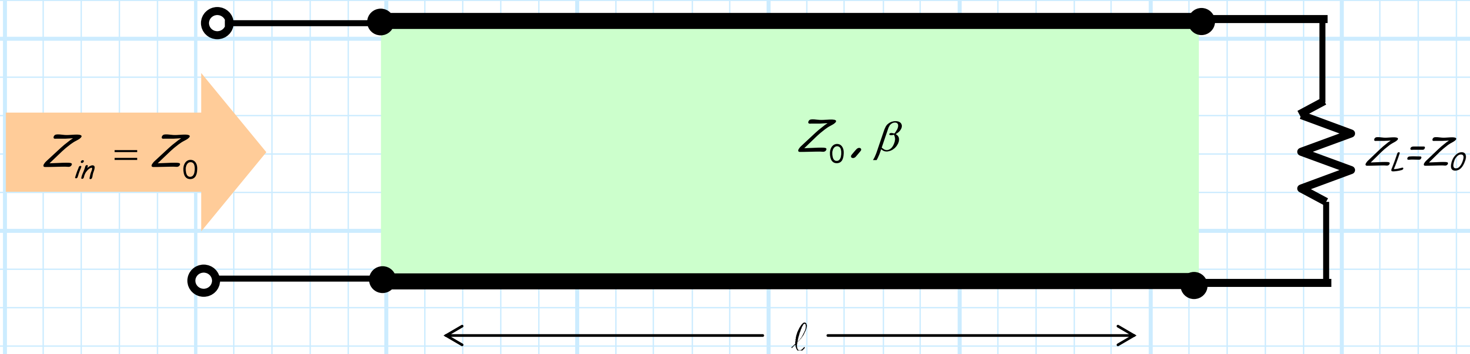
To see why, consider the load Z_L that **minimizes** the reflected power.

We know this load **must** have one specific value—the **matched** load $Z_L = Z_0$.

If the load is Z_0 then so is Z_{in}

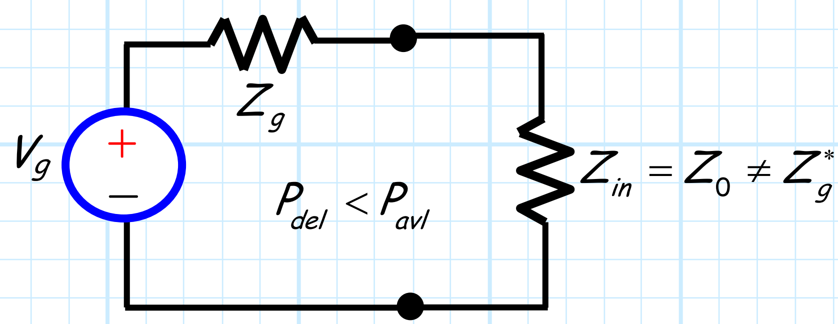
But, **if** the load is matched to the transmission line (i.e., $Z_L = Z_0$, then the **input** impedance will likewise be equal to Z_0:

$$Z_{in} = Z_0$$



And, **generally** speaking, an input impedance of $Z_{in} = Z_0$ will **not** be equal to the complex conjugate of the source impedance:

$$Z_0 \neq Z_g^* \quad !!!!$$



Look closer at this result

Recall that we **just** determined that for $Z_L = Z_0$, the absorbed power is:

$$P_{abs} = \frac{|V_g|^2}{2} \frac{Z_0}{|Z_0 + Z_g|^2}$$

It can be shown that this value is **less** than the available power of the source:

$$\frac{|V_g|^2}{2} \frac{Z_0}{|Z_0 + Z_g|^2} \leq \frac{|V_g|^2}{8\, R_g} = P_{avl}$$

Q: *Huh!? This makes **no** sense!*

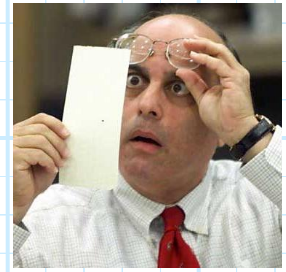
*After all, just **look** at the expression for absorbed power:*

$$P_{abs} = \frac{|V_0^+|^2}{2\, Z_0}\left(1 - |\Gamma_L|^2\right)$$

***Clearly**, this value is maximized when $\Gamma_L = 0$ (i.e., when $Z_L = Z_0$)!!!*

Remember, the load affects V^+

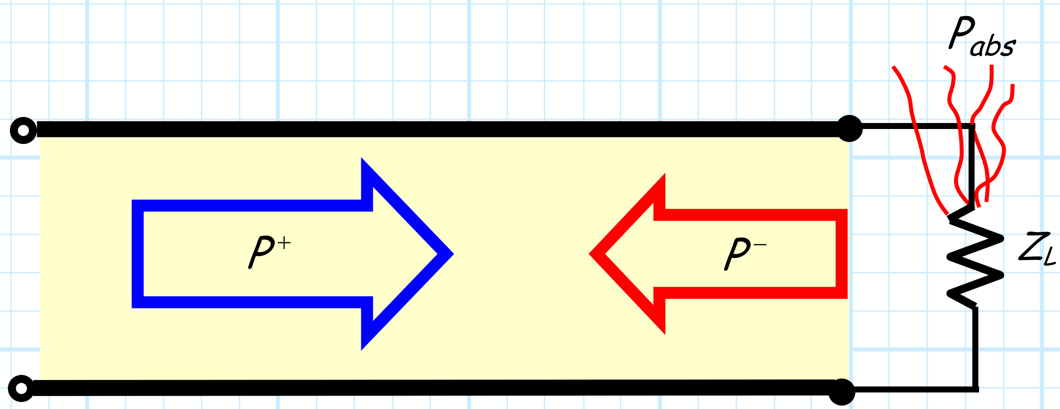
A: Let's **look closer** at this result:



$$P_{abs} = \frac{|V_0^+|^2}{2\,Z_0}\left(1-|\Gamma_L|^2\right)$$

Remember, we can rewrite this to show that the **absorbed** power is simply the **difference** between the power of the **incident** and **reflected** waves:

$$P_{abs} = \frac{|V_0^+|^2}{2\,Z_0}\left(1-|\Gamma_L|^2\right) = \frac{|V_0^+|^2}{2\,Z_0} - \frac{|V_0^+\Gamma_L|^2}{2\,Z_0} = \frac{|V_0^+|^2}{2\,Z_0} - \frac{|V_0^-|^2}{2\,Z_0} = P^+ - P^-$$



Absorbed power is maximized if the *difference* between incident and reflected is maximized!

Of course, it **is** true that the load impedance Z_L affects the **minus-wave** amplitude V_0^-, and thus affects likewise the **reflected** wave power P^-.

Remember however, that the value of Z_L **likewise** affects the **plus**-wave amplitude V_0^+, and thus affects likewise the **incident** wave power P^-!

* Thus, the value of Z_L that minimizes P^- will **not** generally maximize P^+!

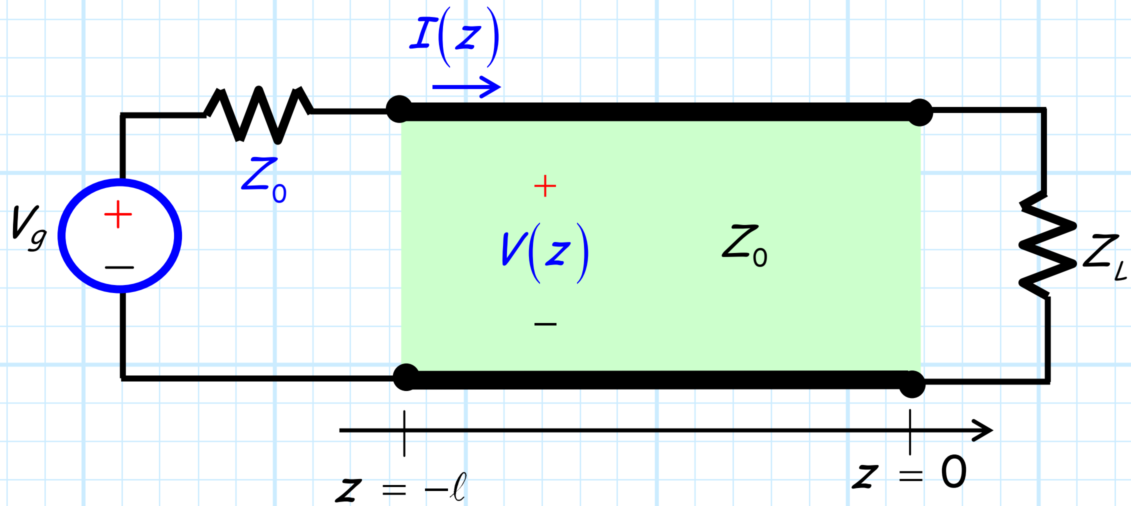
* **Likewise** the value of Z_L that maximizes P^+ will **not** generally minimize P^-.

* Instead, the value of Z_L that maximizes the **absorbed** power P_{abs} is, by definition, the value that **maximizes** the **difference** $P^+ - P^-$.

We find that this **ideal** impedance Z_L is the value that results in the **ideal** case of $Z_{in} = Z_g^*$!

The third special case: the matched source

$$Z_g = Z_0 + j0$$



For this case, we find that V_0^+ **simplifies** greatly:

$$\begin{aligned} V_0^+ &= V_g \, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_g\left(1-\Gamma_{in}\right)} \\ &= V_g \, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right)+Z_0\left(1-\Gamma_{in}\right)} \\ &= V_g \, e^{-j\beta\ell} \frac{1}{1+\Gamma_{in}+1-\Gamma_{in}} \\ &= \frac{1}{2} V_g \, e^{-j\beta\ell} \end{aligned}$$

Look at what **this** says!

This is the answer—regardless of the load!

It says that the incident wave in this case is **independent** of the load attached at the other end (the value Γ_{in} is nowhere to be found)!

Thus, for the **one** case $Z_g = Z_0$, we in fact **can** consider $V^+(z)$ as being the **source wave**.

And then the reflected wave $V^-(z)$ is the **causal result** of this stimulus.

Specifically, for this case we can write **directly** the plus-wave—without knowing **anything** about the load Z_L:

$$V^+(z) = V_0^+ e^{-j\beta z} = \left(\frac{1}{2} V_g \, e^{-j\beta\ell}\right) e^{-j\beta z} = \frac{V_g}{2} \, e^{-j\beta(z+\ell)}$$

This blue box equation will come in quite handy!

Therefore, the value at the **load** (i.e., $z = 0$) is:

$$V^{+}(z=0) = \frac{V_g}{2}\, e^{-j\beta\ell}$$

And the value of the **plus**-wave at the **source** (i.e., $z = -\ell$) is:

$$\boxed{V^{+}(z=-\ell) = \frac{V_g}{2}}$$

This last result is **very** important.

Remember this result, it will come in **quite handy** later in the course!

All I can say is: wow!

Now consider another **really** important result.

Recall the power associated with the plus-wave is:

$$P^+ = \frac{|V_0^+|^2}{2\,Z_0}$$

Inserting the **simplified** expression for V_0^+ (i.e., when $Z_g = Z_0$):

$$P^+ = \frac{|V_0^+|^2}{2\,Z_0} = \frac{|V_g\, e^{-j\beta\ell}|^2}{8\,Z_0} = \frac{|V_g|^2}{8\,Z_0} \quad \leftarrow \text{Wow!}$$

Q: *I don't see **why** this is "wow". Am I **missing** something?*

A: Apparently you are. Look **closer** at the above result.

Do I have to explain everything?

Remember, this result is for the **special case** where the source impedance is a **real value**, numerically equal to Z_0:

$$Z_g = Z_0 + j0$$

Therefore:

$$R_g = \text{Re}\{Z_g\} = Z_0$$

The **source** resistance is **numerically equal** to transmission line characteristic impedance Z_0.

Thus, the **power** of the plus-wave can be alternatively written as:

$$P^+ = \frac{|V_g|^2}{8\,Z_0} = \frac{|V_g|^2}{8\,R_g} \qquad \leftarrow \text{Wow!}$$

Q: *This result looks vaguely **familiar**; haven't we seen this **before**?*

A: Sigh. This result is the **available power of the source**!!!!!!!

The incident power is the available power (wow)!

Therefore:

$$P^{+} = \frac{|V_g|^2}{8\,Z_0} = \frac{|V_g|^2}{8\,R_g} = P_{avl} \qquad \leftarrow \text{Wow!}$$

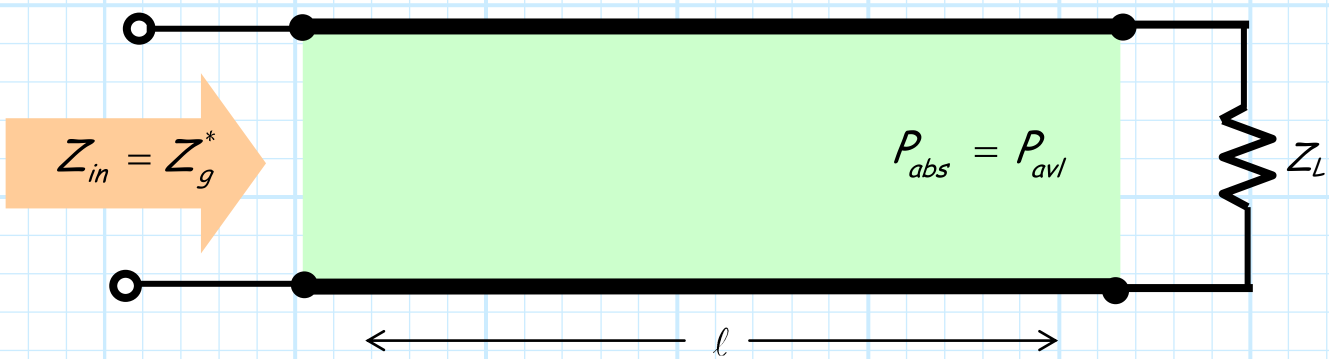
For the special case $Z_g = Z_0 + j0$, the power P^{+} associated with the plus-wave (i.e., the **incident** wave) is **equal** to the **available** power P_{avl} of the **source**.

Q: ***Wow!*** *Does this mean that the power* ***absorbed*** *by the load is* ***equal*** *to the* ***available*** *power P_{avl} of the source?*

A: **Absolutely not!**

But some of this incident power is reflected (Doh!)

Remember, **only** if $Z_{in} = Z_g^*$ does the power absorbed by the load **equal** the **available** power.



* Making $Z_g = Z_0 + j0$ causes the power of the **incident** wave to be equal to the **available** power.

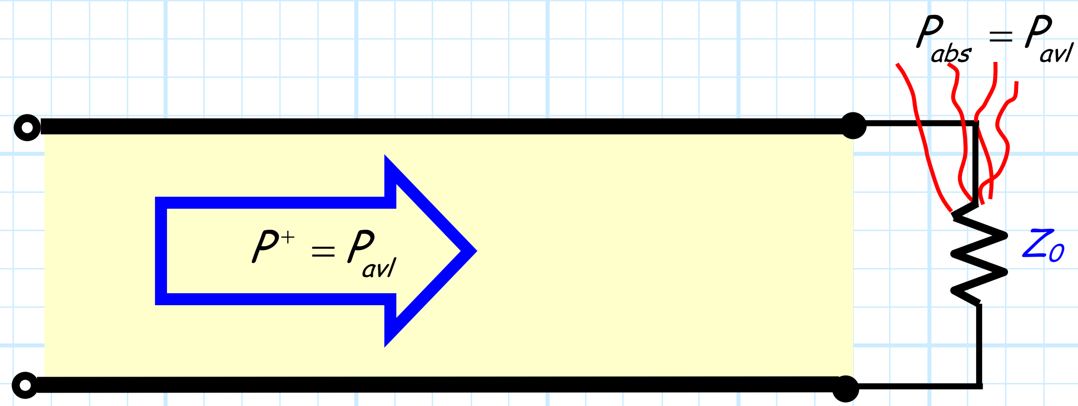
* But, the value Z_L of the **load impedance** determines how much of that incident power is absorbed—and how much gets **reflected**.

The *really* special case!

Q: *Hey, that's right!*

If *the load is matched (i.e., $Z_L = Z_0 + j0$), then wouldn't **none** of the incident power be **reflected** (i.e., $P^- = 0$).*

*And so wouldn't **all** the incident (i.e., available!) power be **absorbed** by the load?*



A: That's **exactly correct**!

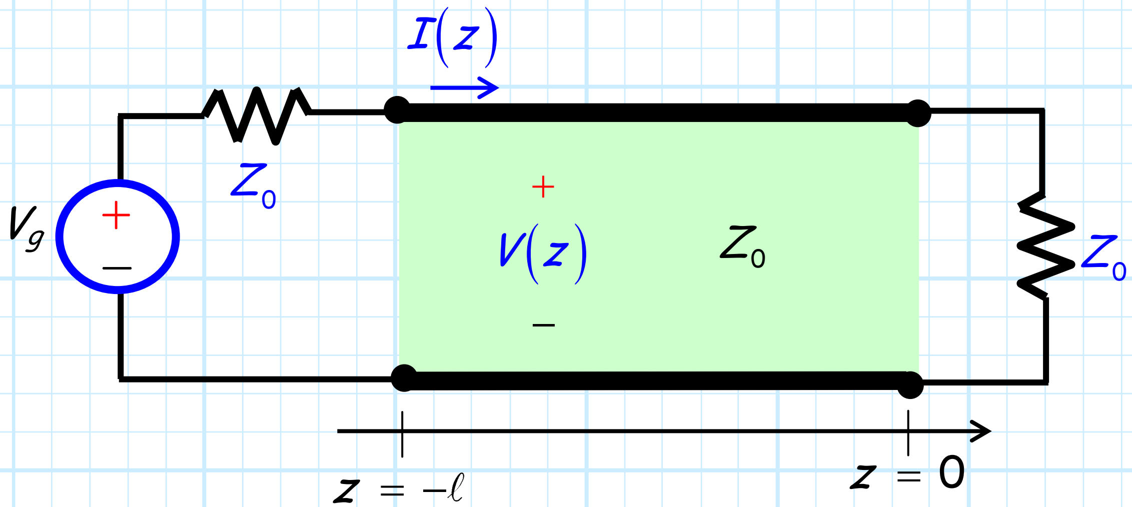
Q: *But you said earlier that **minimizing** the **reflected** power would **not** result in **maximizing** the **absorbed** power. Aren't you **contradicting** yourself?*

A: Not at all.

A conjugate match!

Generally speaking, minimizing the **reflected** power (i.e., $Z_L = Z_0$) will **not** result in maximizing the **absorbed** power.

However, we **are** considering the **special case** where also $Z_g = Z_0$!



Thus, maximum power transfer will occur **if** there is a conjugate match $Z_{in} = Z_g^*$.

Note for this **special** case $Z_g^* = Z_0$, meaning that **input impedance** Z_{in} must be equal to Z_0 for **maximum** power delivery.

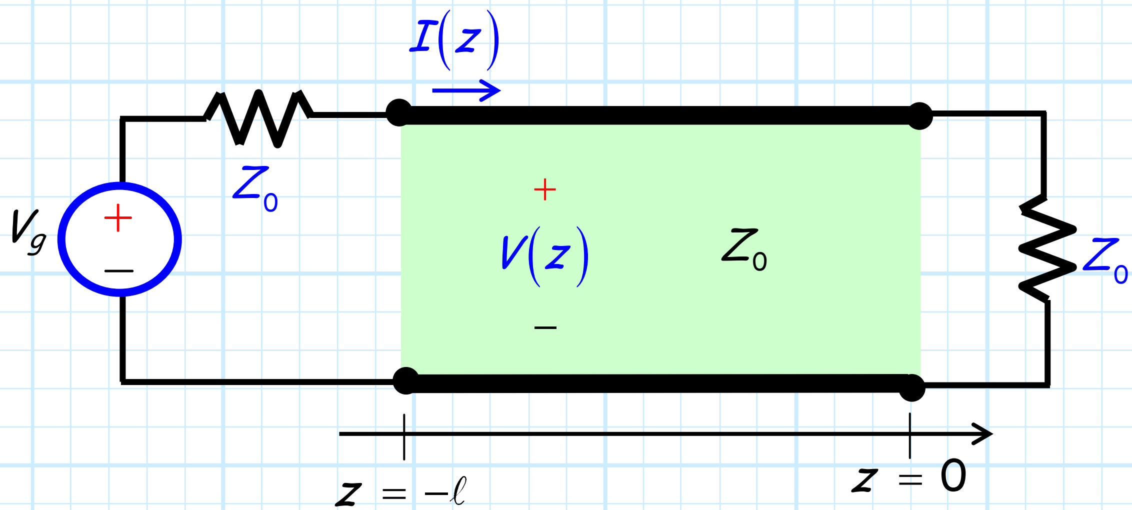
Of course, the input impedance Z_{in} **will** be equal to Z_0 when the **load impedance is matched** (i.e., $Z_L = Z_0$)!

"Zee-zeros" everywhere

Thus, in many ways, the case:

$$Z_g = Z_0 = Z_L$$

(i.e., **both** source and load impedances are numerically equal to Z_0) is **ideal**.



Let's summarize this "ideal case":

* A **conjugate match** occurs ($Z_{in} = Z_g^*$), and so **all** the **available power** from the source is **absorbed** by the load:

$$P_{del} = P_{avl} = P_{abs}$$

It's just *so* simple; you hardly deserve credit for this class

* The **incident** (plus) wave is **independent** of the **load** impedance:

$$V^+(z) = \frac{V_g}{2}\, e^{-j\beta(z+\ell)} \qquad V^+(z=0) = \frac{V_g}{2}\, e^{-j\beta\ell} \qquad V^+(z=-\ell) = \frac{V_g}{2}$$

* And the **power** associated with this **incident** wave is **equal** to the **available power** of the source:

$$P^+ = \frac{\left|V_g\right|^2}{8\, Z_0} = P_{avl}$$

* The **reflected** (minus) wave is **zero**:

$$V^-(z) = 0$$

* **So** of course, the **power** associated with the reflected wave is likewise **zero**.

$$P^- = 0$$

Load AND source is matched—it's a very good thing

* This **again** allows us to verify that **all available power** is absorbed by the **load**:

$$P_{abs} = P^{+} - P^{-} = P_{avl} - 0 = P_{avl}$$

* Finally, the **total** current and voltage simplifies **nicely**:

$$V(z) = V^{+}(z) + V^{-}(z) = \frac{V_g}{2}\, e^{-j\beta(z+\ell)} \qquad I(z) = \frac{V^{+}(z) - V^{-}(z)}{Z_0} = \frac{V_g}{2Z_0}\, e^{-j\beta(z+\ell)}$$