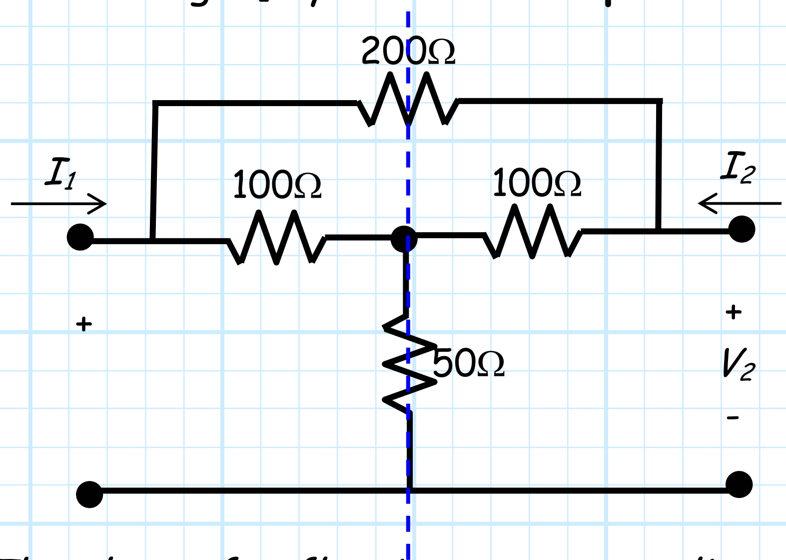
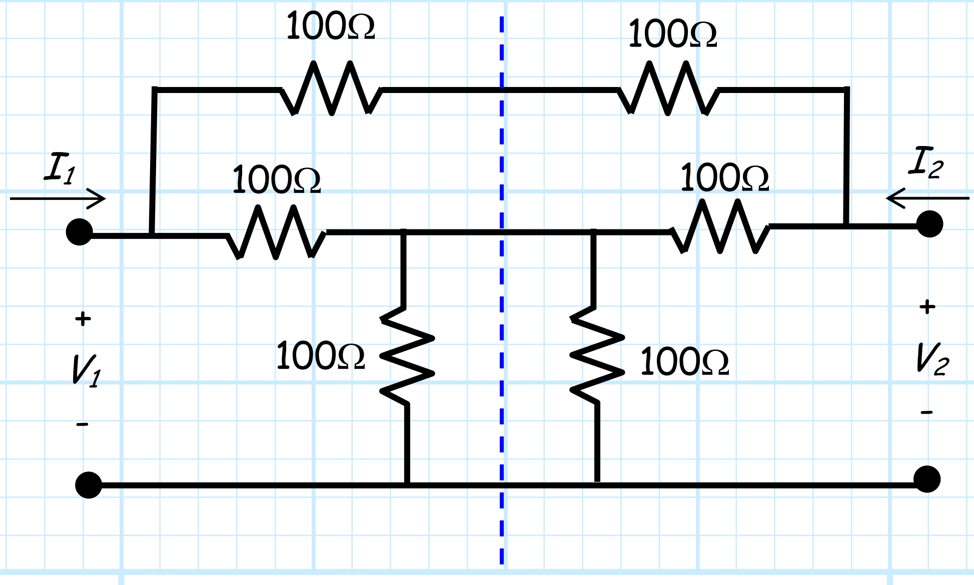
Symmetric Circuit Analysis

Consider the following D_1 symmetric **two-port** device:



Q: *Yikes! The plane of reflection symmetry slices through two resistors. What can we do about that?*

A: Resistors are easily split into two equal pieces: the 200Ω resistor into two 100Ω resistors in **series**, and the 50Ω resistor as two 100 Ω resistors in **parallel**.



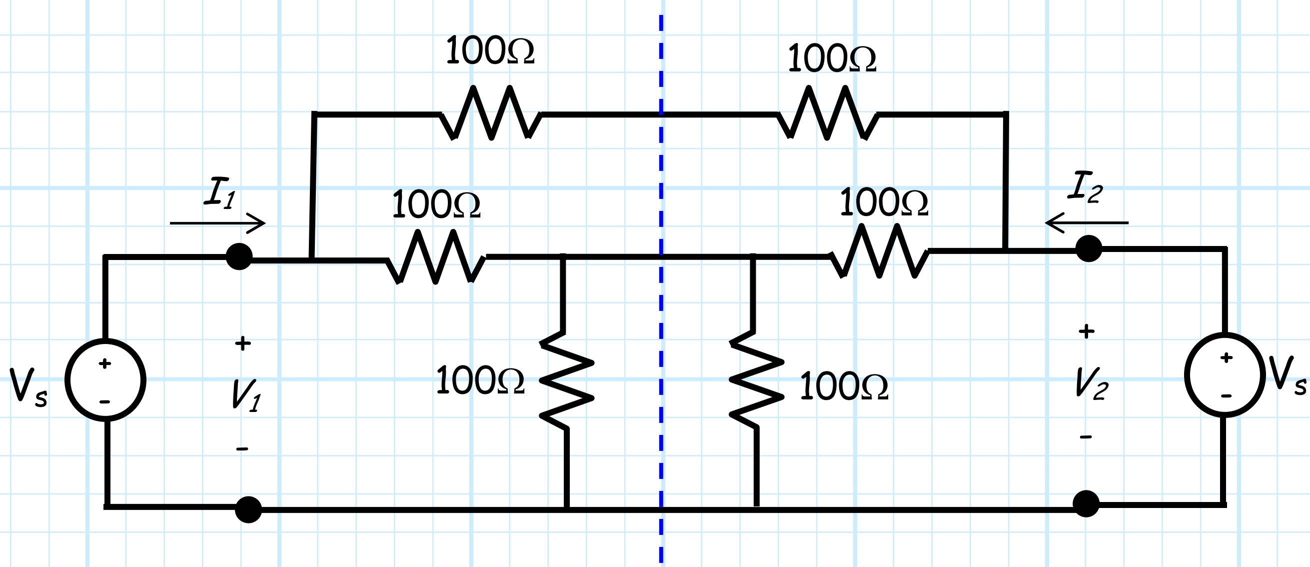
Recall that the **symmetry** of this 2-port device leads to **simplified** network matrices:

$$\mathcal{S}=\begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix} \qquad \mathcal{Z}=\begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix} \qquad \mathcal{Y}=\begin{bmatrix} Y_{11} & Y_{21} \\ Y_{21} & Y_{11} \end{bmatrix}$$

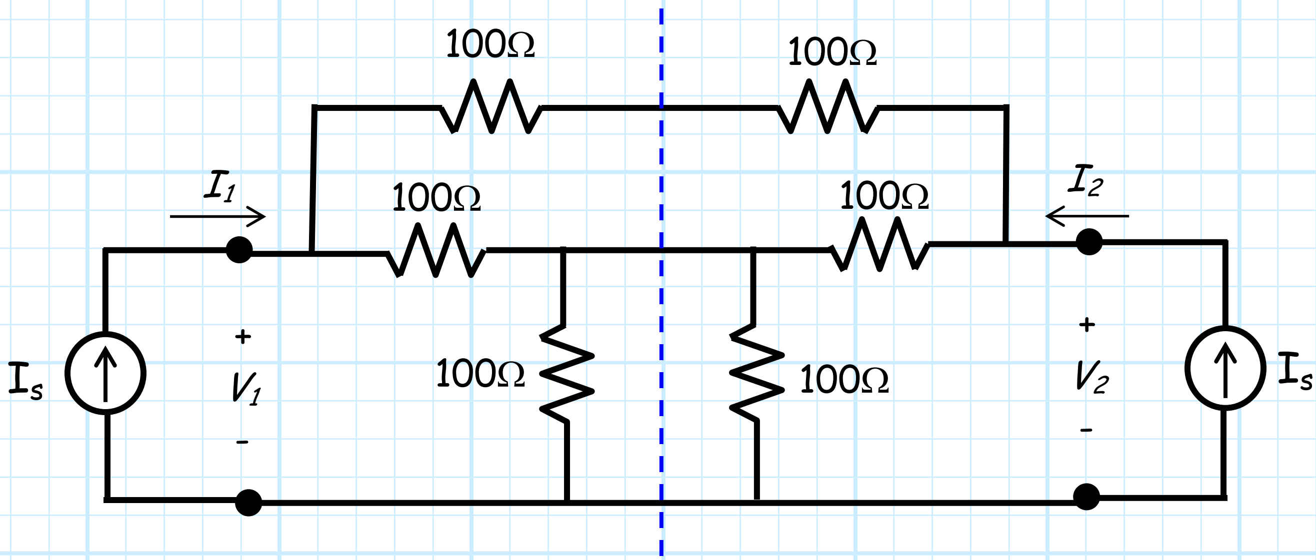
Q: *Yes, but can circuit symmetry likewise simplify the procedure of **determining** these elements? In other words, can symmetry be used to **simplify circuit analysis**?*

A: You bet!

First, consider the case where we **attach sources** to circuit in a way that **preserves** the circuit **symmetry**:



Or,



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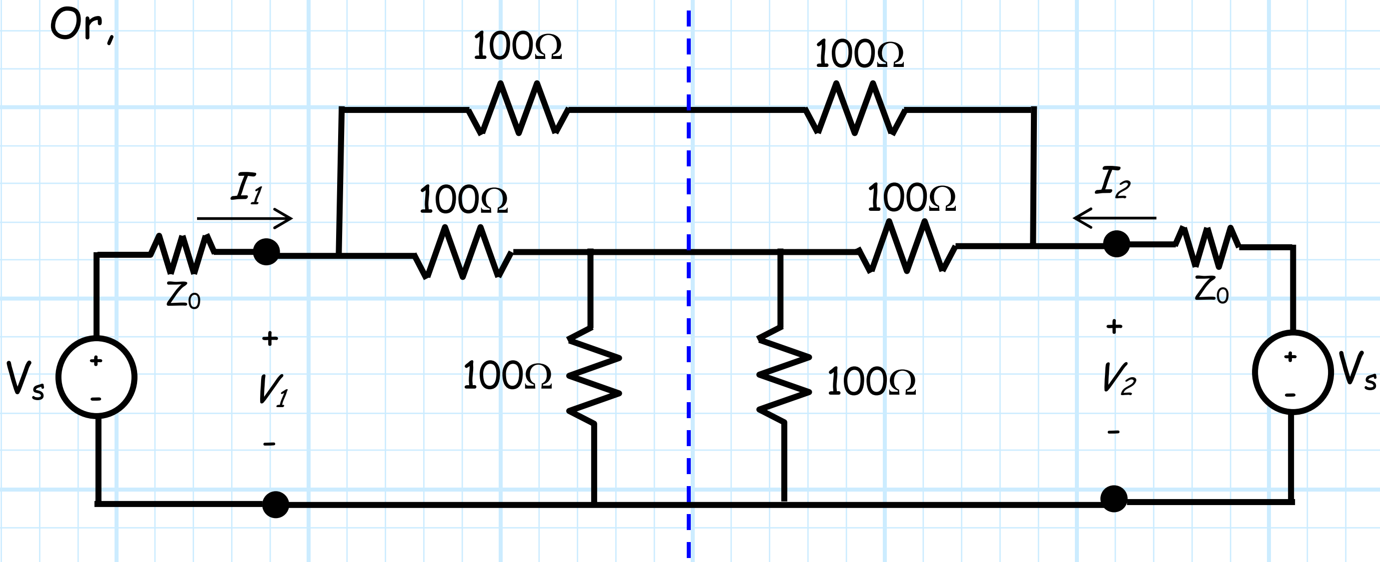
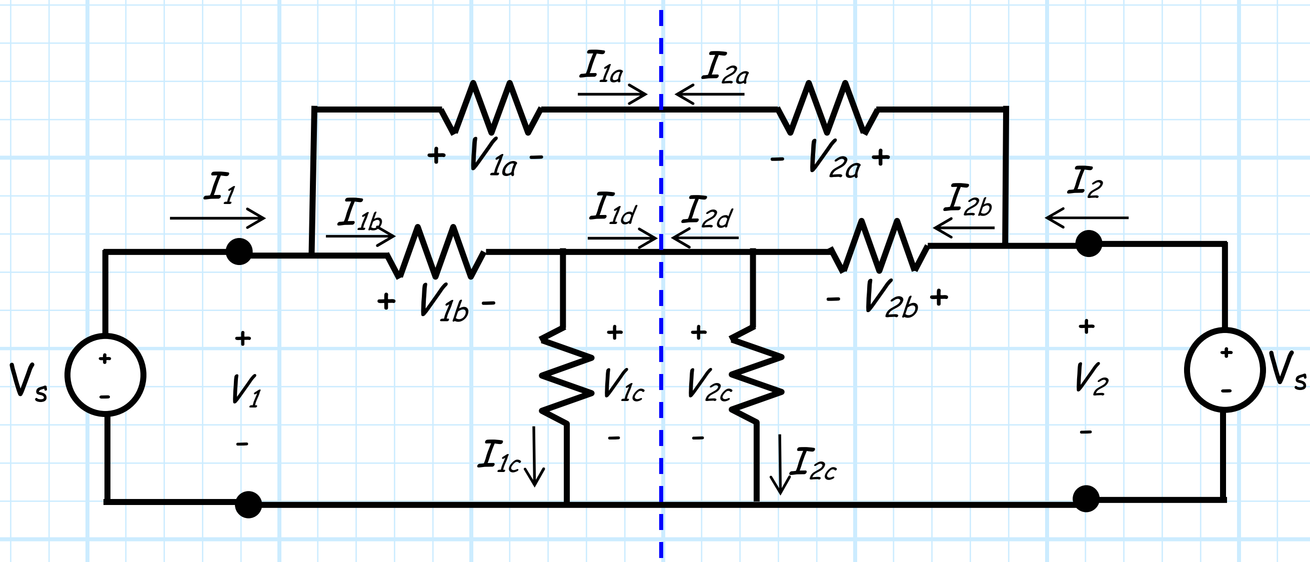


But remember! In order for **symmetry to be preserved**, the source values on both sides (i.e, I_s, V_s, Z_0) must be **identical**!

Now, consider the **voltages** and **currents** within this circuit under this symmetric configuration:



Since this circuit possesses **bilateral** (reflection) symmetry $(1 \rightarrow 2, 2 \rightarrow 1)$, symmetric currents and voltages must be equal:

$$V_1 = V_2 \qquad I_1 = I_2$$
$$V_{1a} = V_{2a} \qquad I_{1a} = I_{2a}$$
$$V_{1b} = V_{2b} \qquad I_{1b} = I_{2b}$$
$$V_{1c} = V_{2c} \qquad I_{1c} = I_{2c}$$
$$I_{1d} = I_{2d}$$

Q: ***Wait!*** *This **can't** possibly be correct! Look at currents I_{1a} and I_{2a}, as well as currents I_{1d} and I_{2d}. From KCL, **this** must be true:*

$$I_{1a} = -I_{2a} \qquad I_{1d} = -I_{2d}$$

*Yet **you** say that **this** must be true:*

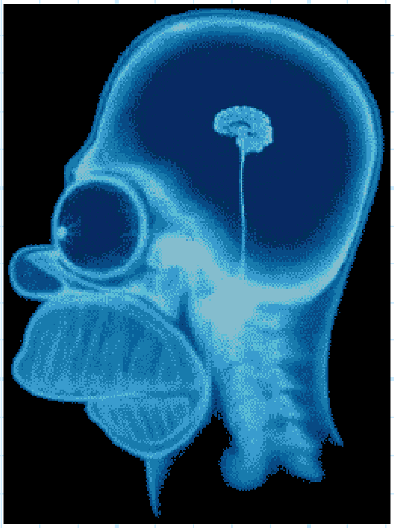
$$I_{1a} = I_{2a} \qquad I_{1d} = I_{2d}$$

*There is an **obvious contradiction** here! There is **no way** that both sets of equations can simultaneously be correct, **is there**?*

A: Actually there **is**! There is **one** solution that will satisfy **both** sets of equations:

$$I_{1a} = I_{2a} = 0 \qquad\qquad I_{1d} = I_{2d} = 0$$

The currents are **zero**!

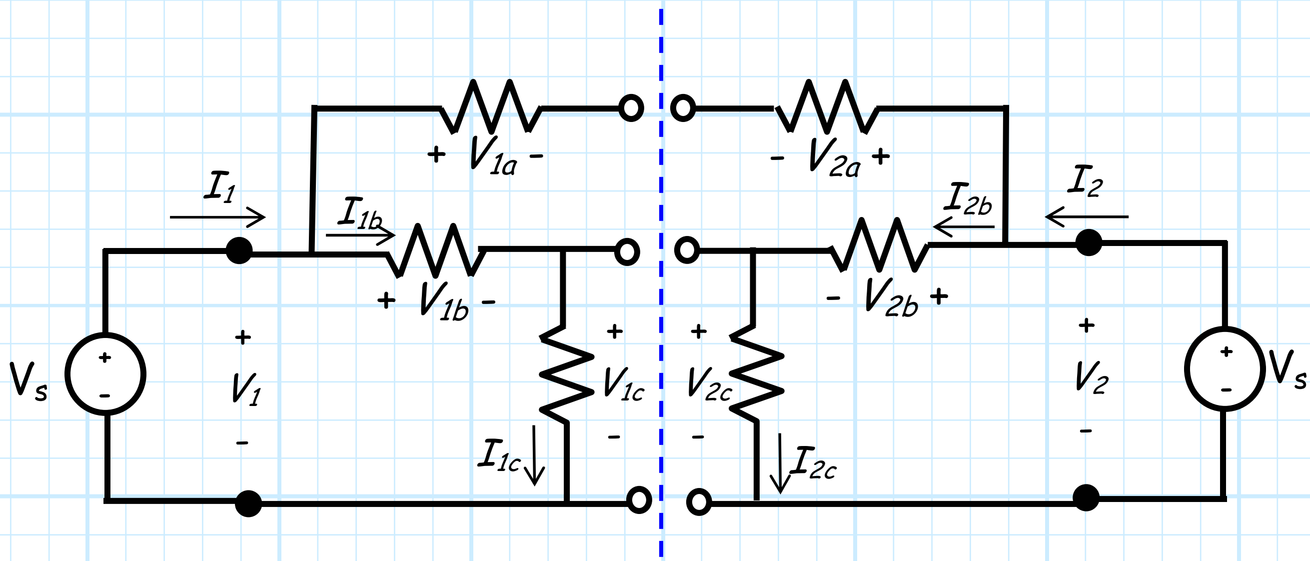


If you **think** about it, this makes **perfect sense**! The result says that **no current** will flow from one side of the symmetric circuit into the other.

If current **did** flow across the symmetry plane, then the circuit symmetry would be **destroyed**—one side would effectively become the "**source side**", and the other the "**load side**" (i.e., the source side delivers current to the load side).

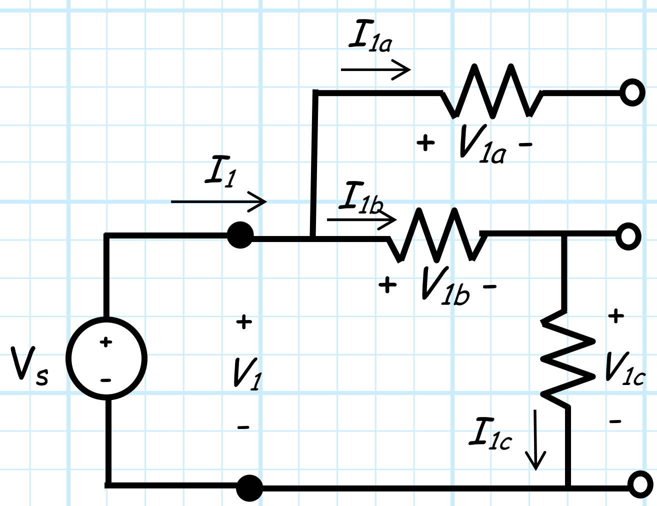
Thus, **no current** will flow **across** the reflection symmetry plane of a **symmetric circuit**—the symmetry plane thus acts as a **open circuit**!

The plane of symmetry thus becomes a **virtual open**!



Q: *So what?*

A: So what! This means that our circuit can be **split apart** into **two separate** but **identical** circuits. Solve **one** half-circuit, and you have **solved** the other!



$$V_1 = V_2 = V_s$$

$$V_{1a} = V_{2a} = 0$$

$$V_{1b} = V_{2b} = V_s/2$$

$$V_{1c} = V_{2c} = V_s/2$$

$$I_1 = I_2 = V_s/200$$

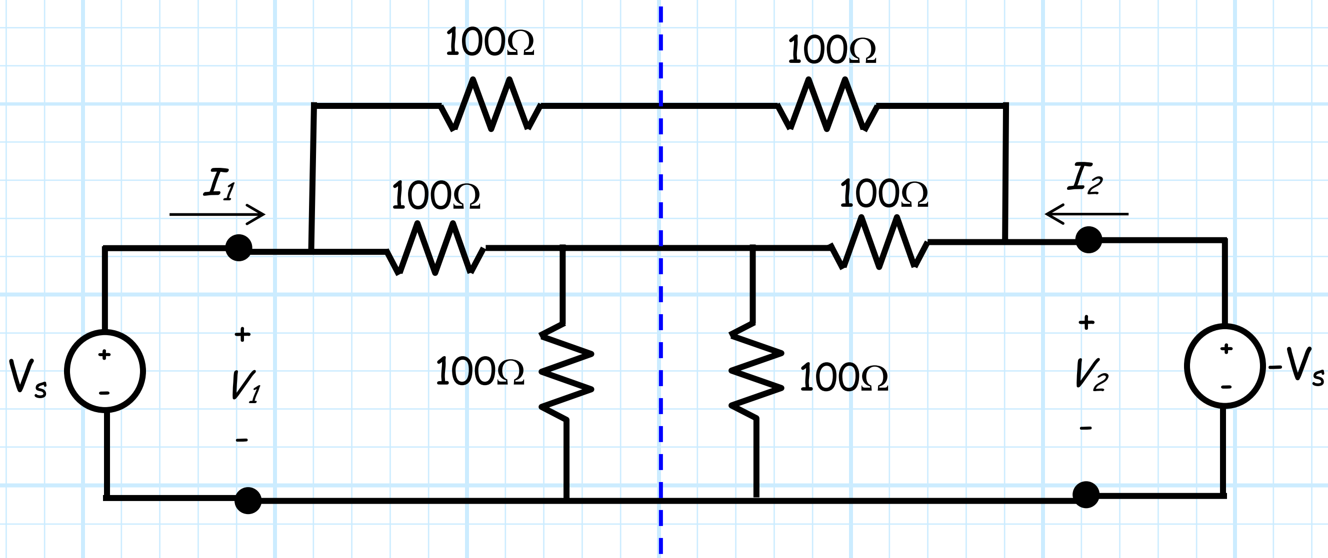
$$I_{1a} = I_{2a} = 0$$

$$I_{1b} = I_{2b} = V_s/200$$

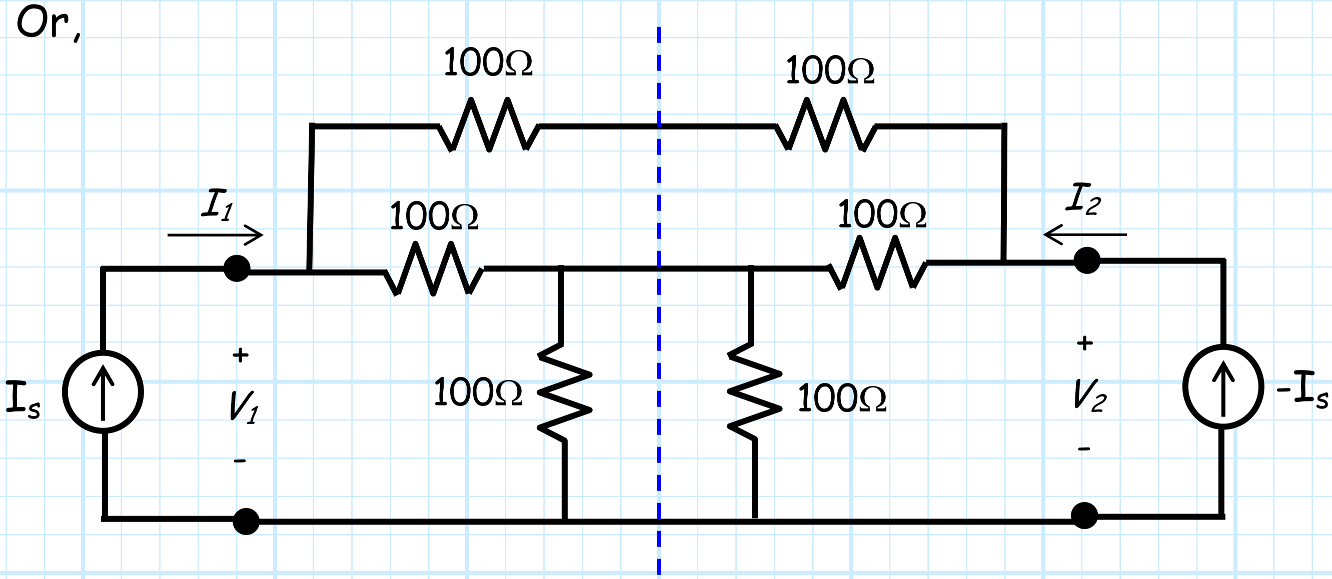
$$I_{1c} = I_{2c} = V_s/200$$

$$I_{1d} = I_{2d} = 0$$

Now, consider **another** type of symmetry, where the sources are **equal** but **opposite** (i.e., **180 degrees** out of phase).



Or,



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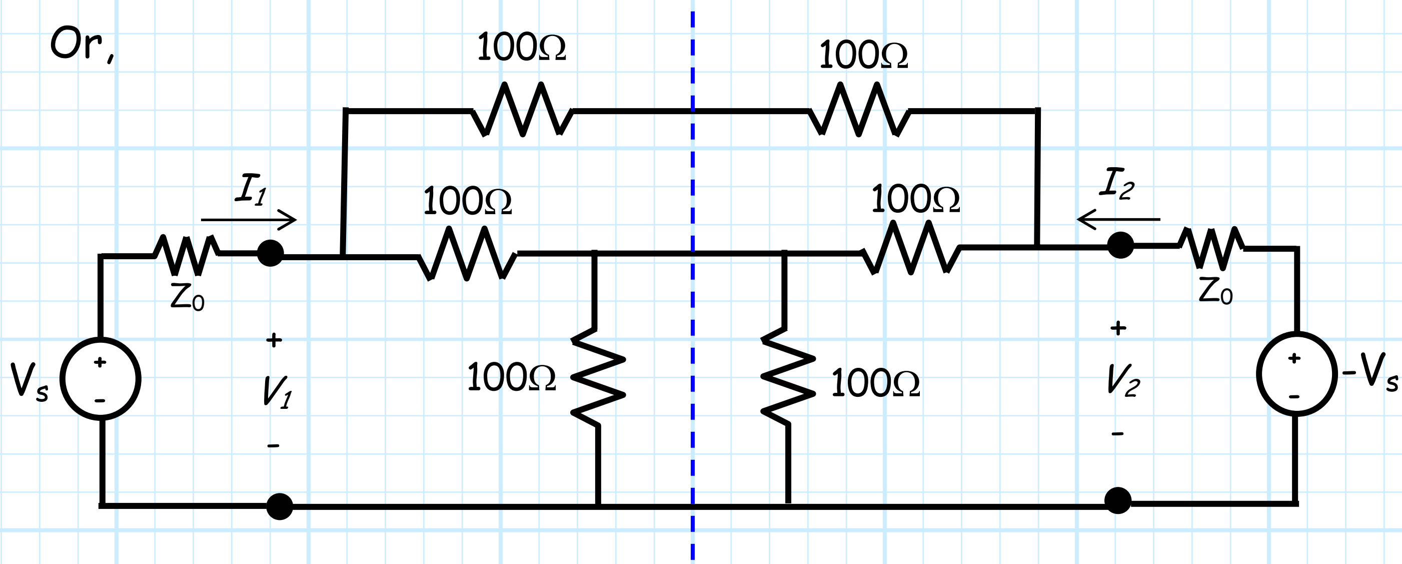
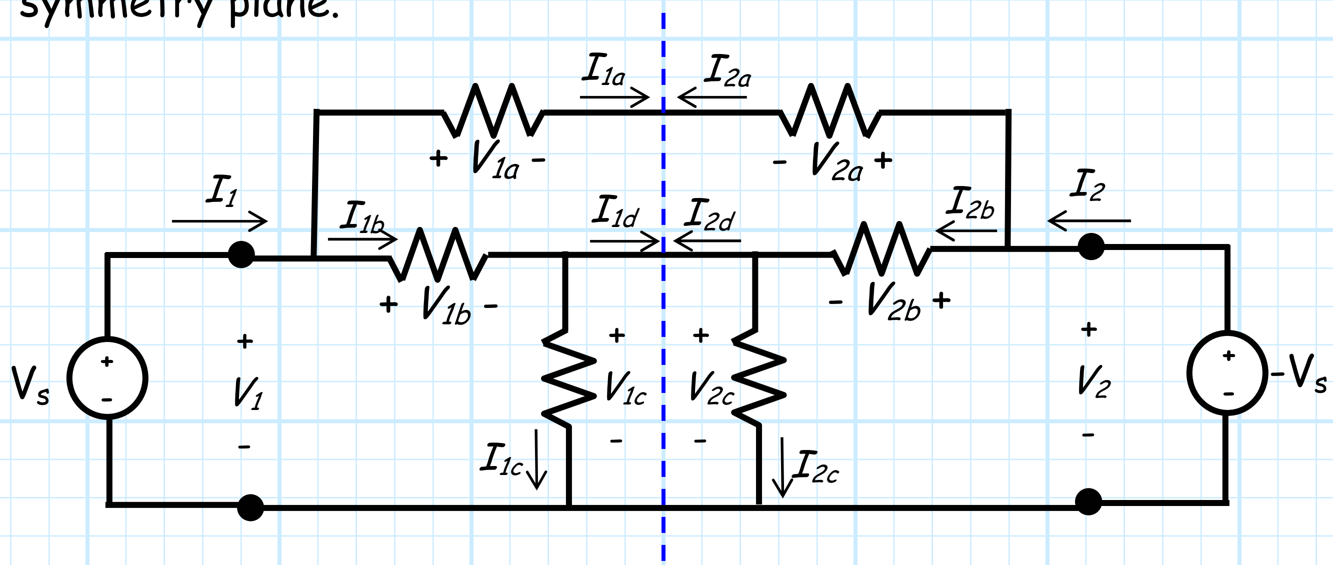


This situation still preserves the **symmetry** of the circuit—**somewhat.** The **voltages** and **currents** in the circuit will now posses **odd symmetry**—they will be **equal but opposite** (180 degrees out of phase) at symmetric points across the symmetry plane.



$$V_1 = -V_2$$

$$V_{1a} = -V_{2a}$$

$$V_{1b} = -V_{2b}$$

$$V_{1c} = -V_{2c}$$

$$I_1 = -I_2$$

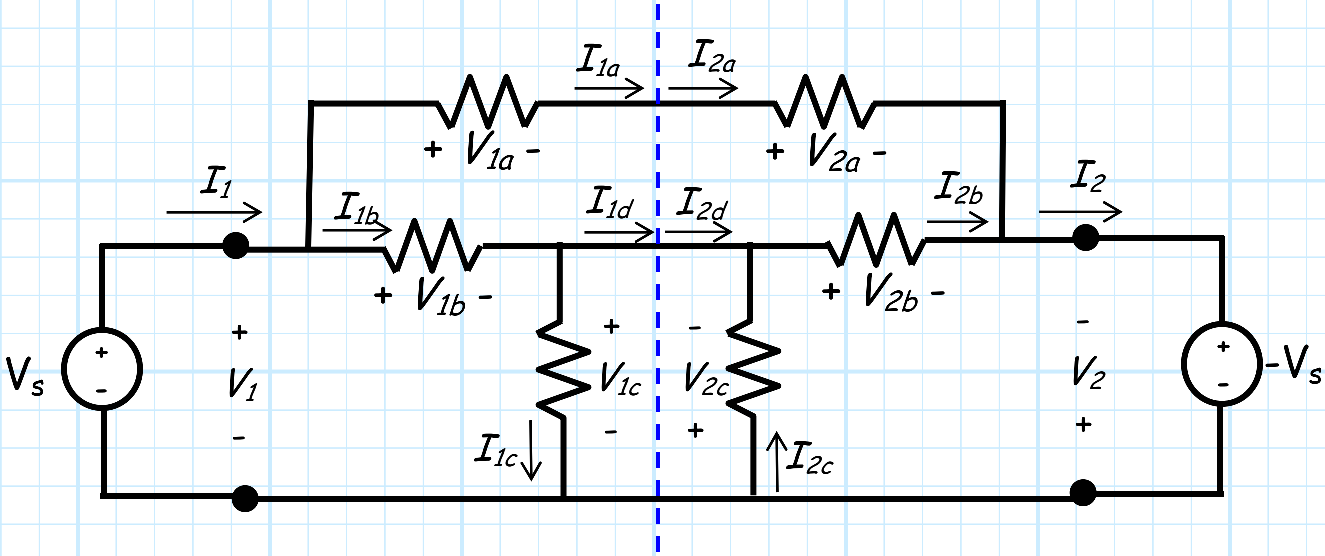
$$I_{1a} = -I_{2a}$$

$$I_{1b} = -I_{2b}$$

$$I_{1c} = -I_{2c}$$

$$I_{1d} = -I_{2d}$$

Perhaps it would be easier to **redefine** the circuit variables as:



$$V_1 = V_2$$
$$V_{1a} = V_{2a}$$
$$V_{1b} = V_{2b}$$
$$V_{1c} = V_{2c}$$

$$I_1 = I_2$$
$$I_{1a} = I_{2a}$$
$$I_{1b} = I_{2b}$$
$$I_{1c} = I_{2c}$$
$$I_{1d} = I_{2d}$$

Q: *But wait! **Again** I see a problem. By **KVL** it is evident that:*

$$V_{1c} = -V_{2c}$$

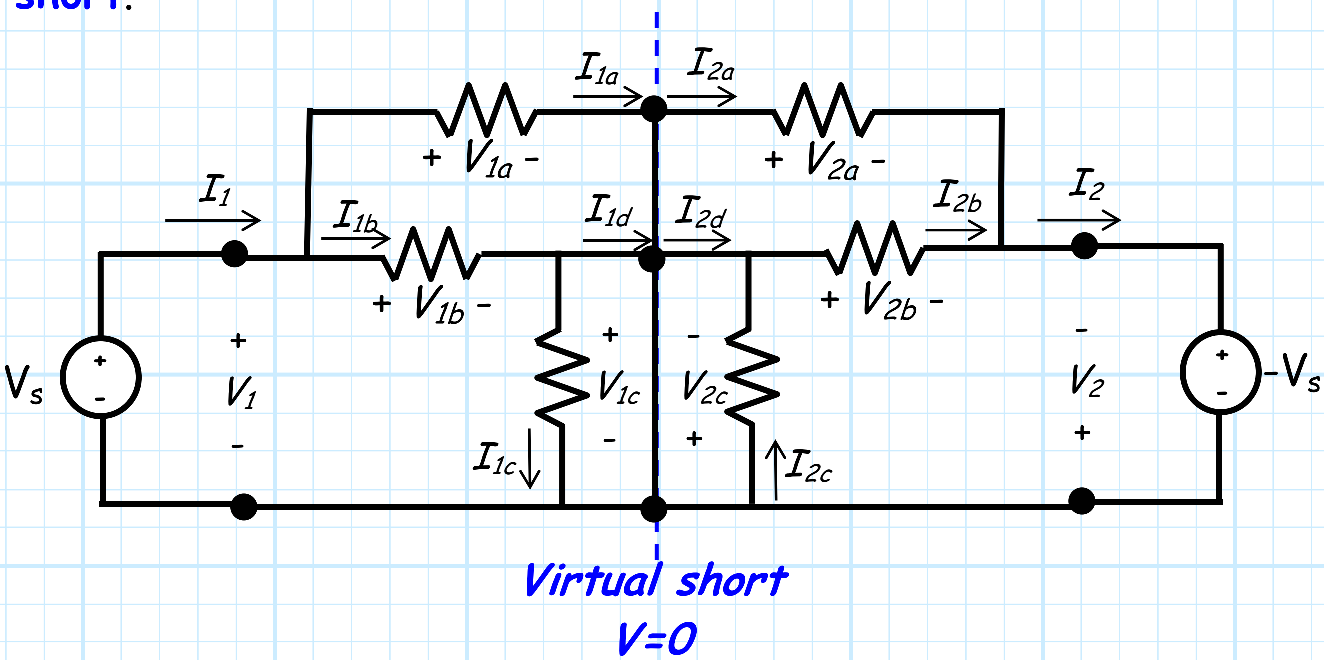
*Yet **you** say that $V_{1c} = V_{2c}$ must be true!*

A: Again, the solution to **both** equations is **zero**!

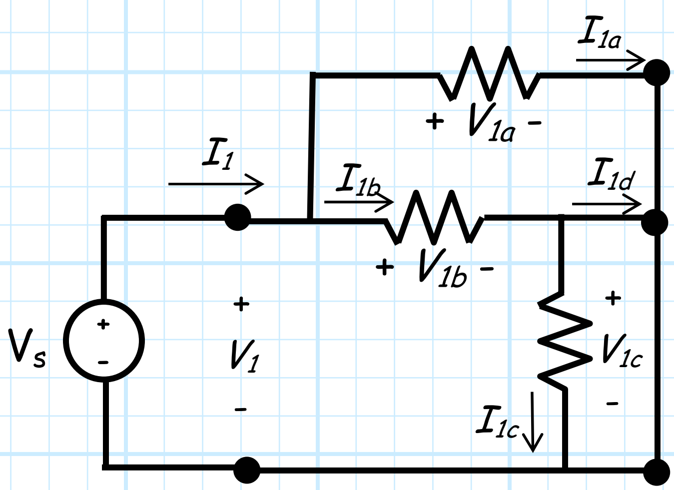
$$V_{1c} = V_{2c} = 0$$

For the case of **odd symmetry**, the symmetric plane must be a plane of **constant potential** (i.e., constant voltage)—just like a **short circuit**!

Thus, for odd symmetry, the symmetric plane forms a **virtual short**.



This **greatly** simplifies things, as we can again **break** the circuit into **two** independent and (effectively) identical circuits!



$$V_1 = V_s$$
$$V_{1a} = V_s$$
$$V_{1b} = V_s$$
$$V_{1c} = 0$$

$$I_1 = V_s/50$$
$$I_{1a} = V_s/100$$
$$I_{1b} = V_s/100$$
$$I_{1c} = 0$$
$$I_{1d} = V_s/100$$