

The 180° Hybrid Coupler

The 180° **Hybrid Coupler** (sometimes know as the “ring”, “rat-race”, or “Magic-T” hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the **anti-symmetric** form (D_1 symmetry):

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Just like the quadrature coupler, however, we find that:

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

So that the scattering matrix for this device is:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Hence, this coupler is likewise a **3dB coupler**—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.

Note the relative **phase** between the outputs, however, is **dependent** on which port is the input.

For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be **180° out of phase** ($e^{j\pi} = -1$)!

An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3 (with ports 1 and 4 terminated in matched loads). Note the signal out of port 1 would therefore be:

$$\begin{aligned} V_1^-(z) &= S_{12} V_2^+(z) + S_{13} V_3^+(z) \\ &= \frac{1}{\sqrt{2}} (V_3^+(z) + V_2^+(z)) \end{aligned}$$

while the signal out of port 4 is:

$$\begin{aligned} V_4^-(z) &= S_{42} V_2^+(z) + S_{43} V_3^+(z) \\ &= \frac{1}{\sqrt{2}} (V_3^+(z) - V_2^+(z)) \end{aligned}$$

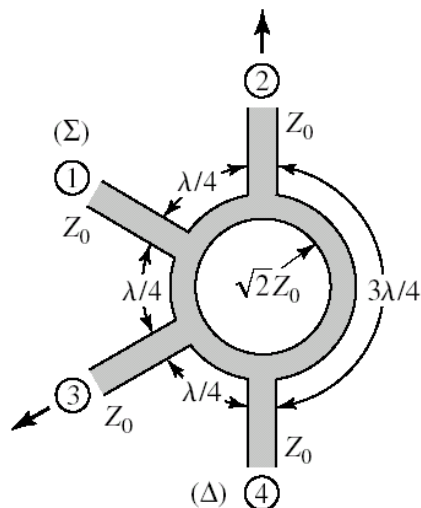
Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180° Hybrid Coupler is thus often referred to as the **sum** (Σ) port.

Likewise, port 4 is proportional to the **difference** between the two inputs. Port 4 a 180° Hybrid Coupler is thus often referred to as the **delta** (Δ) port.

There are **many** applications where we wish to take the sum and/or difference between two signals!

The 180° Hybrid Coupler can likewise be used in the **opposite** manner. If we have **both** the sum and difference of two signals available, we can use this device to separate the signals into their separate components!

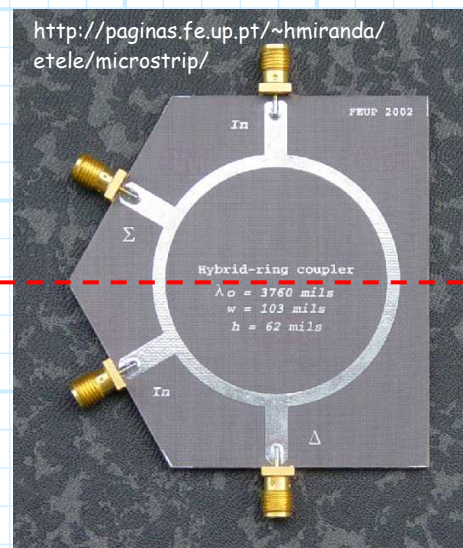
Q: *How is this hybrid coupler constructed?*



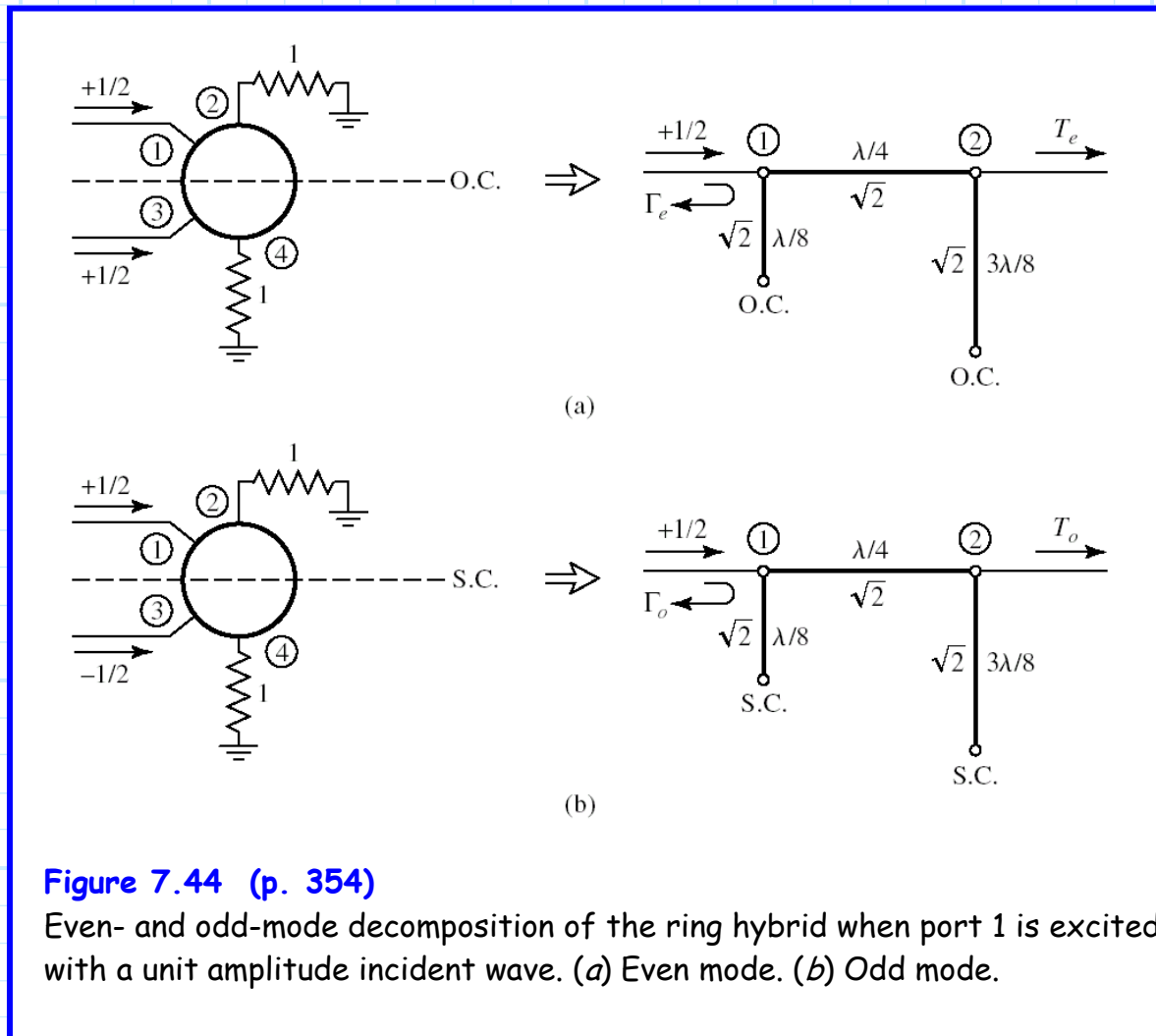
A: Like the quadrature hybrid, it is simply made of **lengths** of transmission lines. However, unlike the quadrature hybrid, the characteristic impedance of each line is **identical** ($\sqrt{2}Z_0$), but the lengths of the lines are dissimilar.

Q: *How can we possibly analyze this mess?*

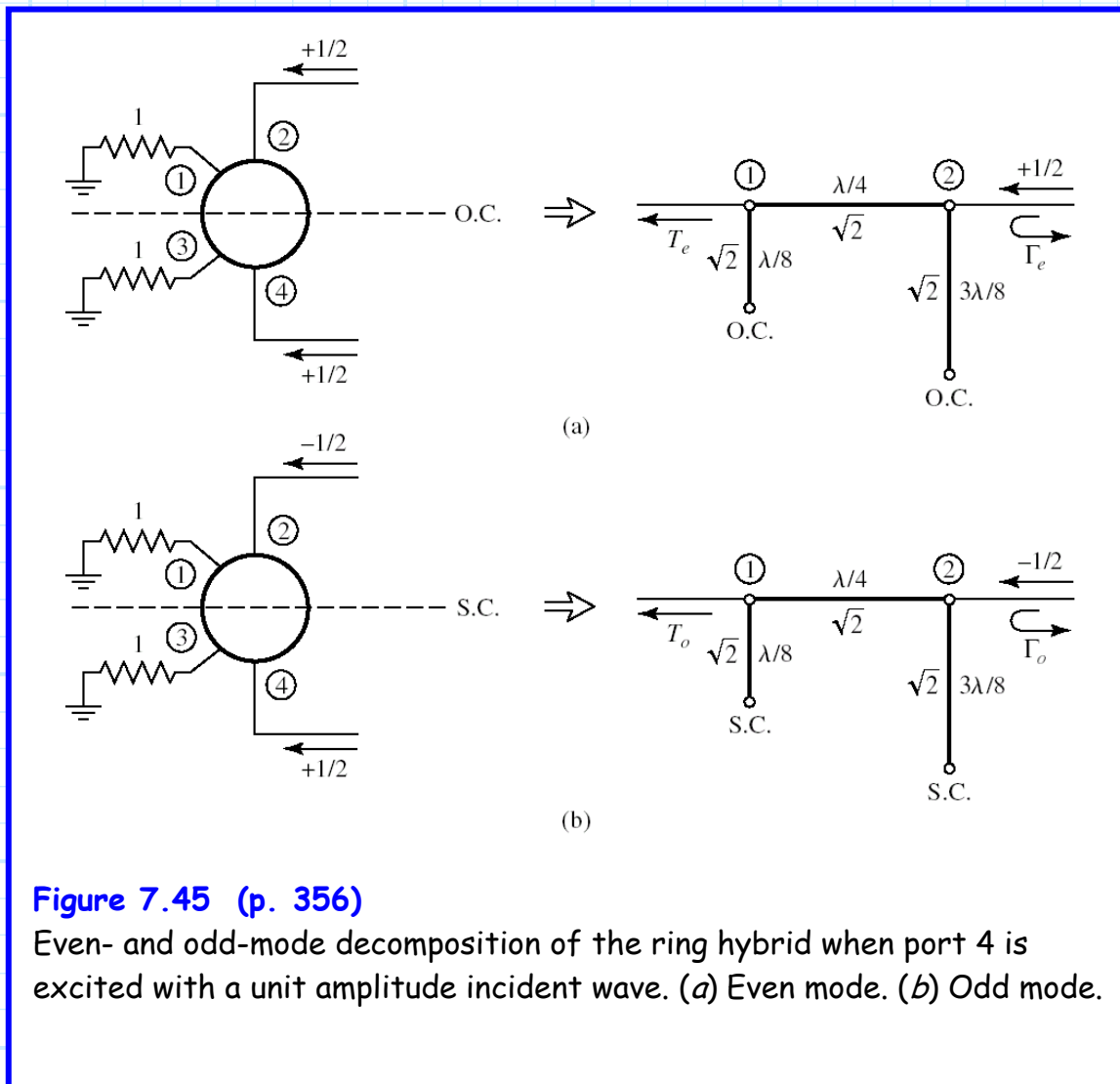
A: Note there is one plane of bilateral **symmetry** (D_1) in this circuit—we can use even/odd mode analysis!



However, we must perform **two** separate analysis—one using sources on ports **1 and 3**:



While the **other** uses sources on ports **2 and 4**:



Finally, because of the transmission line lengths, we find that the ring hybrid is a **narrow-band** device:

