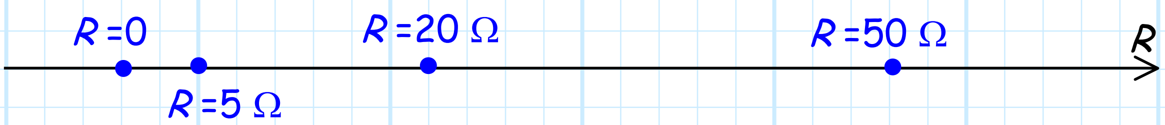
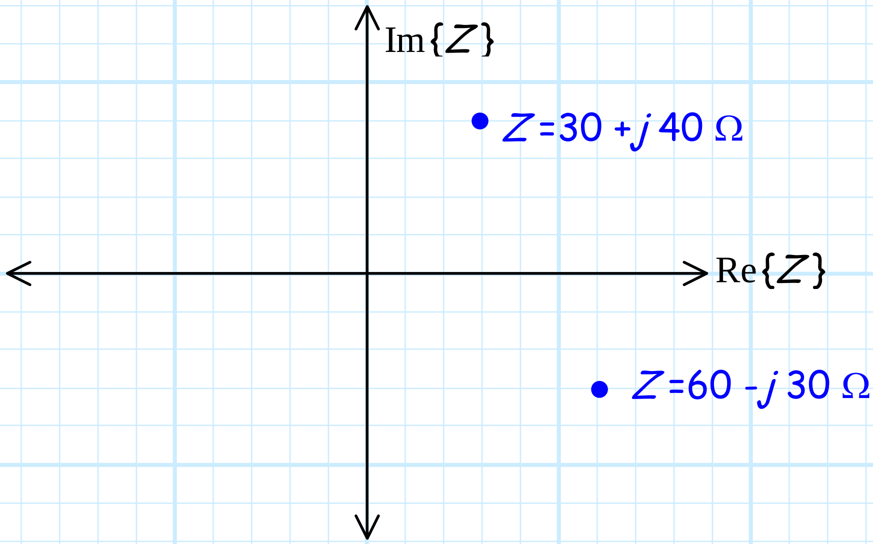
The Complex Γ Plane

Resistance R is a **real** value, thus we can indicate specific resistor values as points on the **real line**:



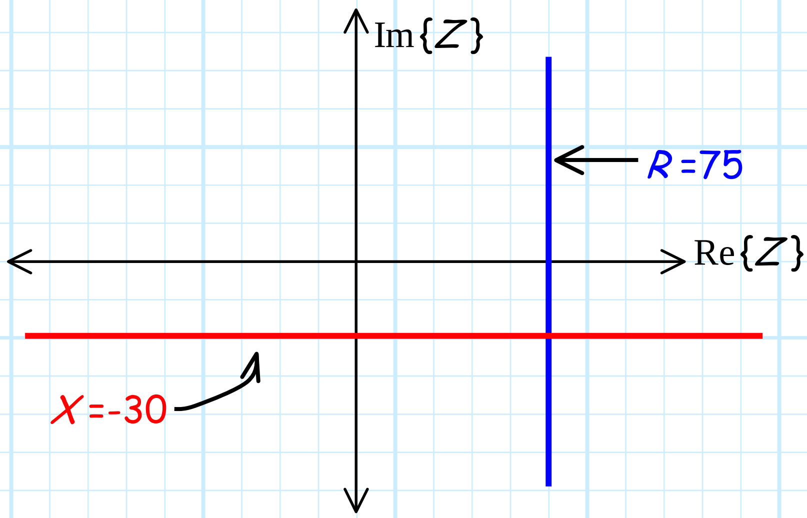
Likewise, since impedance Z is a **complex** value, we can indicate specific impedance values as point on a two dimensional **complex plane**:



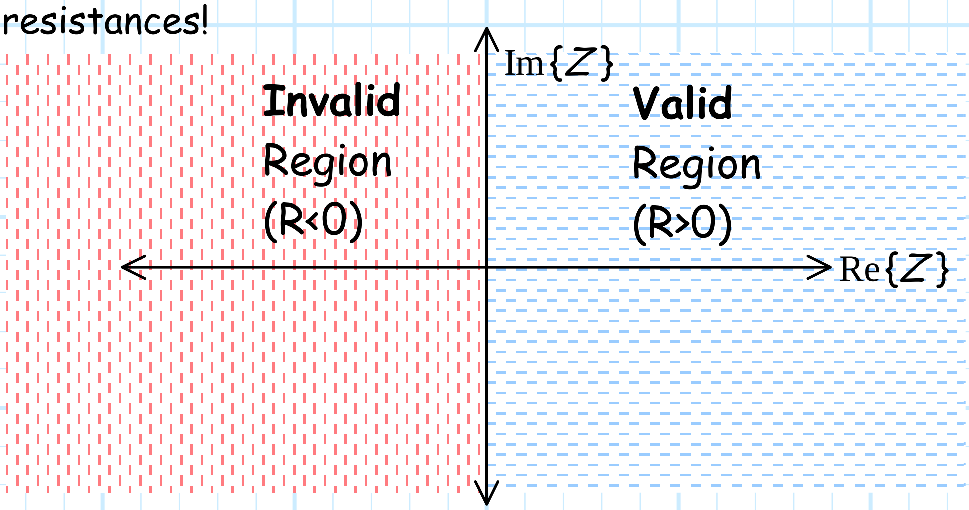
Note each dimension is defined by a single real line: the **horizontal** line (axis) indicating the **real** component of Z (i.e., $\text{Re}\{Z\}$), and the **vertical** line (axis) indicating the **imaginary** component of impedance Z (i.e., $\text{Im}\{Z\}$). The **intersection** of these two lines is the point denoting the impedance Z **= 0**.

* Note then that a **vertical line** is formed by the locus of **all** points (impedances) whose **resistive** (i.e., real) component is equal to, say, 75.

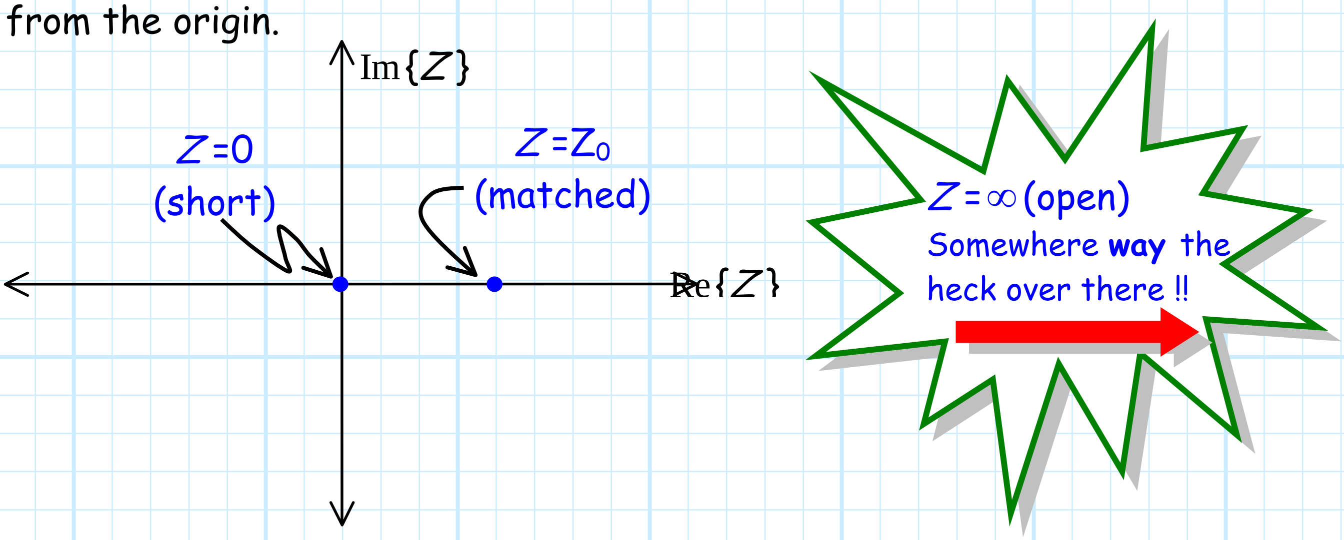
* Likewise, a **horizontal line** is formed by the locus of **all** points (impedances) whose **reactive** (i.e., imaginary) component is equal to -30.



If we assume that the **real** component of **every** impedance is **positive**, then we find that **only the right side** of the plane will be useful for plotting impedance Z—points on the left side indicate impedances with **negative** resistances!



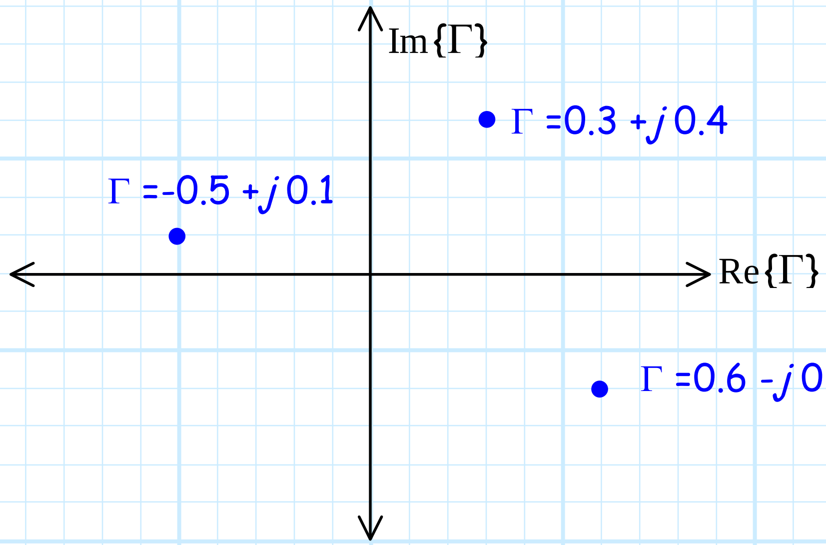
Moreover, we find that common impedances such as $Z = \infty$ (an open circuit!) **cannot** be plotted, as their points appear an **infinite** distance from the origin.



Q: *Yikes! The complex Z plane does* ***not*** *appear to be a very helpful. Is there some graphical tool that* ***is*** *more useful?*

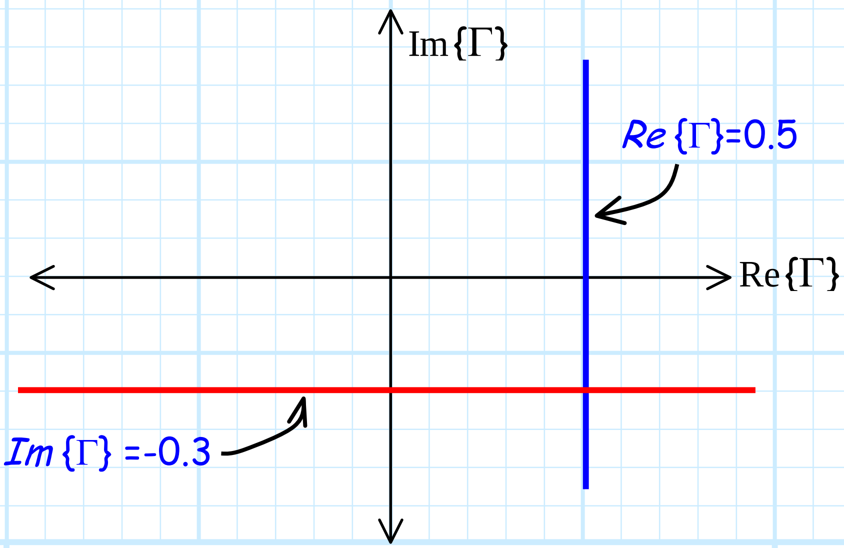
A: Yes! Recall that impedance Z and reflection coefficient Γ are **equivalent complex values**—if you know **one**, you know the **other**.

We can therefore define a **complex** Γ **plane** in the same manner that we defined a complex impedance plane. We will find that there are **many** advantages to plotting on the complex Γ plane, as opposed to the complex Z plane!



Note that the **horizontal** axis indicates the **real** component of Γ ($\mathrm{Re}\{\Gamma\}$), while the **vertical** axis indicates the **imaginary** component of Γ ($\mathrm{Im}\{\Gamma\}$).

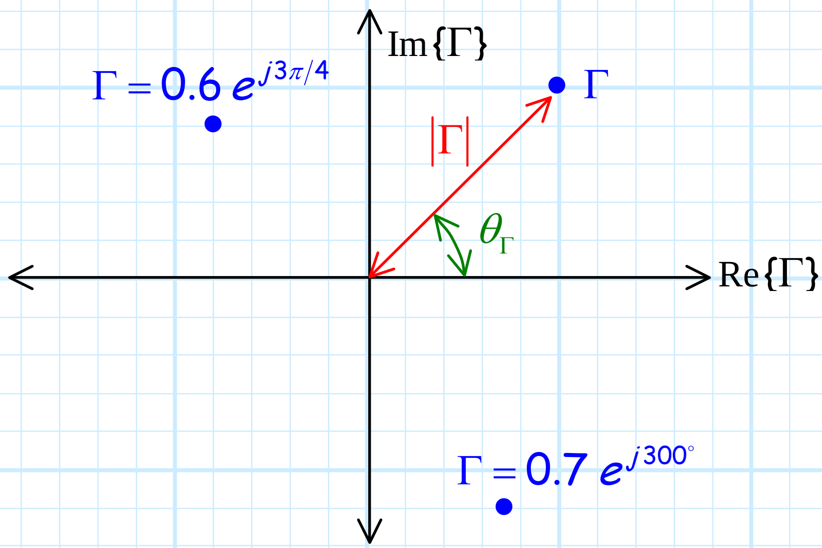
We **could** plot points and lines on this plane **exactly as before**:



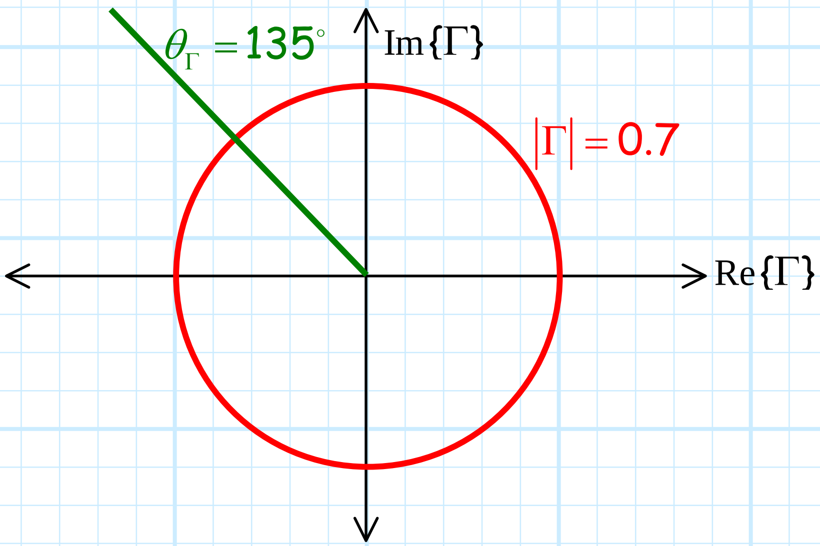
However, we will find that the utility of the complex Γ pane as a graphical tool becomes apparent **only** when we represent a **complex** reflection coefficient in terms of its **magnitude** ($|\Gamma|$) and **phase** (θ_Γ):

$$\Gamma = |\Gamma| e^{j\theta_\Gamma}$$

In other words, we express Γ using **polar coordinates**:



Note then that a **circle** is formed by the locus of all points whose **magnitude** $|\Gamma|$ equal to, say, 0.7. Likewise, a **radial line** is formed by the locus of all points whose **phase** θ_Γ is equal to 135°.



Perhaps the most important aspect of the complex Γ plane is its **validity region**. Recall for the complex Z plane that this validity region was the **right-half plane**, where $\text{Re}\{Z\} > 0$ (i.e., **positive** resistance).

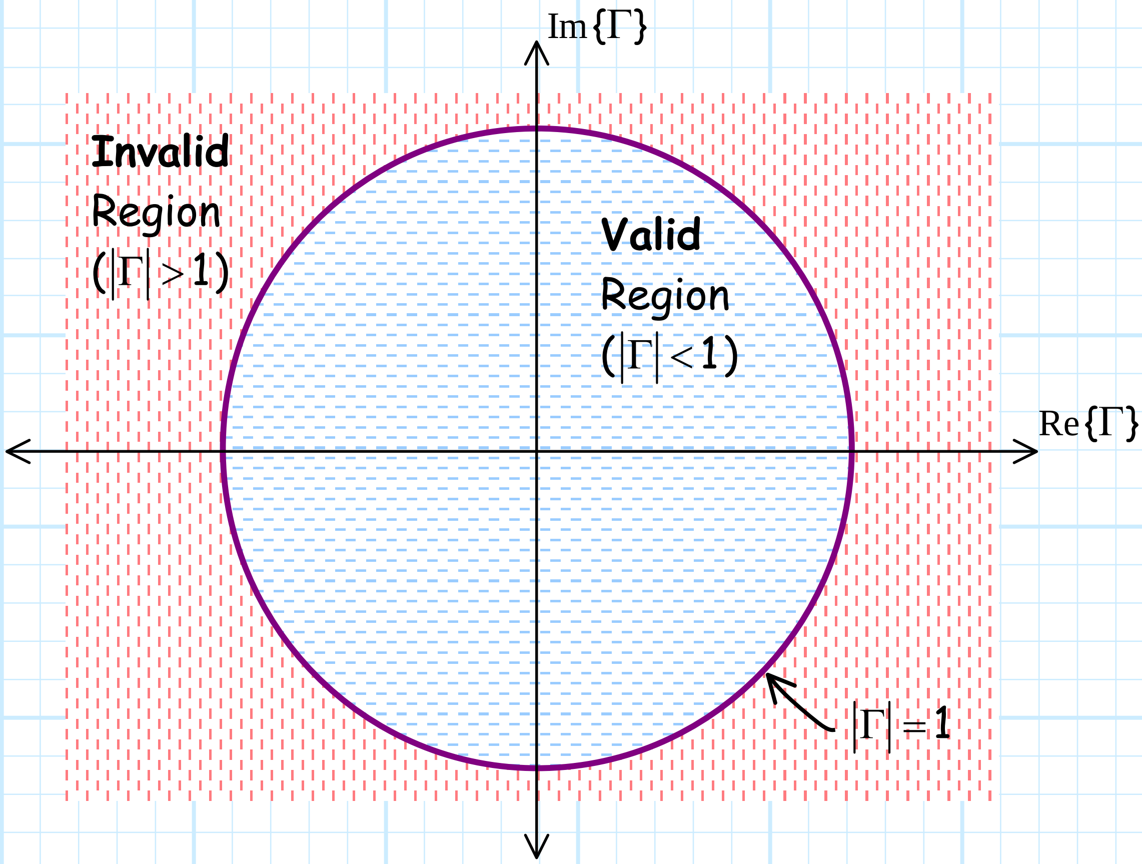
The **problem** was that this validity region was **unbounded** and **infinite** in extent, such that many important impedances (e.g., open-circuits) could **not** be plotted.

Q: *What is the validity region for the complex Γ plane?*

A: Recall that we found that for $\text{Re}\{Z\} > 0$ (i.e., positive resistance), the **magnitude** of the reflection coefficient was **limited**:

$$0 < |\Gamma| < 1$$

Therefore, the **validity region** for the complex Γ plane consists of all points **inside the circle** $|\Gamma| = 1$--a finite **and** bounded area!



Note that we can plot **all** valid impedances (i.e., $R > 0$) within this **finite** region!