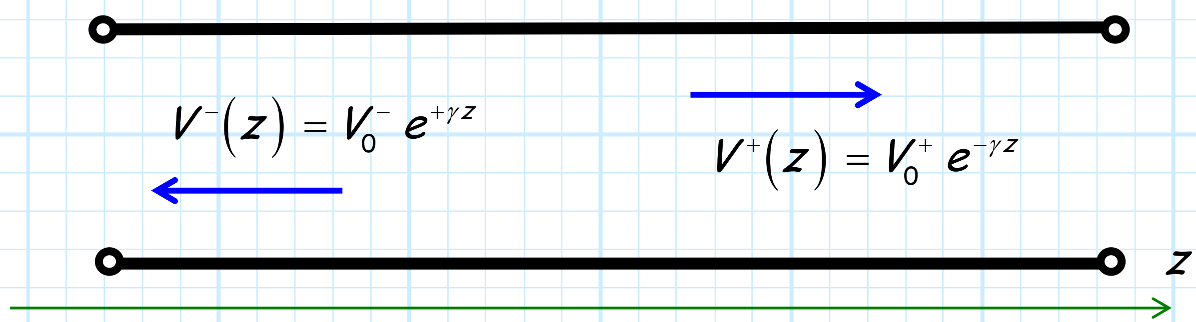
Complex Propagation Constant γ

Recall that the activity along a transmission line can be expressed in terms of two functions, functions that we have described as **wave** functions:



where γ is a **complex constant** that describe the properties of a transmission line. Since γ is complex, we can consider both its **real** and **imaginary** components.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \doteq \alpha + j\beta$$

where $\alpha = \mathrm{Re}\{\gamma\}$ and $\beta = \mathrm{Im}\{\gamma\}$. Therefore, we can write:

$$V^{+}(z) = V_0^{+} e^{-\gamma z} = V_0^{+} e^{-(\alpha + j\beta)z} = V_0^{+} e^{-\alpha z} e^{-j\beta z}$$

The value α

Q: *What **are** these constants α and β? What do they **physically** represent?*

A: Remember, a complex value can be expressed in terms of its **magnitude** and **phase**.

For example:

$$V_0^+ = \left|V_0^+\right| e^{j\varphi_0^+}$$

Likewise:

$$V^+(z) = \left|V^+(z)\right| \, e^{j\,\varphi^+(z)}$$

And since:

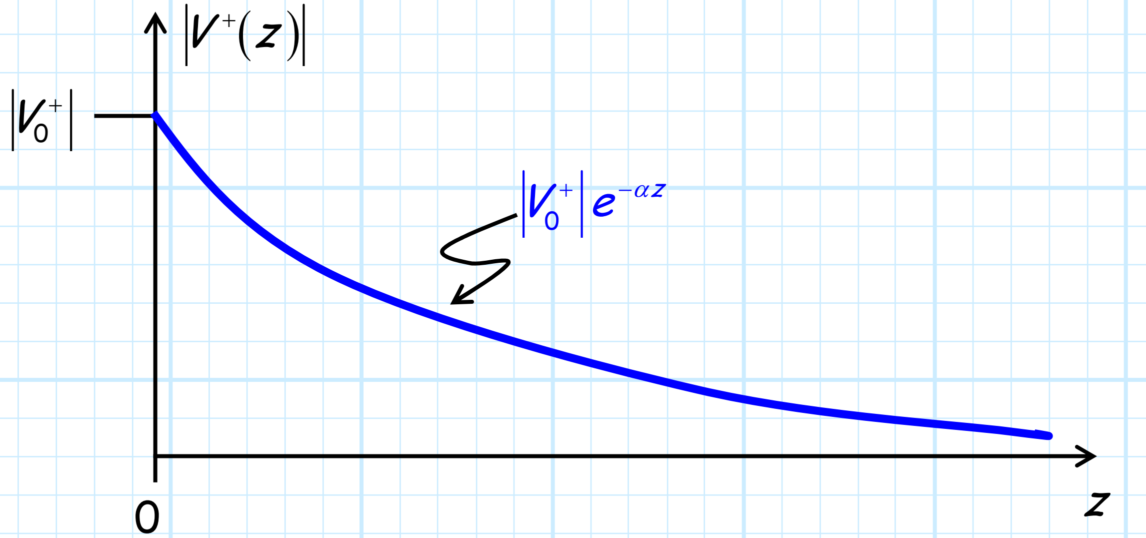
$$\begin{aligned} V^+(z) &= V_0^+ \, e^{-\alpha z} \, e^{-j\beta z} \\ &= \left|V_0^+\right| e^{j\varphi_0^+} \, e^{-\alpha z} \, e^{-j\beta z} \\ &= \left|V_0^+\right| e^{-\alpha z} \, e^{j\left(\varphi_0^+ - \beta z\right)} \end{aligned}$$

we find:

$$\left|V^+(z)\right| = \left|V_0^+\right| e^{-\alpha z} \qquad\qquad \varphi^+(z) = \varphi_0^+ - \beta z$$

The value α specifies attenuation

It is thus evident that $e^{-\alpha z}$ **alone** determines the **magnitude** of wave $V^+(z) = V_0^+ e^{-\gamma z}$ as a function of position z.



Therefore, α expresses the **attenuation** of the signal due to the **loss** in the transmission line.

The **larger** the value of α, the **greater** the exponential attenuation.

Q: *So just **why** does the wave attenuate as it propagates down the transmission line?*

A:

The value β

Q: So *what* ***is*** *the constant* β*? What does* ***it*** *physically mean?*

A: Recall the function:

$$\varphi^{+}(z) = \varphi_0^{+} - \beta z$$

represents the relative **phase** of wave $V^{+}(z)$; a **function** of transmission line **position** z.

Since phase φ is expressed in **radians**, and z is distance (in meters), the value β must have **units** of:

$$\beta = \frac{\varphi}{z} \qquad \frac{\text{radians}}{\text{meter}}$$

Thus, if the value β is **small**, we will need to move a **significant distance** Δz down the transmission line in order to observe a change in the relative phase of the oscillation.

Conversely, if the value β is **large**, a significant change in relative phase can be observed if traveling a **short** distance $\Delta z_{2\pi}$ down the transmission line.

The Wavelength λ

Q: *How far must we move along a transmission line in order to observe a change in relative phase of 2π radians?*

A: We can easily determine this distance ($\Delta z_{2\pi}$, say) from the transmission line characteristic β.

$$2\pi = \varphi\left(z + \Delta z_{2\pi}\right) - \varphi(z) = \beta\,\Delta z_{2\pi}$$

or, rearranging:

$$\Delta z_{2\pi} = \frac{2\pi}{\beta} \qquad \Rightarrow \qquad \beta = \frac{2\pi}{\Delta z_{2\pi}}$$

The **distance** $\Delta z_{2\pi}$ over which the relative phase changes by 2π **radians**, is more specifically known as the **wavelength** λ of the propagating wave (i.e., $\lambda \doteq \Delta z_{2\pi}$):

$$\lambda = \frac{2\pi}{\beta} \qquad \Rightarrow \qquad \beta = \frac{2\pi}{\lambda}$$

β is Spatial Frequency

The value β is thus essentially a **spatial frequency**, in the same way that ω is a **temporal** frequency:

$$\omega = \frac{2\pi}{T}$$

Note T is the **time** required for the phase of the oscillating signal to change by a value of 2π radians, i.e.:

$$\omega T = 2\pi$$

And the **period** of a sinewave, and related to its **frequency** in Hertz (cycles/second) as:

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Compare these results to:

$$\beta = \frac{2\pi}{\lambda} \qquad 2\pi = \beta\lambda \qquad \lambda = \frac{2\pi}{\beta}$$

Propagation Velocity

Q: *So, just how **fast** does this wave propagate down a transmission line?*

A: We describe wave velocity in terms of its **phase velocity**—in other words, how **fast** does a specific value of absolute phase φ seem to **propagate** down the transmission line.

It can be shown that this velocity is:

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

From this we can conclude:

$$v_p = f\lambda$$

as well as:

$$\beta = \frac{\omega}{v_p}$$