The Directional Coupler

A **directional coupler** is a 4-port network that is designed to **divide** and **distribute** power.

Although this would seem to be a particularly **mundane** and simple task, these devices are both very **important** in microwave systems, and very **difficult** to design and construct.

Two of the **reasons** for this difficulty are our desire for the device to be:

1. Matched

B. Lossless

Thus, we require a **matched**, **lossless**, and (to make it simple) **reciprocal** 4-port device!

Recall that a matched, lossless, reciprocal, 4-port device was difficult to even **mathematically** determine, as the resulting scattering matrix must be (among other things) **unitary**.

You must remember this…

However, we were able to determine two possible mathematical solutions, which we called the **symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

And the **asymmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

wherein for both cases, the relationship:

$$|\alpha|^2 + |\beta|^2 = 1$$

must be true in order for the device to be **lossless** (i.e., for $\mathcal{S}$ to be unitary).

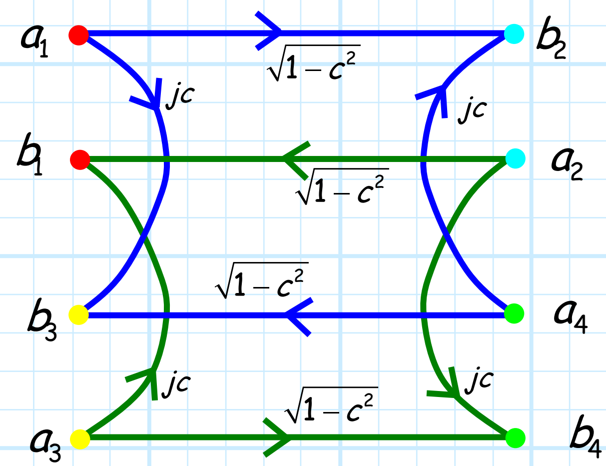
The coupling coefficient defines all!

For most couplers we will find that α and β can (at least ideally) be represented by a real value c, known as the **coupling coefficient**.

$$\beta = c \qquad \alpha = \sqrt{1-c^2}$$

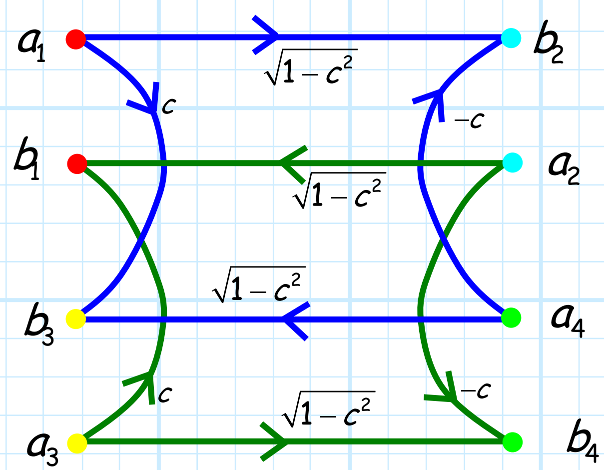
The **symmetric** solution is thus described as:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$



And the **asymmetric** solution is:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & c & 0 \\ \sqrt{1-c^2} & 0 & 0 & -c \\ c & 0 & 0 & \sqrt{1-c^2} \\ 0 & -c & \sqrt{1-c^2} & 0 \end{bmatrix}$$



Let's see how power is divided!

Additionally, for a directional coupler, the coupling coefficient c will be less than $1/\sqrt{2}$ **always**. Therefore, we find that:

$$0 \leq c \leq \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \leq \sqrt{1-c^2} \leq 1$$

Let's see what this means in terms of the **physical behavior** of a directional coupler!

First, consider the case where some signal is incident on **port 1**, with power P_1^+. If all other ports are matched, we find that the power flowing out of **port 1** is **zero**:

$$P_1^- = \left|S_{11}\right|^2 P_1^+ = 0^2 P_1^+ = 0$$

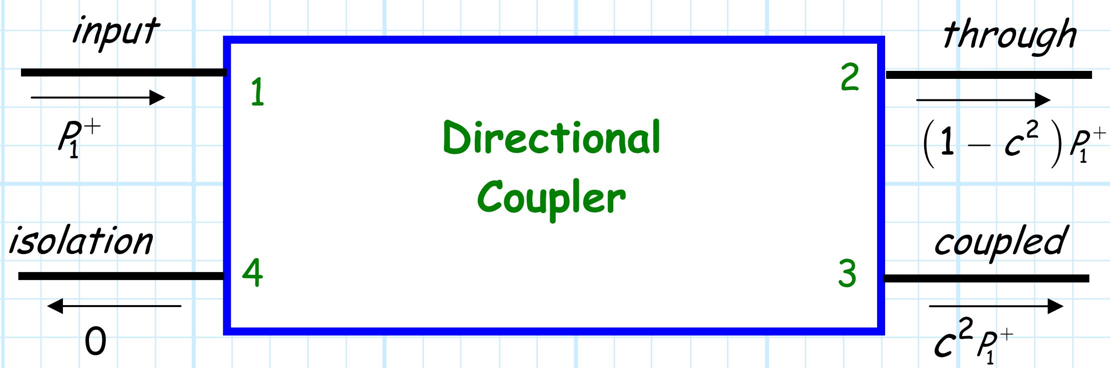
while **a lot** of power leaves **port 2**: $P_2^- = \left|S_{21}\right|^2 P_1^+ = \left(1-c^2\right) P_1^+$

and **a little** power is coupled out of **port 3**: $P_3^- = \left|S_{31}\right|^2 P_1^+ = c^2\, P_1^+$

Finally, we find there is **no power** flowing out of **port 4**: $P_4^- = \left|S_{41}\right|^2 P_1^+ = 0^2 P_1^+ = 0$

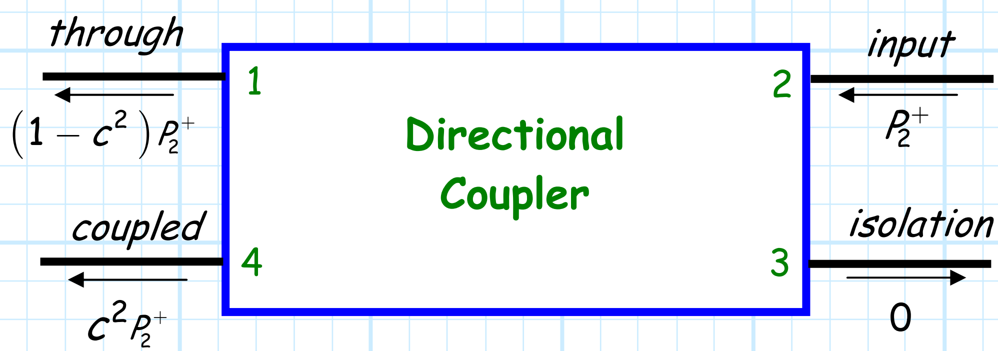
Remember, the device has D_2 symmetry

In the terminology of the directional coupler, we say that port 1 is the **input** port, port 2 is the **through** port, port 3 is the **coupled** port, and port 4 is the **isolation** port.



Note however, that **any** of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, **if** a signal is incident on **port 2**, while all other ports are matched, we find that:



A big table to reinforce the point

Thus, from the scattering matrix of a directional coupler, we can form the following table:

Input	*Through*	*Coupled*	*Isolation*
Port 1	Port 2	Port 3	Port 4
Port 2	Port 1	Port 4	Port 3
Port 3	Port 4	Port 1	Port 2
Port 4	Port 3	Port 2	Port 1

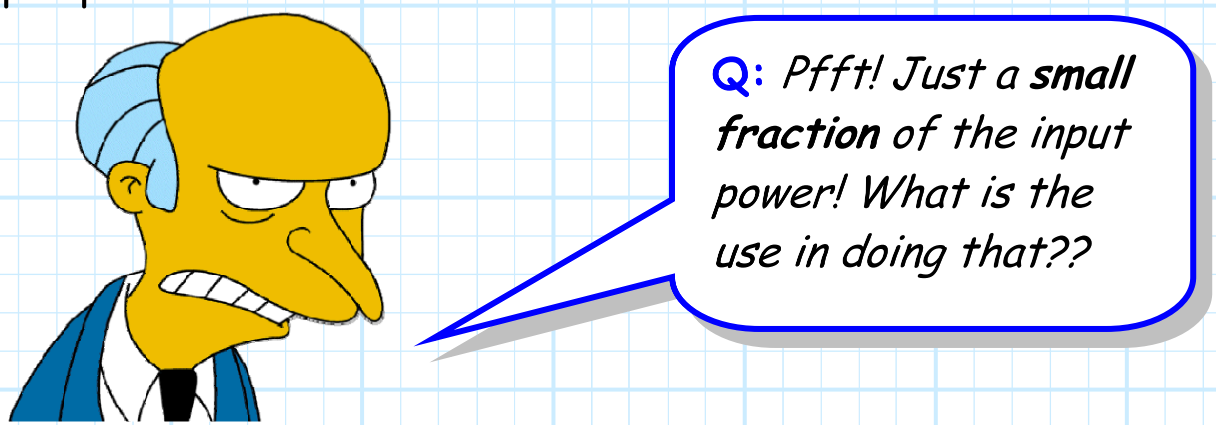
Really; is this thing at all useful?

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

$$0.25 > c^2 > 0.0001$$

As a result, we find that $\sqrt{1-c^2} \approx 1$. What this means is that the power out of the **through** port is just **slightly smaller** (typically) than the power incident on the input port.

Likewise, the power out of the **coupling** port is typically a **small fraction** of the power incident on the input port.



A: A directional coupler is often used for **sampling** a small portion of the signal power. For example, we might **measure** the output power of the **coupled** port (e.g., P_3^-) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^- / c^2$)

Alas, the ideal device is a mythical device

Unfortunately, the **ideal** directional coupler **cannot** be built! For example, the input match is never **perfect**, so that the diagonal elements of the scattering matrix, although **very small**, are not zero.

Likewise, the isolation port is never **perfectly** isolated, so that the values S_{41}, S_{32}, S_{23} and S_{14} are also non-zero—some **small** amount of power leaks out!

As a result, the through port will be **slightly less** than the value $\sqrt{1-c^2}$. The scattering matrix for a **non-ideal coupler** would therefore be:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{21} & jc & S_{41} \\ S_{21} & S_{11} & S_{41} & jc \\ jc & S_{41} & S_{11} & S_{21} \\ S_{41} & jc & S_{21} & S_{11} \end{bmatrix}$$

From **this** scattering matrix, we can extract **some important parameters** about directional couplers!

Coupling C

The **coupling** value is the ratio of the coupled output power (P_3^-) to the input power (P_1^+), expressed in decibels:

$$C\,(dB) = 10\log_{10}\left[\frac{P_1^+}{P_3^-}\right] = -10\log_{10}|jc|^2$$

This is the **primary** specification of a directional coupler!

Note the **larger** the coupling value, the **smaller** the coupled power!

For example:

A **6 dB** coupler couples out **25%** of the input power.

A **10 dB** coupler couples out **10%** of the input power.

A **20 dB** coupler couples out **1.0%** of the input power.

A **30 dB** coupler couples out **0.1%** of the input power.

Directivity D

The **directivity** is the ratio of the power **out** of the coupling port (P_3^-) to the power **out** of the isolation port (P_4^-), expressed in decibels.

$$D(dB) = 10\log_{10}\left[\frac{P_3^-}{P_4^-}\right] = 10\log_{10}\left[\frac{|jc|^2}{|S_{41}|^2}\right]$$

This value indicates how effective the device is in "**directing**" the coupled energy into the correct port (i.e., into the coupled port, **not** the isolation port).

Ideally this is infinite (i.e.,$P_4^- = 0$) , so the **higher** the directivity, the **better**.

Isolation $\mathcal{I}$

Isolation is the ratio of the **input power** (P_1^+) to the power out of the **isolation** port (P_4^-), expressed in decibels.

$$\mathcal{I}\,(dB) = 10\log_{10}\left[\frac{P_1^+}{P_4^-}\right] = -10\log_{10}\left[|S_{41}|^2\right]$$

This value indicates how "isolated" the isolation port actually is.

Ideally this is infinite (i.e.,$P_4^- = 0$), so the **higher** the isolation, the better.

Note that isolation, directivity, and coupling are **not** independent values! **You** should be able to quickly show that:

$$\mathcal{I}\,(dB) = \mathcal{C}\,(dB) + \mathcal{D}\,(dB)$$

Mainline Loss *ML*

The **mainline loss** is the ratio of the **input** power (P_1^+)to the power out of the **through** port (P_2^-), expressed in decibels.

$$ML(dB) = 10\log_{10}\left[\frac{P_1^+}{P_2^-}\right] = -10\log_{10}\left[|S_{21}|^2\right]$$

It indicates how much power the signal **loses** as it travels from the input to the through port.

Coupling Loss CL

The **coupling loss** indicates the **portion** of the mainline loss that is due to coupling some of the input power into the coupling port.

$$CL(dB) = 10\log_{10}\left[\frac{P_1^+}{P_1^+ - P_3^-}\right] = -10\log_{10}\left[1 - |jc|^2\right]$$

Conservation of energy makes this loss is **unavoidable.**

Note this value can be **very small**, for example:

The coupling loss of a **10dB** coupler is **0.44 dB**

The coupling loss of a **20dB** coupler is **0.044 dB**

The coupling loss of a **30dB** coupler is **0.0044 dB**

Insertion Loss $\mathcal{IL}$

Q: *But wait, shouldn't* $P_1^+ - P_3^- = P_2^-$ *, meaning the coupling loss and the mainline loss will be the* ***same exact value?***

A: **Ideally** this would be true.

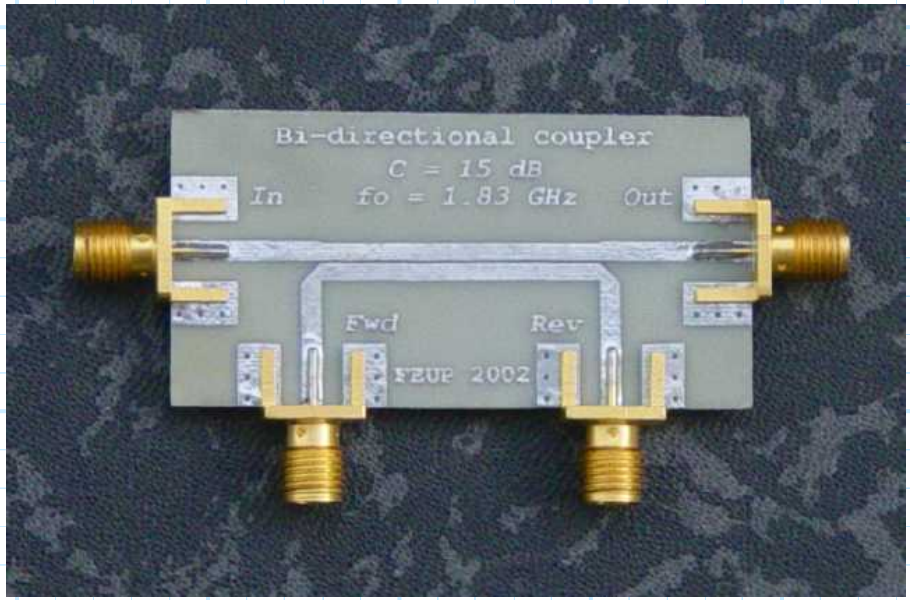
But, the reality is that couplers are **not perfectly lossless**, so there will additionally be loss due to **absorbed** energy (i.e., heat). This loss is called **insertion loss** and is simply the **difference** between the mainline loss and coupling loss:

$$\mathcal{IL}(dB) = \mathcal{ML}(dB) - \mathcal{CL}(dB)$$

The insertion loss thus indicates the portion of the mainline loss that is **not** due to coupling some input power to the coupling port. This insertion loss **is** avoidable, and thus the **smaller** the insertion loss, the better.

For couplers with **very small coupling** coefficients (e.g., $C(dB) > 20$) the coupling loss is so small that the mainline loss is almost entirely due to insertion loss (i.e., $\mathcal{ML} = \mathcal{IL}$)—often then, the two terms are used **interchangeably**.

A photo of a microstrip coupler



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