The Directional Coupler

A directional coupler is a 4-port network that is designed to divide and distribute power.

Although this would seem to be a particularly mundane and simple task, these devices are both very important in microwave systems, and very difficult to design and construct.

Two of the reasons for this difficulty are our desire for the device to be:

1. Matched
2. Lossless

Thus, we require a matched, lossless, and (to make it simple) reciprocal 4-port device!

Recall that a matched, lossless, reciprocal, 4-port device was difficult to even mathematically determine, as the resulting scattering matrix must be (among other things) unitary.
You must remember this...

However, we were able to determine two possible mathematical solutions, which we called the symmetric solution:

\[
S = \begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0
\end{bmatrix}
\]

And the asymmetric solution:

\[
S = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0
\end{bmatrix}
\]

wherein for both cases, the relationship:

\[|\alpha|^2 + |\beta|^2 = 1\]

must be true in order for the device to be lossless (i.e., for \(S\) to be unitary).
The coupling coefficient defines all!

For most couplers we will find that $\alpha$ and $\beta$ can (at least ideally) be represented by a real value $c$, known as the coupling coefficient.

$\beta = c$

The symmetric solution is thus described as:

$$S = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$

And the asymmetric solution is:

$$S = \begin{bmatrix} 0 & \sqrt{1-c^2} & c & 0 \\ \sqrt{1-c^2} & 0 & 0 & -c \\ c & 0 & 0 & \sqrt{1-c^2} \\ 0 & -c & \sqrt{1-c^2} & 0 \end{bmatrix}$$
Let’s see how power is divided!

Additionally, for a directional coupler, the coupling coefficient $c$ will be less than $1/\sqrt{2}$ always. Therefore, we find that:

$$0 \leq c \leq \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \leq \sqrt{1-c^2} \leq 1$$

Let’s see what this means in terms of the physical behavior of a directional coupler!

First, consider the case where some signal is incident on port 1, with power $P_1^+$. If all other ports are matched, we find that the power flowing out of port 1 is zero:

$$P_1^- = |S_{11}|^2 P_1^+ = 0^2 P_1^+ = 0$$

while a lot of power leaves port 2:

$$P_2^- = |S_{21}|^2 P_1^+ = (1 - c^2) P_1^+$$

and a little power is coupled out of port 3:

$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

Finally, we find there is no power flowing out of port 4:

$$P_4^- = |S_{41}|^2 P_1^+ = 0^2 P_1^+ = 0$$
Remember, the device has $D_2$ symmetry

In the terminology of the directional coupler, we say that port 1 is the **input** port, port 2 is the **through** port, port 3 is the **coupled** port, and port 4 is the **isolation** port.

Note however, that **any** of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, if a signal is incident on port 2, while all other ports are matched, we find that:
Thus, from the scattering matrix of a directional coupler, we can form the following table:

<table>
<thead>
<tr>
<th>Input</th>
<th>Through</th>
<th>Coupled</th>
<th>Isolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port 1</td>
<td>Port 2</td>
<td>Port 3</td>
<td>Port 4</td>
</tr>
<tr>
<td>Port 2</td>
<td>Port 1</td>
<td>Port 4</td>
<td>Port 3</td>
</tr>
<tr>
<td>Port 3</td>
<td>Port 4</td>
<td>Port 1</td>
<td>Port 2</td>
</tr>
<tr>
<td>Port 4</td>
<td>Port 3</td>
<td>Port 2</td>
<td>Port 1</td>
</tr>
</tbody>
</table>
Really; is this thing at all useful?

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

\[ 0.25 > c^2 > 0.0001 \]

As a result, we find that \( \sqrt{1 - c^2} \approx 1 \). What this means is that the power out of the through port is just slightly smaller (typically) than the power incident on the input port.

Likewise, the power out of the coupling port is typically a small fraction of the power incident on the input port.

Q: Pfft! Just a small fraction of the input power! What is the use in doing that??

A: A directional coupler is often used for sampling a small portion of the signal power. For example, we might measure the output power of the coupled port (e.g., \( P_3^- \)) and then we can determine the amount of signal power flowing through the device (e.g., \( P_1^+ = P_3^- / c^2 \))
Alas, the ideal device is a mythical device

Unfortunately, the ideal directional coupler cannot be built! For example, the input match is never perfect, so that the diagonal elements of the scattering matrix, although very small, are not zero.

Likewise, the isolation port is never perfectly isolated, so that the values $S_{41}$, $S_{32}$, $S_{23}$ and $S_{14}$ are also non-zero—some small amount of power leaks out!

As a result, the through port will be slightly less than the value $\sqrt{1-c^2}$. The scattering matrix for a non-ideal coupler would therefore be:

$$S = 
\begin{bmatrix}
S_{11} & S_{21} & jc & S_{41} \\
S_{21} & S_{11} & S_{41} & jc \\
jc & S_{41} & S_{11} & S_{21} \\
S_{41} & jc & S_{21} & S_{11}
\end{bmatrix}$$

From this scattering matrix, we can extract some important parameters about directional couplers!
**Coupling C**

The **coupling** value is the ratio of the coupled output power ($P_3^-$) to the input power ($P_1^+$), expressed in decibels:

$$C (dB) = 10 \log_{10} \left( \frac{P_1^+}{P_3^-} \right) = -10 \log_{10} |jc|^2$$

This is the primary specification of a directional coupler!

Note the larger the coupling value, the smaller the coupled power!

For example:

- A **6 dB** coupler couples out **25%** of the input power.

- A **10 dB** coupler couples out **10%** of the input power.

- A **20 dB** coupler couples out **1.0%** of the input power.

- A **30 dB** coupler couples out **0.1%** of the input power.
**Directivity**  \( D \)

The **directivity** is the ratio of the power out of the coupling port \( P_3^- \) to the power out of the isolation port \( P_4^- \), expressed in decibels.

\[
D (\text{dB}) = 10 \log_{10} \left( \frac{P_3^-}{P_4^-} \right) = 10 \log_{10} \left( \frac{|j \epsilon|^2}{|S_{41}|^2} \right)
\]

This value indicates how effective the device is in "directing" the coupled energy into the correct port (i.e., into the coupled port, not the isolation port).

**Ideally** this is infinite (i.e., \( P_4^- = 0 \)), so the **higher** the directivity, the **better**.
**Isolation**

Isolation is the ratio of the input power ($P_1^+$) to the power out of the isolation port ($P_4^-$), expressed in decibels.

$$I (dB) = 10 \log_{10} \left( \frac{P_1^+}{P_4^-} \right) = -10 \log_{10} \left| S_{41} \right|^2$$

This value indicates how “isolated” the isolation port actually is.

Ideally this is infinite (i.e., $P_4^- = 0$), so the higher the isolation, the better.

Note that isolation, directivity, and coupling are not independent values! You should be able to quickly show that:

$$I (dB) = C (dB) + D (dB)$$
Mainline Loss $ML$

The **mainline loss** is the ratio of the input power ($P_1^+$) to the power out of the through port ($P_2^-$), expressed in decibels.

\[
ML \ (dB) = 10 \log_{10} \left( \frac{P_1^+}{P_2^-} \right) = -10 \log_{10} \left( |S_{21}|^2 \right)
\]

It indicates how much power the signal **loses** as it travels from the input to the through port.
**Coupling Loss \( CL \)**

The *coupling loss* indicates the *portion* of the mainline loss that is due to coupling some of the input power into the coupling port.

\[
CL \text{ (dB)} = 10 \log_{10} \left( \frac{P_1^+}{P_1^+ - P_3^-} \right) = -10 \log_{10} \left[ 1 - |jc|^2 \right]
\]

Conservation of energy makes this loss *unavoidable*.

Note this value can be *very small*, for example:

- The coupling loss of a 10dB coupler is 0.44 dB
- The coupling loss of a 20dB coupler is 0.044 dB
- The coupling loss of a 30dB coupler is 0.0044 dB
**Insertion Loss** \( IL \)

**Q:** But wait, shouldn’t \( P_1^+ - P_3^- = P_2^- \), meaning the coupling loss and the mainline loss will be the same exact value?

**A:** Ideally this would be true.

But, the reality is that couplers are not **perfectly lossless**, so there will additionally be loss due to absorbed energy (i.e., heat). This loss is called **insertion loss** and is simply the difference between the mainline loss and coupling loss:

\[
IL (dB) = ML (dB) - CL (dB)
\]

The insertion loss thus indicates the portion of the mainline loss that is not due to coupling some input power to the coupling port. This insertion loss is avoidable, and thus the **smaller** the insertion loss, the better.

For couplers with **very small coupling coefficients** (e.g., \( C(dB) > 20 \)) the coupling loss is so small that the mainline loss is almost entirely due to insertion loss (i.e., \( ML = IL \))—often then, the two terms are used **interchangeably**.
A photo of a microstrip coupler

From: paginas.fe.up.pt/~hmiranda/etele/microstrip/