The Insertion Loss Method

Recall that a **lossless** filter can be described in terms of either its power transmission coefficient $\mathcal{T}(\omega)$ **or** its power reflection coefficient $\Gamma(\omega)$, as the two values are completely **dependent**:

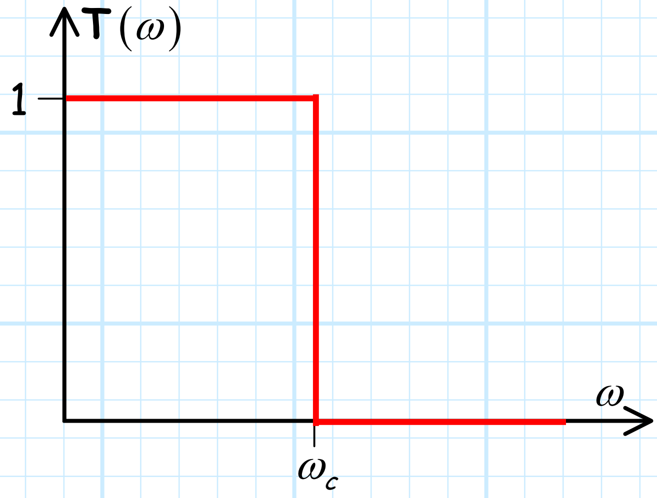
$$\Gamma(\omega) = 1 - \mathcal{T}(\omega)$$

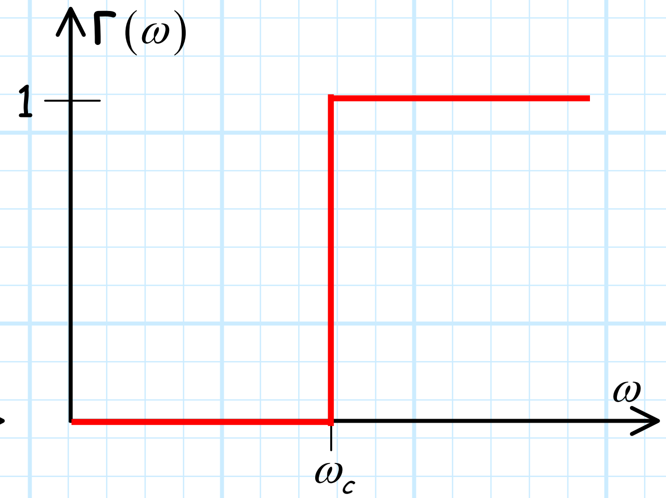
Ideally, these functions would be quite **simple**:

1. $\mathcal{T}(\omega) = 1$ and $\Gamma(\omega) = 0$ for **all** frequencies within the **pass**-band.

2. $\mathcal{T}(\omega) = 0$ and $\Gamma(\omega) = 1$ for **all** frequencies within the **stop**-band.

For example, the **ideal** low-pass filter would be:





Add to this a **linear phase** response, and you have the **perfect** microwave filter!

There's just one small problem with this **perfect** filter→ It's **impossible** to build!

Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

$$\mathbf{T}(\omega) = \frac{a_0 + a_1\,\omega + a_2\,\omega^2 + \cdots}{b_0 + b_1\,\omega + b_2\,\omega^2 + \cdots + b_N\,\omega^{2N}}$$

The **order** N of the (denominator) polynomial is likewise the **order** of the filter.

Instead of the power transmission coefficient, we often use an equivalent function (assuming lossless) called the **power loss ratio** P_{LR}:

$$P_{LR} = \frac{P_1^+}{P_2^-} = \frac{1}{1 - \boldsymbol{\Gamma}(\omega)}$$

Note with this definition, $P_{LR} = \infty$ when $\boldsymbol{\Gamma}(\omega) = 1$, and $P_{LR} = 0$ when $\boldsymbol{\Gamma}(\omega) = 0$.

We likewise note that, for a lossless filter:

$$P_{LR} = \frac{1}{1 - \boldsymbol{\Gamma}(\omega)} = \frac{1}{\mathbf{T}(\omega)}$$

Therefore $P_{LR}(dB)$ is :

$$P_{LR}(dB) = 10\,log_{10} P_{LR} = -10\,log_{10} \mathbf{T}(\omega) \doteq \text{Insertion Loss}$$

The power loss ratio in dB is simply the insertion loss of a lossless filter, and thus filter design using the power loss ratio is also called the Insertion Loss Method.

We find that realizable filters will have a power loss ratio of the form:

$$P_{LR}(\omega) = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

where $M(\omega^2)$ and $N(\omega^2)$ are polynomials with terms $\omega^2, \omega^4, \omega^6, etc.$

By specifying these polynomials, we specify the frequency behavior of a realizable filter. Our job is to first choose a desirable polynomial!

There are many different **types** of polynomials that result in good filter responses, and each type has its own set of **characteristics**.

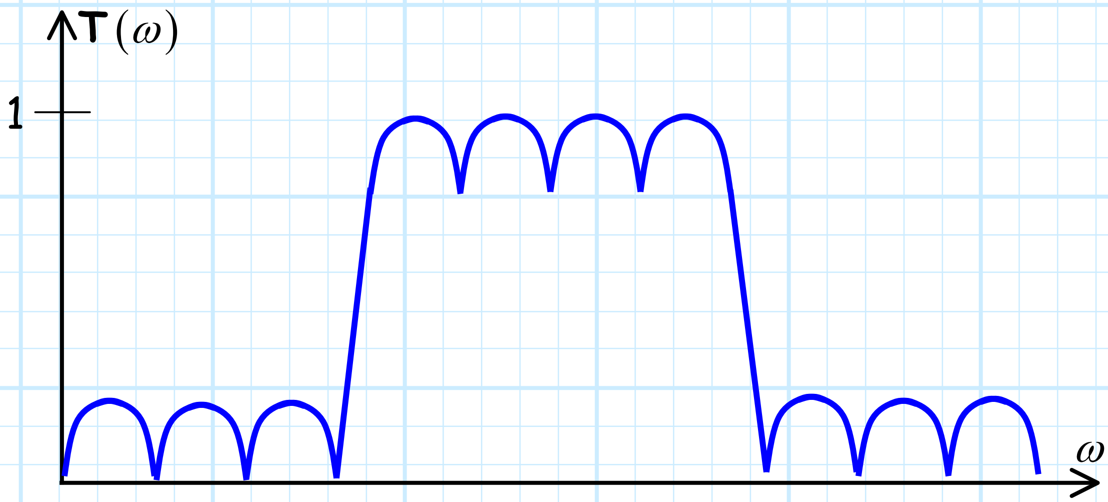
The type of **polynomial** likewise describes the type of microwave **filter**. Let's consider **three** of the most popular types:

1. Elliptical

Elliptical filters have three primary characteristics:

a) They exhibit very **steep** "**roll-off**", meaning that the transition from pass-band to stop-band is very rapid.
b) They exhibit **ripple** in the **pass**-band, meaning that the value of **T** will vary slightly within the pass-band.

c) They exhibit ripple in the **stop**-band, meaning that the value of **T** will vary slightly within the stop-band.



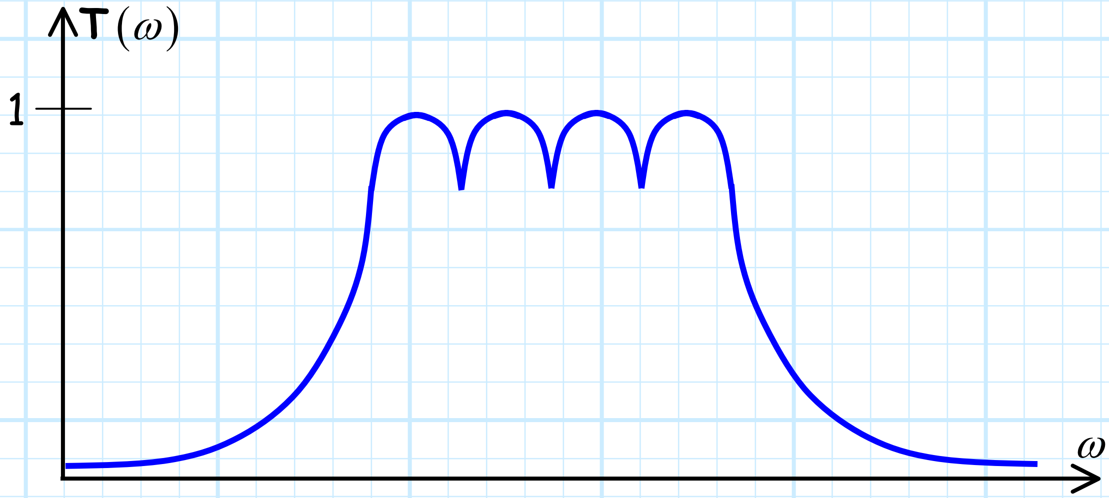
We find that we can make the roll-off **steeper** by accepting more **ripple**.

2. Chebychev

Chebychev filters are also known as **equal-ripple** filters, and have two primary characteristics

a) **Steep** roll-off (but not as steep as Elliptical).

b) Pass-band **ripple** (but not stop-band ripple).



We likewise find that the roll-off can be made steeper by **accepting** more ripple.

We find that Chebychev **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

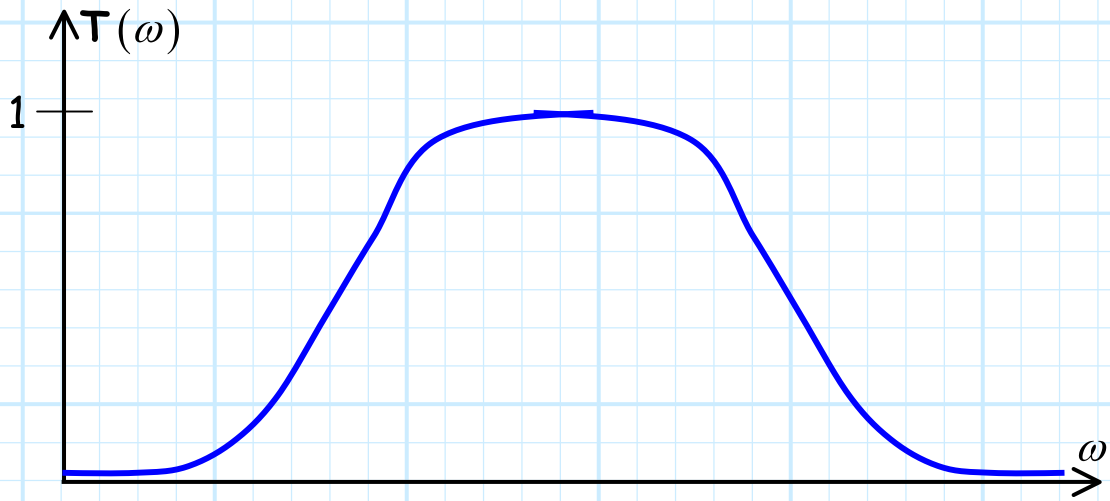
where k specifies the passband **ripple**, $T_N(x)$ is a Chebychev polynomial of **order** N, and ω_c is the low-pass **cutoff frequency**.

3. Butterworth

Also known as **maximally flat** filters, they have two primary characteristics

a) **Gradual** roll-off .

b) **No ripple**—not anywhere.



We find that Butterworth **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

where ω_c is the low-pass **cutoff frequency**, and N specifies the **order** of the filter.

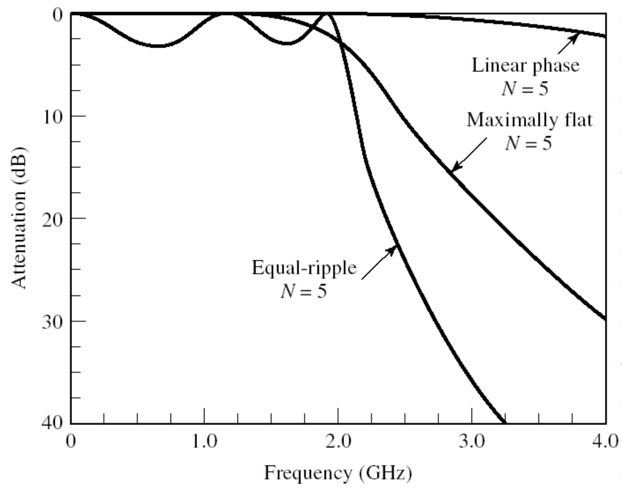
Q: *So we always chose* ***elliptical*** *filters; since they have the steepest roll-off, they are* ***closest*** *to ideal—****right****?*

A: Ooops! I forgot to talk about the **phase response** $\angle S_{21}(\omega)$ of these filters. Let's examine $\angle S_{21}(\omega)$ for each filter type **before** we pass judgment.

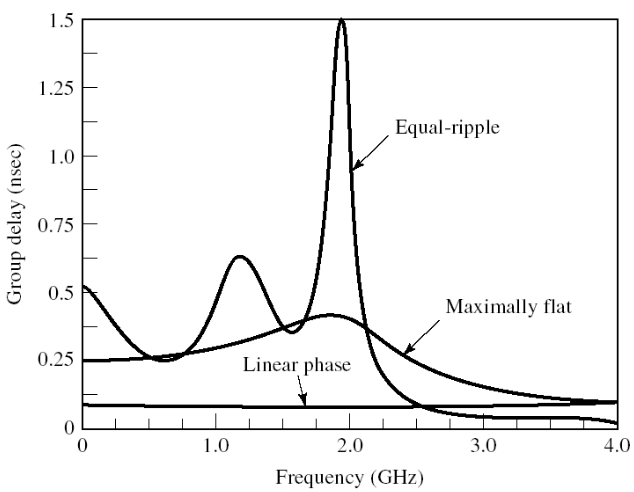
Butterworth $\angle S_{21}(\omega)$ → **Close** to linear phase.

Chebychev $\angle S_{21}(\omega)$ → **Not** very linear.

Elliptical $\angle S_{21}(\omega)$ → A big non-linear **mess**!



(a)



(b)

Thus, it is apparent that as a filter roll-off **improves**, the phase response gets **worse** (watch out for **dispersion!**).

→ There is **no** such thing as the "**best**" filter type!

Q: *So, a filter with **perfectly** linear phase is impossible to construct?*

A: No, it **is** possible to construct a filter with **near** perfect linear phase—but it will exhibit a **horribly** poor roll-off!

Now, for any **type** of filter, we can **improve** roll-off (i.e., increase stop-band attenuation) by **increasing the filter order** N. However, be aware that increasing the filter order likewise has these **deleterious** effects:

1. It makes **phase response** $\angle S_{21}(\omega)$ worse (i.e., more non-linear).

2. It increases filter **cost**, **weight**, and **size**.

3. It increases filter **insertion loss** (this is bad).

4. It makes filter performance more **sensitive** to temperature, aging, etc.

From a **practical** viewpoint, the **order** of a filter should typically be kept to $N < 10$.

Q: *So how do we take these polynomials and make real filters?*

A: Similar to matching networks and couplers, we:

1. Form a general circuit structure with **several** degrees of design freedom.

2. Determine the **general form** of the power loss ratio for these circuits.

3. Use our degrees of design freedom to **equate terms** in the general form to the terms of the **desired** power loss ratio polynomial.