The Linear Phase Filter

Q: *So, narrowband filters should exhibit a **constant** phase delay $\tau(\omega)$. What should the phase function $\angle S_{21}(\omega)$ be for this **dispersionless** case?*

A: We can express this problem mathematically as requiring:

$$\tau(\omega) = \tau_c$$

where τ_c is some **constant**.

Recall that the definition of **phase delay** is:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

and thus **combining** these two equations, we find ourselves with a **differential equation**:

$$-\frac{\partial \angle S_{21}(\omega)}{\partial \omega} = \tau_c$$

The **solution** to this differential equation provides us with the necessary phase function $\angle S_{21}(\omega)$ for a **constant** phase delay τ_c.

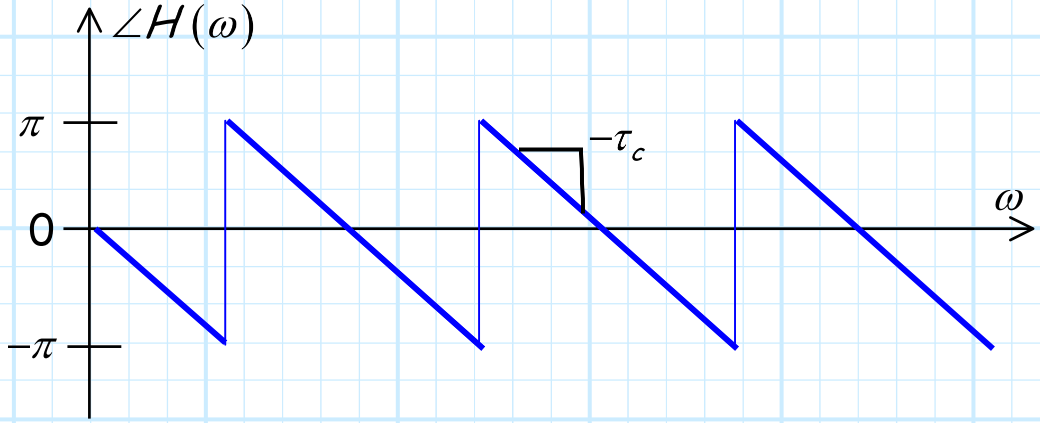
Fortunately, this differential equation is **easily** solved!

The solution is:

$$\angle S_{21}(\omega) = -\omega\,\tau_c + \phi_c$$

where ϕ_c is an arbitrary **constant**.

Plotting this phase function (with $\phi_c = 0$):



As **you** likely expected, this phase function is **linear**, such that it has a **constant slope** ($-\tau_c$).

Filters with this phase response are called **linear phase filters**, and have the desirable trait that they cause **no** dispersion distortion.