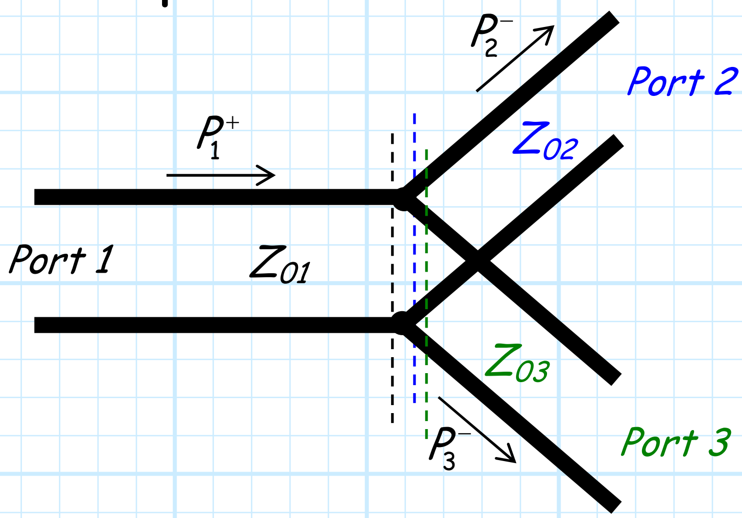
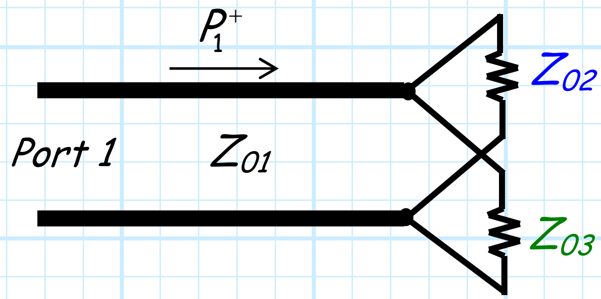
The Lossless Divider

Consider the **lossless power divider**:



To be ideal, we want $S_{11} = 0$. Thus, when ports 2 and port 3 are **terminated** in matched loads, the input impedance at port 1 must be equal to Z_{01}. This will only be true if the values Z_{02} and Z_{03} are selected such that:

$$Z_{01} = \left(\frac{1}{Z_{02}} + \frac{1}{Z_{03}} \right)^{-1} = \frac{Z_{02}\, Z_{03}}{Z_{02} + Z_{03}}$$



Note however that this circuit is **not** symmetric, thus we find that $S_{22} \neq 0$ and $S_{33} \neq 0$!

It is evident that this divider is **lossless** (no resistive components), so that:

$$P_1^+ = P_2^- + P_3^-$$

where P_1^+ is the power incident (and absorbed if $S_{11} = 0$) on port 1, and P_2^- and P_3^- is the power absorbed by the matched loads of ports 2 and 3.

Unless $Z_{02} = Z_{03}$, the power will not be divide equally between P_2^- and P_3^-. With a little microwave circuit analysis, it can be shown that the **division ratio** α is :

$$\alpha = \frac{P_2^-}{P_3^-} = \frac{Z_{03}}{Z_{02}}$$

Thus, if we desire an **ideal** ($S_{11} = 0$) divider with a specific division ratio α, we will find that:

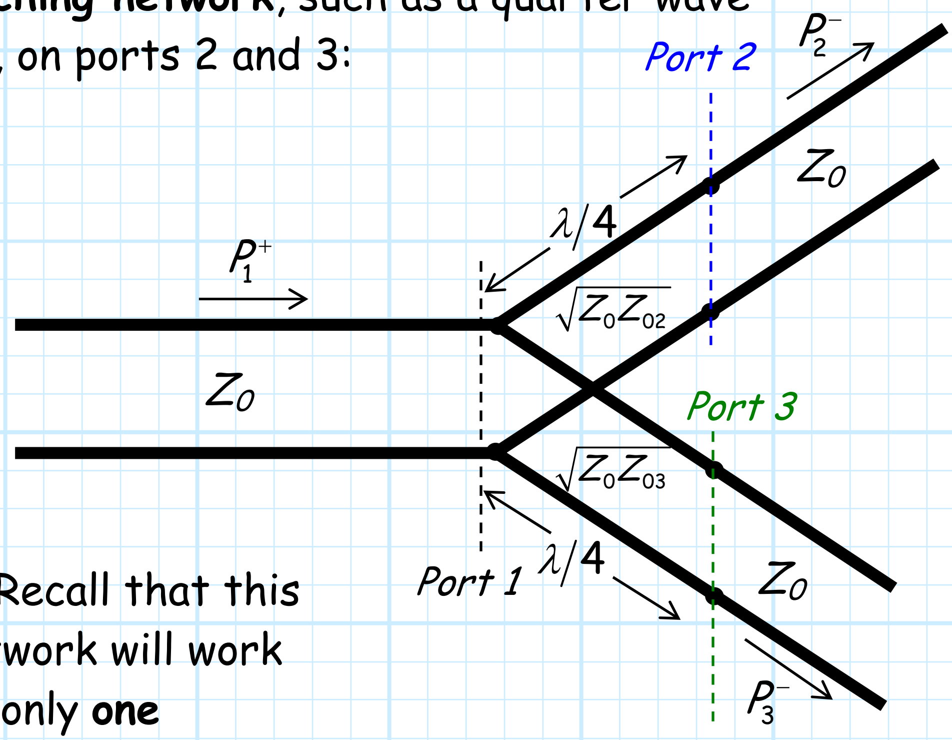
$$Z_{02} = Z_{01}\left(1 + \frac{1}{\alpha}\right)$$

and:

$$Z_{03} = Z_{01}\left(1 + \alpha\right)$$

Q: *I don't understand how this is helpful. Don't we typically want the characteristic impedance of all three ports to be equal to the* ***same*** *value (e.g., $Z_{01} = Z_{02} = Z_{03} = Z_0$)?*

A: True ! A more practical way to implement this divider is to use a **matching network**, such as a quarter wave transformer, on ports 2 and 3:



But beware! Recall that this matching network will work perfectly at only **one** frequency.

This lossless divider has a scattering matrix (at the design frequency) of this form:

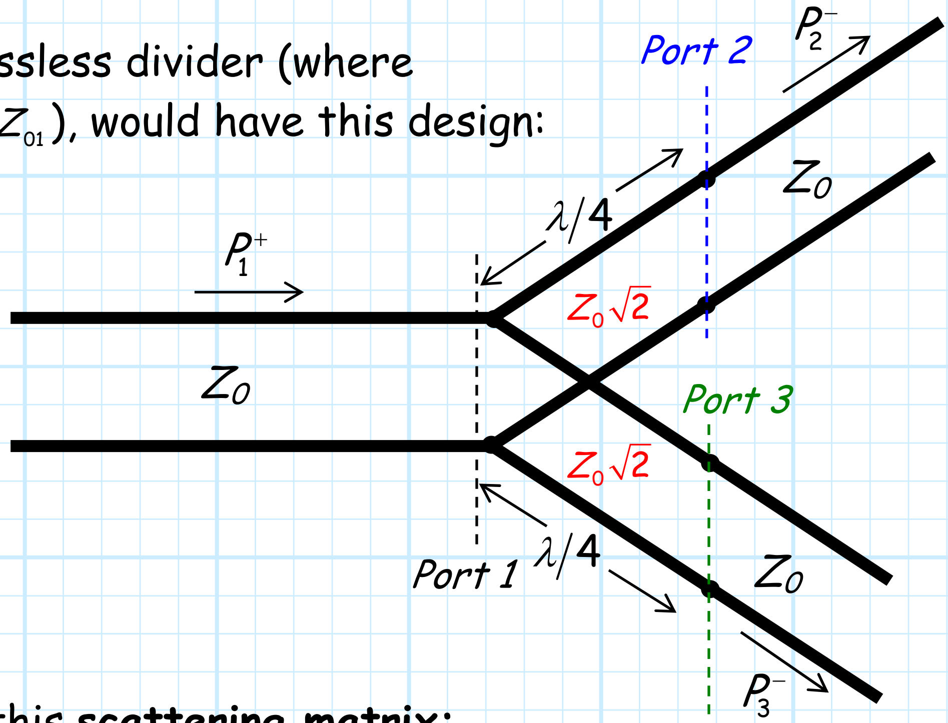
$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}/_{\sqrt{2}} & {}^{-j}/_{\sqrt{2}} \\ {}^{-j}/_{\sqrt{2}} & S_{22} & S_{23} \\ {}^{-j}/_{\sqrt{2}} & S_{32} & S_{33} \end{bmatrix}$$

where the (non-zero!) values of S_{22}, S_{23}, S_{32}, and S_{33} depend on the division ratio α.

Note that if we desire a **3 dB** divider (i.e., $\alpha = 1$), then:

$$Z_{02} = Z_{03} = 2\,Z_{01}$$

This **3dB** lossless divider (where $Z_{02} = Z_{03} = 2\,Z_{01}$), would have this design:



Along with this **scattering matrix:**

$$\mathcal{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$