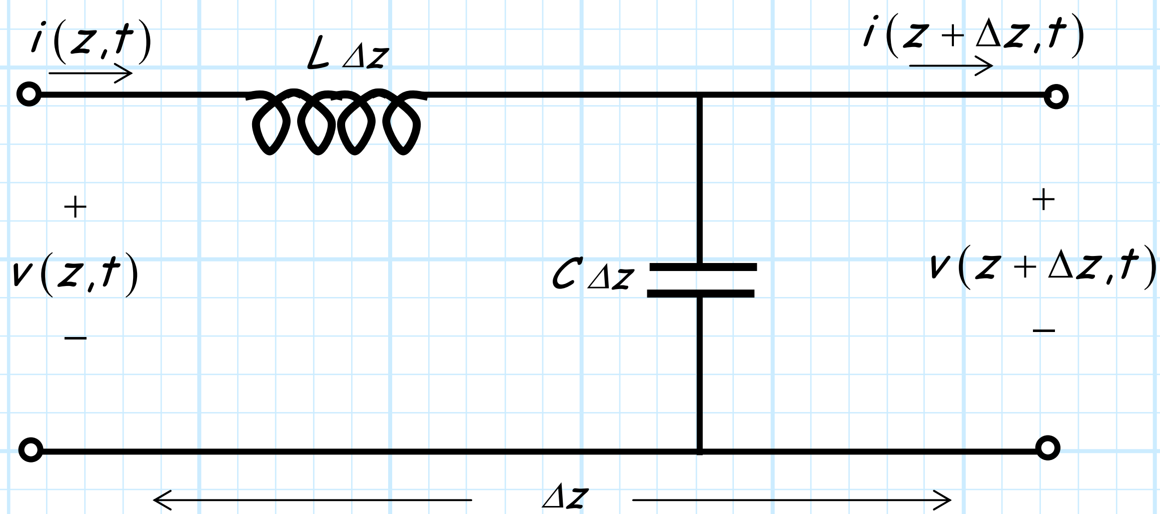
The Lossless Transmission Line

Say a transmission line is **lossless** (i.e., $R = G = 0$).

Thus, this lossless transmission line is a **purely reactive** two port device—it exhibits only **capacitance** and **inductance**!!!



As a result, the transmission line equations are then **significantly** simplified!

The characteristic impedance of the lossless transmission line

For example, the **characteristic impedance** of a lossless lines simply becomes:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

Ironically, the characteristic **impedance** of a **lossless** (i.e., purely reactive) transmission line is—purely **real**!

The propagation constant

Moreover, the **propagation constant** of a lossless line is purely **imagingary**:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(j\omega L)(j\omega C)} = \sqrt{-\omega^2 LC} = j\,\omega\sqrt{LC}$$

In other words, for a **lossless** transmission line:

$$\alpha = 0 \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

Note that since $\alpha = 0$, **neither** propagating wave is **attenuated** as they travel down the line—a wave at the **end** of the line is as large as it was at the **beginning**!

→ And this **makes sense**!

Wave attenuation occurs when **energy is extracted** from the propagating wave and turned into **heat**.

This can **only** occur if resistance and/or conductance are present in the line.

If $R = G = 0$, then **no attenuation** occurs—that why we call the line **lossless**.

Velocity and Wavelength

The **complex functions** describing the magnitude and phase of the **voltage/current** at every location z along a transmission line are for a **lossless** line are:

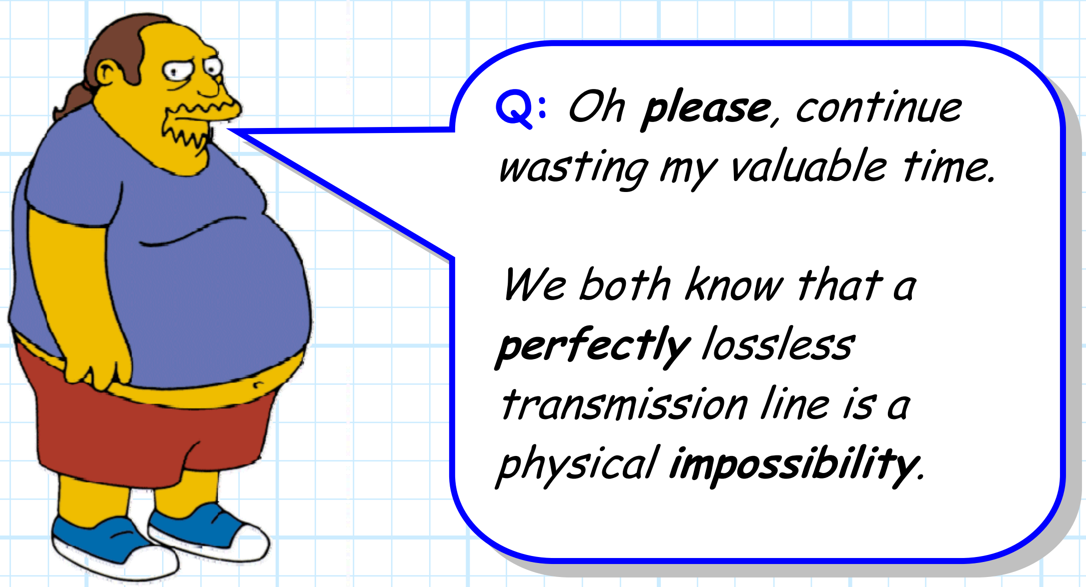
$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

We can now **explicitly** write the **wavelength** and propagation **velocity** of the two transmission line waves in terms of transmission line parameters L and C:

$$\lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{LC}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

The low-loss approximation



A: True! However, a **low-loss** line **is** possible—in fact, it is **typical**!

If $R \ll \omega L$ and $G \ll \omega C$, we find that the lossless transmission line equations are excellent **approximations**!

Unless **otherwise** indicated, **we will use the lossless equations** to **approximate** the behavior of a **low-loss** transmission line.

The lone **exception** is when determining the attenuation of a **long** transmission line. For that case we will use the approximation:

$$\alpha \approx \frac{1}{2}\left(\frac{R}{Z_0} + G Z_0\right)$$

where $Z_0 = \sqrt{L/C}$.