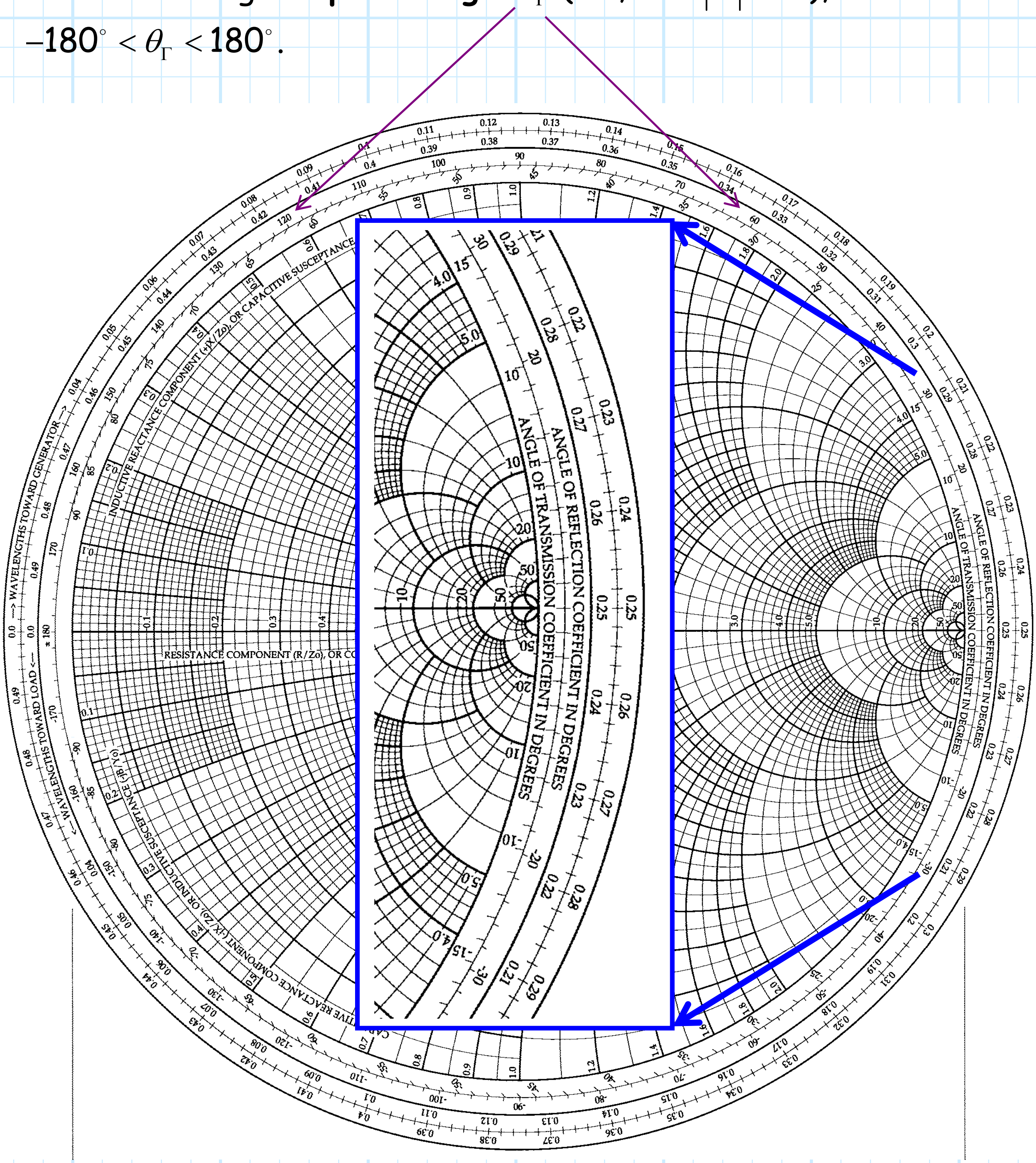
The Outer Scale

Note that around the **outside** of the Smith Chart there is a scale indicating the **phase angle** θ_Γ (i.e., $\Gamma = |\Gamma| e^{j\theta_\Gamma}$), from $-180^\circ < \theta_\Gamma < 180^\circ$.



Recall however, for a **terminated** transmission line, the reflection coefficient function is:

$$\Gamma(z) = \Gamma_0\, e^{j2\beta z} = |\Gamma_0|\, e^{j\,2\beta z + \theta_0}$$

Thus, the **phase** of the reflection coefficient function depends on transmission line **position** z as:

$$\theta_\Gamma(z) = 2\beta z + \theta_0 = 2\left(\frac{2\pi}{\lambda}\right) z + \theta_0 = 4\pi\left(\frac{z}{\lambda}\right) + \theta_0$$

As a result, a **change** in line position z (i.e., Δz) results in a **change** in reflection coefficient phase θ_Γ (i.e., $\Delta\theta_\Gamma$):

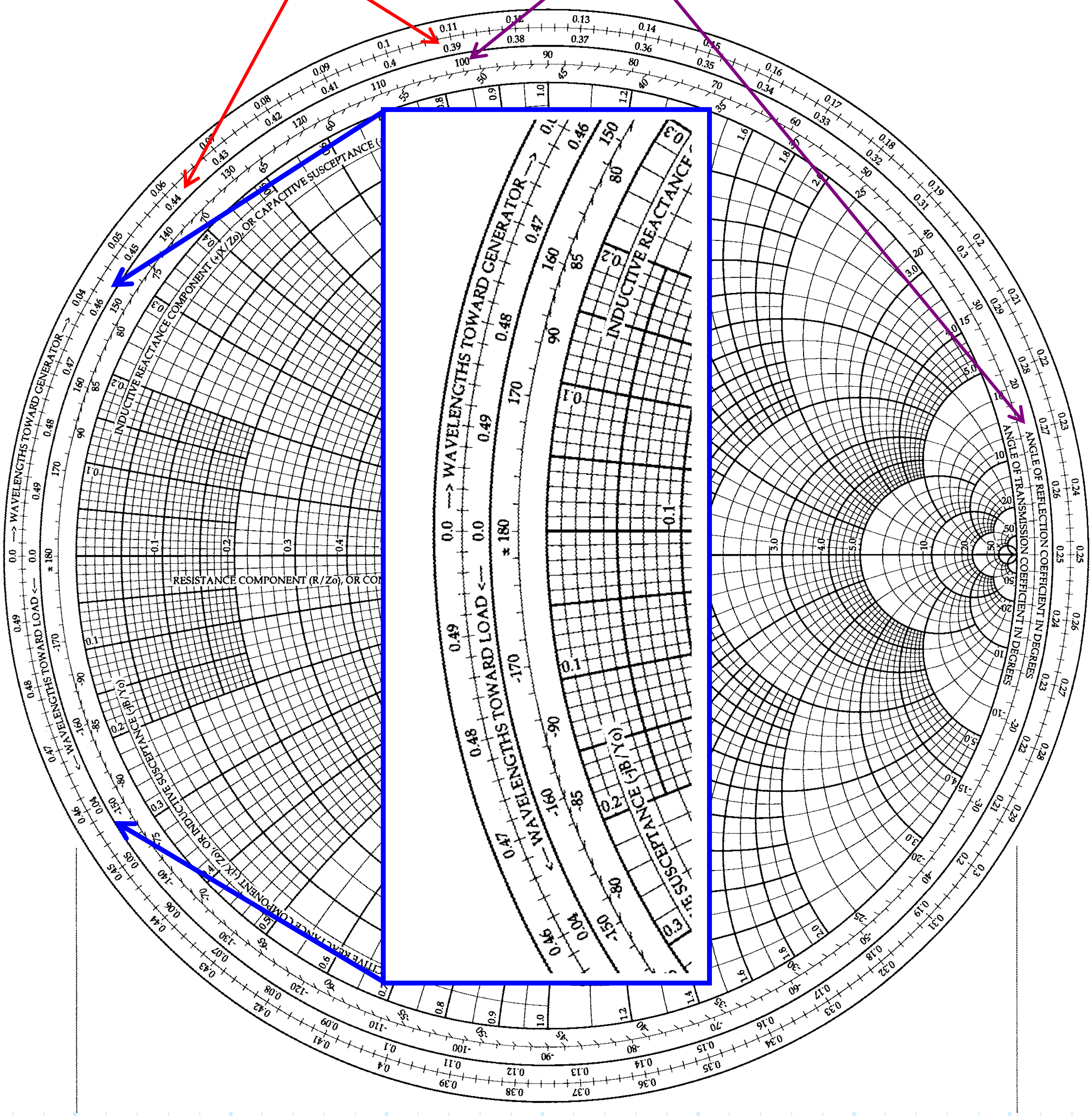
$$\Delta\theta_\Gamma = 4\pi\left(\frac{\Delta z}{\lambda}\right)$$

For example, a change of position equal to one-quarter wavelength $\Delta z = \lambda/4$ results in a phase change of π radians—we rotate **half-way** around the complex Γ plane (otherwise known as the Smith Chart).

Or, a change of position equal to one-half wavelength $\Delta z = \lambda/2$ results in a phase change of 2π radians—we rotate **completely** around the complex Γ plane (otherwise known as the Smith Chart).

The Smith Chart then has a **second scale** (besides θ_Γ) that surrounds it—one that relates transmission line position in **wavelengths** (i.e., $\Delta z/\lambda$) to the reflection coefficient phase:

$$\frac{z}{\lambda} = \frac{1}{4} + \frac{\theta_\Gamma}{4\pi} \qquad \Leftrightarrow \qquad \theta_\Gamma = 4\pi\left(\frac{z}{\lambda} - \frac{1}{4}\right)$$



Since the phase scale on the Smith Chart extends from $-180^\circ < \theta_\Gamma < 180^\circ$ (i.e., $-\pi < \theta_\Gamma < \pi$), this **electrical length scale** extends from:

$$0 < z/\lambda < 0.5$$

Note for this mapping the reflection coefficient phase at location $z = 0$ is $\theta_L = -\pi$. Therefore, $\theta_0 = -\pi$, and we find that:

$$\Gamma_0 = |\Gamma_0| e^{j\theta_0} = |\Gamma_0| e^{-j\pi} = -|\Gamma_0|$$

In other words, Γ_0 is a **negative real** value.

Q: *But, Γ_0 could be **anything!** What is the likelihood of Γ_0* being a real and negative value? *Most of the time this is **not** the case—this second Smith Chart scale seems to **be nearly useless**!?*

A: Quite the contrary! This electrical length scale is in fact **very useful**—you just need to understand how to utilize it!

This electrical length scale is very much like the **mile markers** you see along an interstate highway; although the specific numbers are quite arbitrary, they are still very useful.

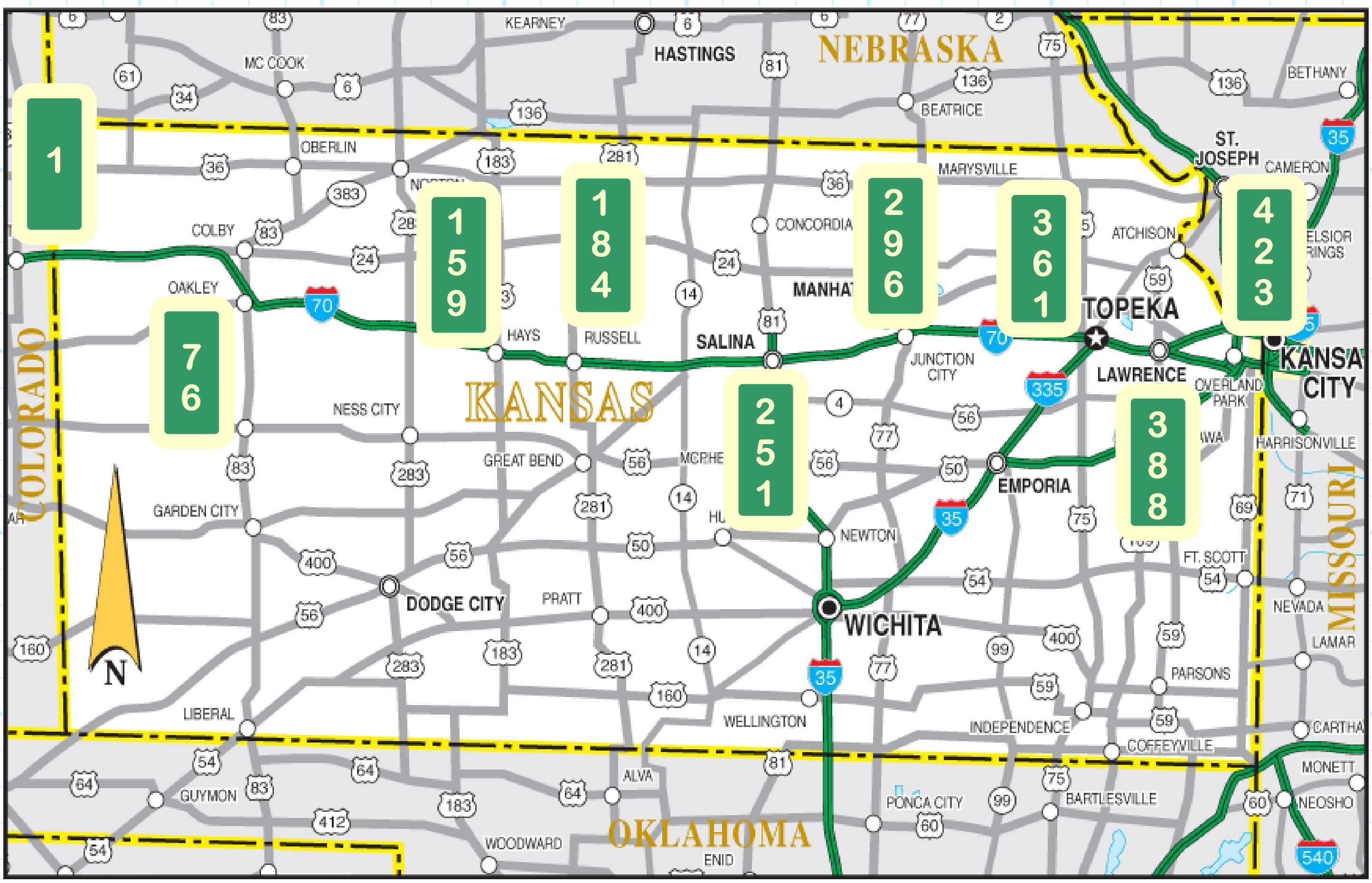
Take for example **Interstate 70**, which stretches across Kansas. The **western end** of I-70 (at the Colorado border) is denoted as **mile 1**.

At each mile along I-70 a new marker is placed, such that the **eastern end** of I-70 (at the **Missouri** border) is labeled **mile 423**— Interstate 70 runs for 423 miles across Kansas!

The location of various towns and burgs along I-70 can thus be specified in terms of these mile markers. For example, along I-70 we find:

Oakley at mile marker 76

Hays at mile marker 159
Russell at mile marker 184
Salina at mile marker 251
Junction City at mile marker 296
Topeka at mile marker 361
Lawrence at mile marker 388



MILES
0 50

So say you are traveling **eastbound** (→) along I-70, and you want to know the distance to **Topeka**. Topeka is at mile marker **361**, but this does **not** of course mean you are **361 miles** from Topeka.

Instead, you **subtract from 361** the value of the mile marker denoting **your position** along I-70.

For **example**, if you find yourself in the lovely borough of **Russell** (mile marker **184**), you have precisely 361-184 = **177 miles** to go before reaching Topeka!

Q: *I'm confused! Say I'm in **Lawrence** (mile marker 388); using **your** logic I am a distance of 361-388 = **-27 miles** from Topeka! How can I be a **negative** distance from something??*

A: The mile markers across Kansas are arranged such that their value **increases** as we move from west to east across the state. Take the value of the mile marker denoting to where you are traveling, and **subtract** from it the value of the mile marker where you are.

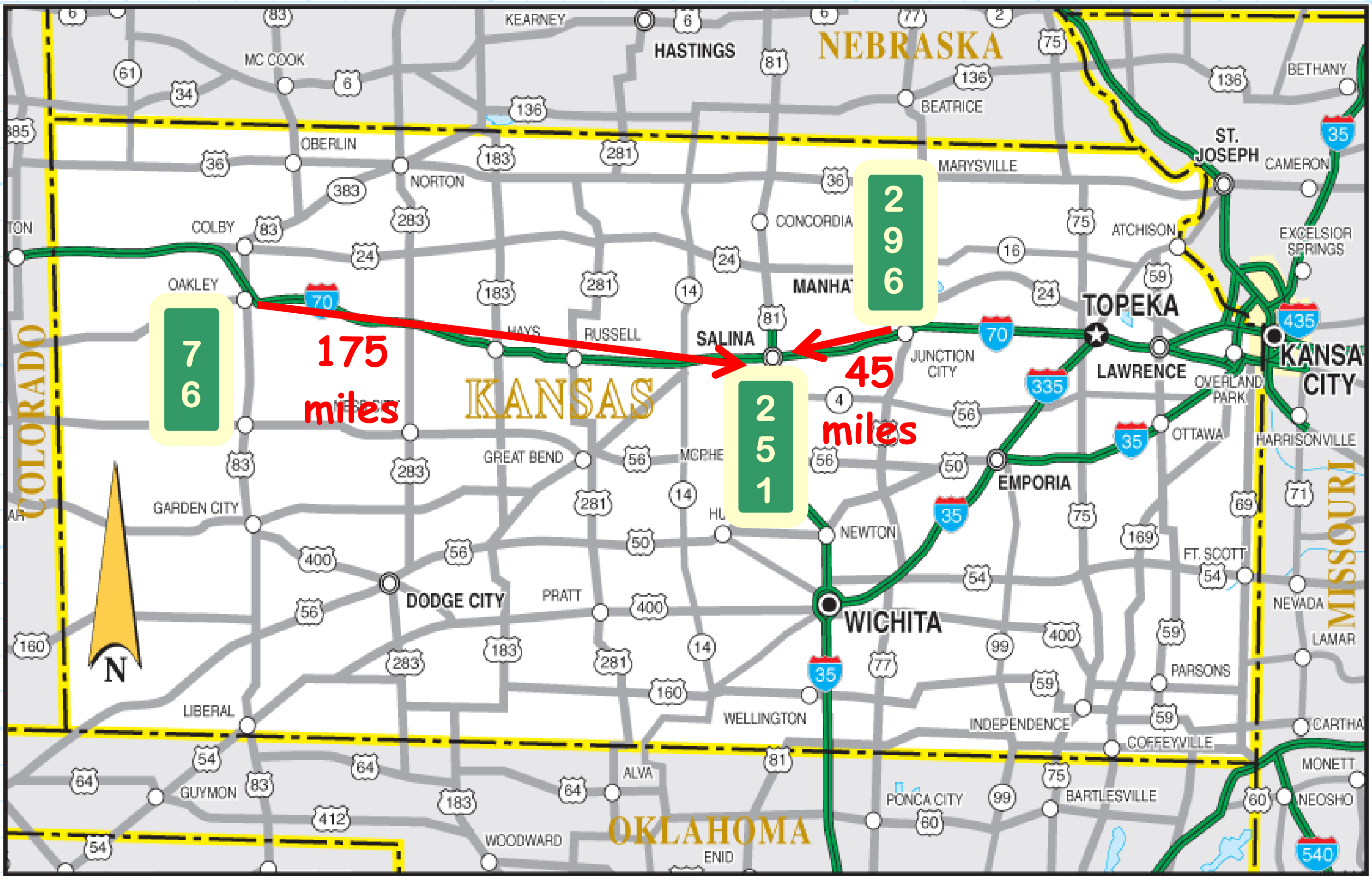
If this value is **positive,** then your destination is **East** of you; if this value is **negative**, it is **West** of your current position (hopefully you're in the westbound lane!).

For example, say you're traveling to Salina (mile marker 251). If you are in Oakley (mile marker 76) then:

251 – 76 = **175** → Salina is 175 miles **East** of Oakley

If, on the other hand, you begin your journey from Junction City (mile marker 296), we find:

251 – 296 = **-45** → Salina is 45 miles **West** of Junction City



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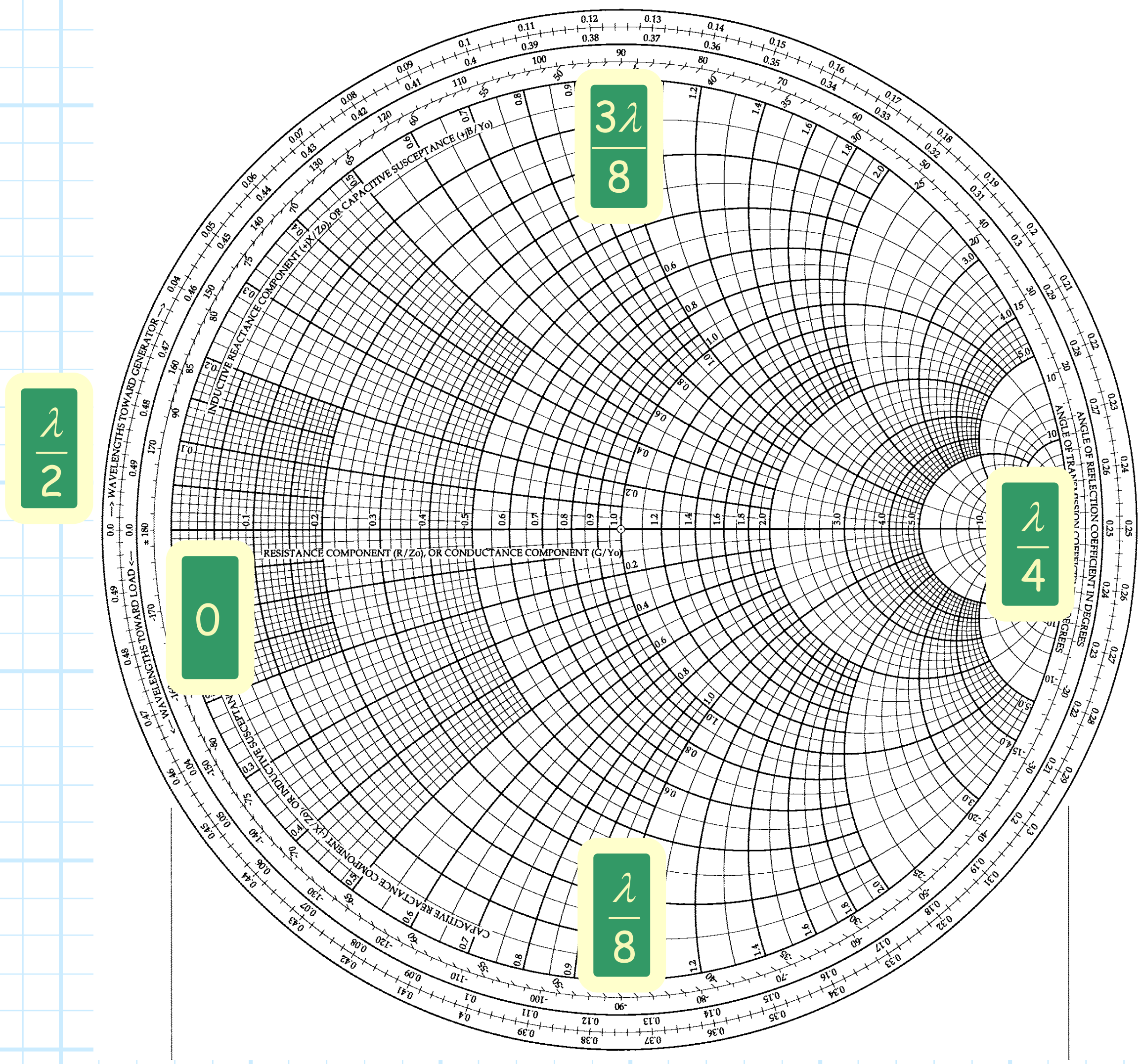
MILES
0 50

Q: *But just what the &()#$@% does this discussion have to do with* ***SMITH CHARTS*** *!!?!?*

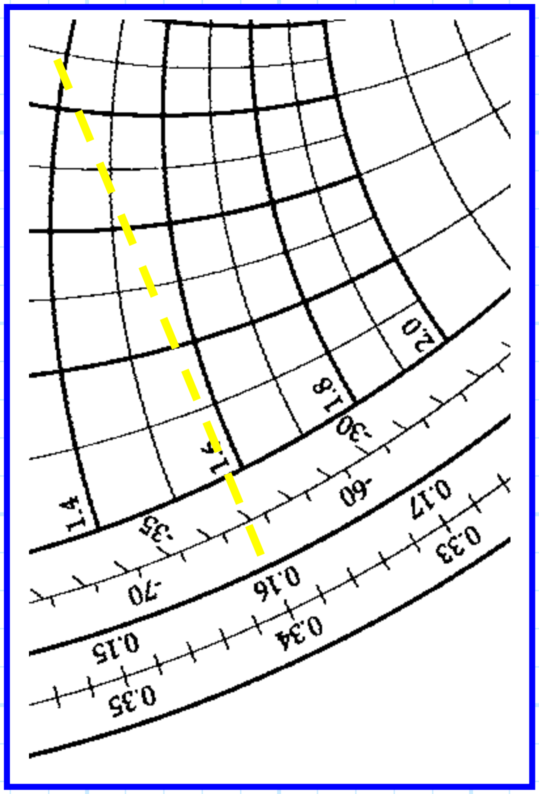
A: The electrical length scale (z/λ) around the perimeter of the Smith Chart is precisely **analogous** to mile markers along an interstate!



Recall that the change in **phase** ($\Delta\theta_\Gamma$) of the reflection coefficient function is related to the change in **distance** (Δz) along a transmission line as:

$$\Delta\theta_\Gamma = 4\pi\left(\frac{\Delta z}{\lambda}\right)$$

The value $\Delta z/\lambda$ can be determined from the **outer scale** of the Smith Chart, simply by taking the **difference** of the two "mile markers" values.



For **example**, say you're at some location $z = z_1$ along a transmission line. The value of the **reflection coefficient** function at that point happens to be:

$$\Gamma(z = z_1) = 0.685\, e^{-j65^\circ}$$

Finding the **phase angle** of $\theta_\Gamma = -65^\circ$ on the **outer scale** of the Smith Chart, we note that the corresponding **electrical length** value is:

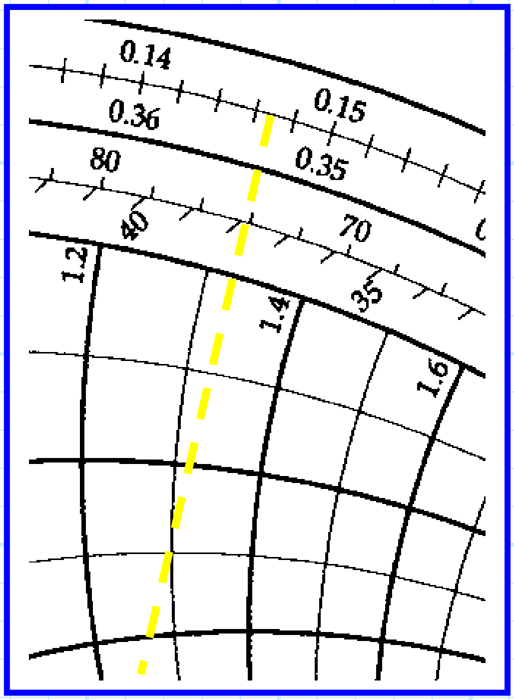
$$0.160\lambda$$

Note this tells us **nothing** about the location $z = z_1$. This does **not** mean that $z_1 = 0.160\lambda$, for example!

Now, say we **move a short distance** Δz (i.e., a distance less than $\lambda/2$) along the transmission line, to a **new location** denoted as $z = z_2$.

We find that this new location that the **reflection coefficient** function has a value of:

$$\Gamma(z = z_2) = 0.685\, e^{+j74^\circ}$$



Now finding the **phase angle** of $\theta_\Gamma = +74^\circ$ on the **outer scale** of the Smith Chart, we note that the corresponding **electrical length** value is:

$$0.353\lambda$$

Note this tells us **nothing** about the location $z = z_2$. This does **not** mean that $z_1 = 0.353\lambda$, for example!

Q: *So what do the values* 0.160λ *and* 0.353λ *tell us?*

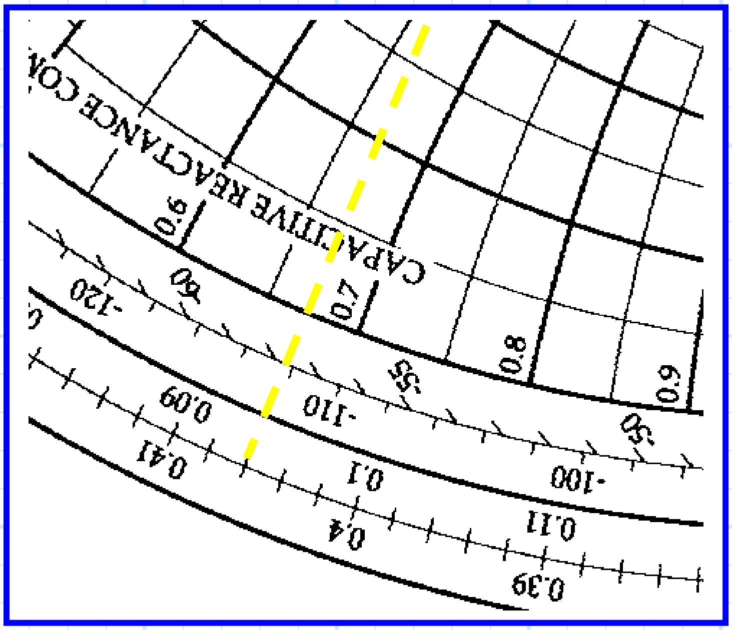
A: They allow us to determine the **distance between** points z_2 and z_1 on the transmission line:

$$\frac{\Delta z}{\lambda} = \frac{z_2}{\lambda} - \frac{z_1}{\lambda} \quad !!!$$

Thus, for this example, the **distance between** locations z_2 and z_1 is:

$$\Delta z = 0.353\lambda - 0.160\lambda = 0.193\lambda$$

→ The transmission line location z_2 is a distance of 0.193λ from location z_1!



Q: *But, say the reflection coefficient at some point z_3 has a phase value of $\theta_\Gamma = -112^\circ$. This maps to a value of:*

$$0.094\lambda$$

on the outer scale of the Smith Chart.

*The **distance** between z_3 and z_1 would then turn out to be:*

$$\frac{\Delta z}{\lambda} = 0.094 - 0.160 = -0.066$$

*What does the **negative** value mean??*

A: Just like our I-70 mile marker analogy, the **sign** (plus or minus) indicates the **direction** of movement from one point to another.

In the first example, we find that $\Delta z > 0$, meaning $z_2 > z_1$:

$$z_2 = z_1 + 0.094\lambda$$

Clearly, the location z_2 is further down the transmission line (i.e., **closer to the load**) than is location z_1.

For the second example, we find that $\Delta z < 0$, meaning $z_3 < z_1$:

$$z_3 = z_1 - 0.066\lambda$$

Conversely, in this second example, the location z_3 is **closer to the beginning** of the transmission line (i.e., farther from the load) than is location z_1.

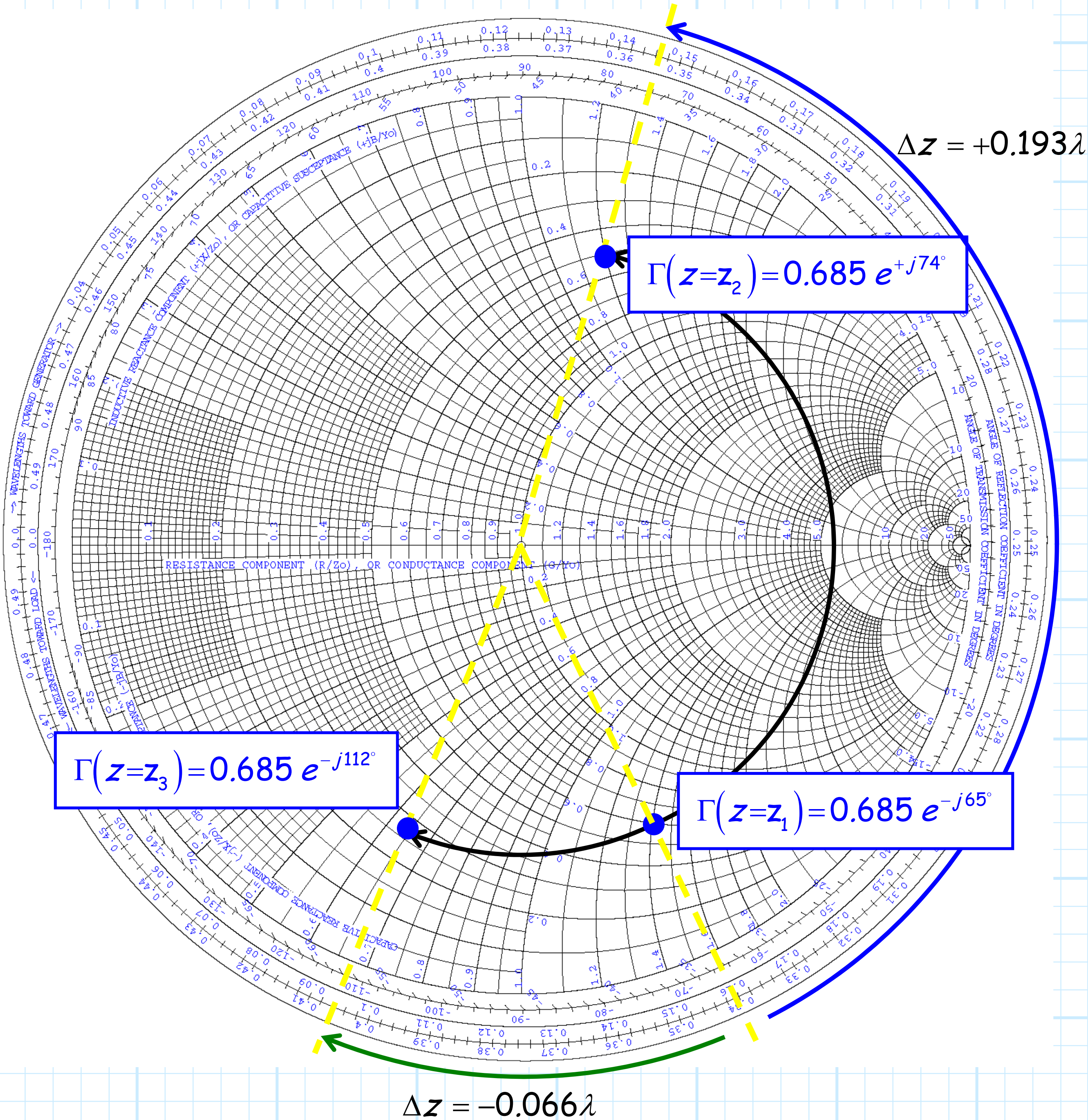
This is completely **consistent** with what we **already** know to be true!

In the first case, the **positive** value $\Delta z = 0.193\lambda$ maps to a phase change of $\Delta\theta_\Gamma = 74^\circ - \left(-65^\circ\right) = 139^\circ$.

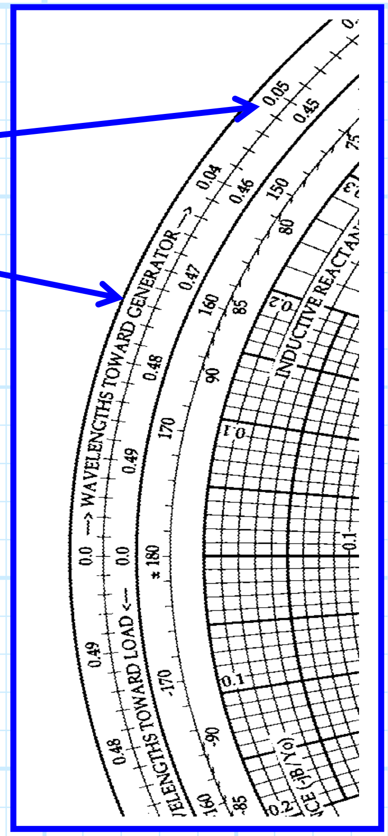
In other words, as we move **toward the load** from location z_1 to location z_2, we **rotate counter-clockwise** around the Smith Chart.

Likewise, the **negative** value $\Delta z = -0.066\lambda$ maps to a phase change of $\Delta\theta_\Gamma = -112^\circ - \left(-65^\circ\right) = -47^\circ$.

In other words, as we move **away from the load** (toward the source) from a location z_1 to location z_3, we **rotate clockwise** around the Smith Chart.



$\Delta z = +0.193\lambda$
$\Gamma(z=z_2) = 0.685\, e^{+j74^\circ}$
$\Gamma(z=z_3) = 0.685\, e^{-j112^\circ}$
$\Gamma(z=z_1) = 0.685\, e^{-j65^\circ}$
$\Delta z = -0.066\lambda$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)



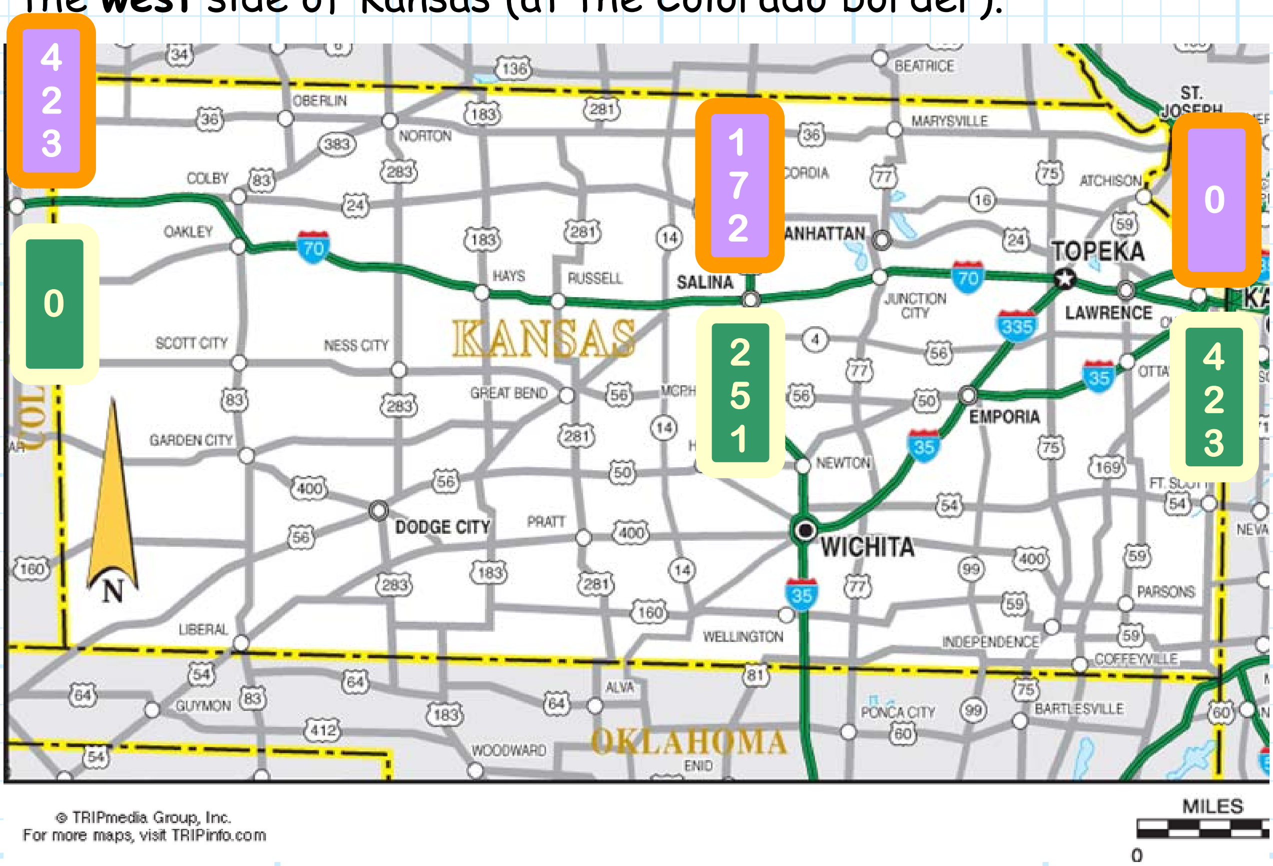
Q: *I notice that there is a **second** electrical length scale on the Smith Chart. Its values increase as we move **clockwise** from an initial value of zero to a maximum value of* 0.5λ *.*

What's up with that?

A: This scale uses an **alternative mapping** between θ_Γ and z/λ:

$$\frac{z}{\lambda} = \frac{1}{4} - \frac{\theta_\Gamma}{4\pi} \qquad \Leftrightarrow \qquad \theta_\Gamma = 4\pi\left(\frac{1}{4} - \frac{z}{\lambda}\right)$$

This scale is **analogous** to a situation wherein a **second set** of mile markers were placed along I-70. These mile markers **begin** at the **east** side of Kansas (at the Missouri border), and **end** at the **west** side of Kansas (at the Colorado border).



Q: *What* ***good*** *would this second set do? Would it serve any purpose?*

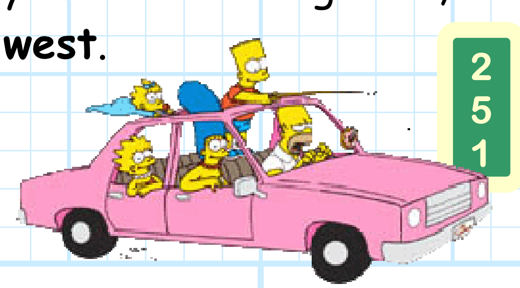
A: **Not much** really. After all, this second set is **redundant**—it does not provide any **new** information that the original set already provides.

Yet, if we were to place this new set along I-70, we almost certainly would place the **original** mile markers along the **eastbound** lanes, and this new set along the **westbound** lanes.

In this manner, all I-70 motorists (eastbound or westbound) would see an **increase** in the mile markers as they traverse the **Sunflower State**.

As a result, a **positive** distance to their destination indicates to **all** drivers that their destination is in **front** of them (in the direction they are driving), while a **negative** distance indicates to **all** drivers that their destination is **behind** the (they better **turn around**!).

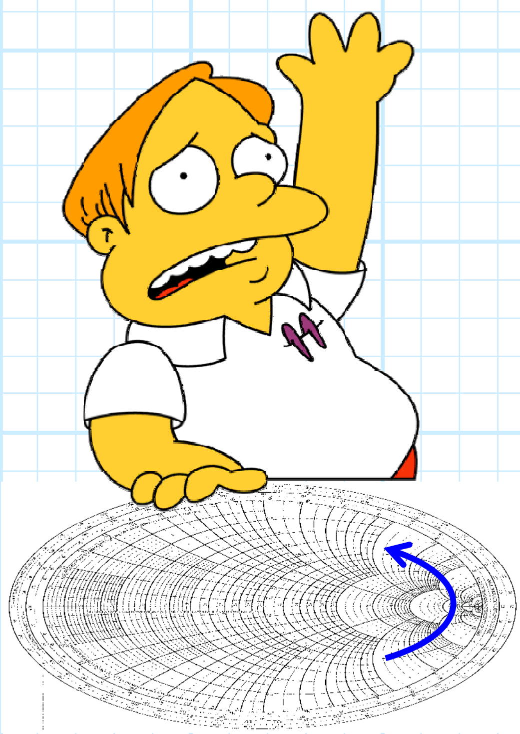
Thus, it could be argued that each set of mile markers is **optimized for a specific direction** of travel—the original set if you are traveling **east**, and this second set if you are traveling **west**.



Similarly, the two electrical length scales on the Smith Chart are meant for two different "directions of travel". If we move down the transmission line **toward the load**, the value Δz will be **positive**.

Conversely, if we move up the transmission line and **away from the load** (i.e., "toward the generator"), this second electrical length scale will also provide a **positive** value of Δz.

Again, these two electrical length scales are **redundant**—you will get the correct answer **regardless** of the scale you use, but be careful to interpret negative signs properly.



Q: *Wait! I just used a Smith Chart to analyze a transmission line problem in the manner you have just explained. At one point on my transmission line the phase of the reflection coefficient is* $\theta_\Gamma = +170^\circ$*, which is denoted as* 0.486λ *on the **"wavelengths toward load"** scale.*

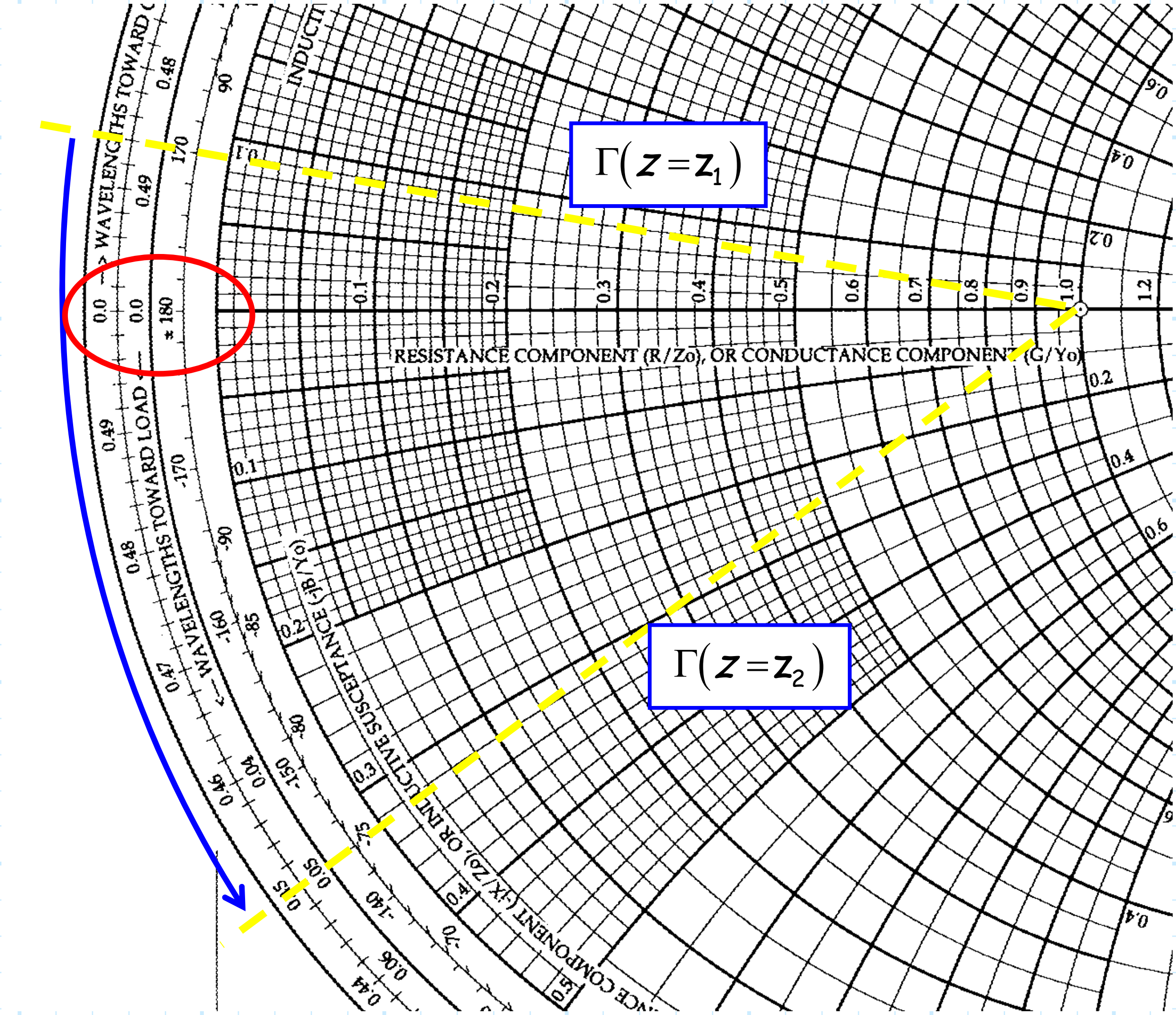
*I then moved a short distance along the line **toward the load**, and found that the reflection coefficient phase was* $\theta_\Gamma = -144^\circ$*, which is denoted as* 0.050λ *on the "wavelengths toward load" scale.*

*According to **your** "instruction", the distance between these two points is:*

$$\Delta z = 0.050\lambda - 0.486\lambda = -0.436\lambda$$

*A large **negative** value! This says that I moved nearly a half wavelength **away** from the load, but I know that I moved just a short distance **toward** the load! **What happened?***

A: Note the electrical length scales on the Smith Chart **begin and end** where $\theta_\Gamma = \pm\pi$ (by the short circuit!).



In your example, when rotating counter-clockwise around the chart (i.e., moving toward the load) you **passed by** this **transition**. This makes the calculation of Δz a bit more problematic.

To see why, let's again consider our **I-70 analogy**. Say we are Lawrence, and wish to drive eastbound on Interstate 70 until we reach **Columbia, Missouri**.

The mile marker for Lawrence is of course **388**, and Columbia Missouri is located at mile marker **126**. We **might** conclude that the distance from Lawrence to Columbia is:

$$126-388=-262 \text{ miles}$$

Q: *Yikes! According to this, Columbia is 262 miles **west** of Lawrence—should we turn the **car around**?*

A: Columbia, Missouri is most decidedly **east** of Lawrence, Kansas. The calculation above is **incorrect**. The problem is that mile markers **"reset"** once we reach a **state border**. Once we hit the Missouri-Kansas border, the mile markers reset to **zero**, and then again **increase** as we travel eastward.

Thus, to **accurately** determine the **distance** between Lawrence and Columbia, we need to break the problem into **two steps**:

Step 1: Determine the distance between **Lawrence** (mile marker 388) , and the **last mile marker** before the state line (mile marker 423):

$$423 - 388 = 35 \text{ miles}$$

Step 2: Determine the distance between the **first mile marker** after the state line (mile marker 0) and **Columbia** (mile marker 126):

$$126 - 0 = 126 \text{ miles}$$

Thus, the distance between Lawrence and Columbia is the distance between Lawrence and the state line (35 miles), **plus** the distance from the state line to Columbia (126 miles):

$$35 + 126 = 161 \text{ miles}$$

Columbia, Missouri is **161 miles east** of Lawrence, Kansas!

Now back to the **Smith Chart problem**; as we rotate counter-clockwise around the Smith Chart, the "wavelengths toward load" scale increases in value, until it reaches a **maximum** value of 0.5λ (at $\theta_\Gamma = \pm\pi$) .

At that point, the scale "resets" to its **minimum** value of **zero**. We have **metaphorically** "crossed the state line" of this scale.

Thus, to accurately determine the electrical length moved along a transmission line, we must divide the problem into **two steps**:

Step 1: Determine the electrical length from the **initial** point to the **"end"** of the scale at 0.5λ.

Step 2: Determine the electrical distance from the **"beginning"** of the scale (i.e., 0) and the **second location** on the transmission line.

Add the results of steps 1 and 2, and you have your answer!

For **example**, let's look at the case that originally gave us the erroneous result. The distance from the initial location to the **end of the scale** is:

$$0.500\lambda - 0.486\lambda = +0.014\lambda$$

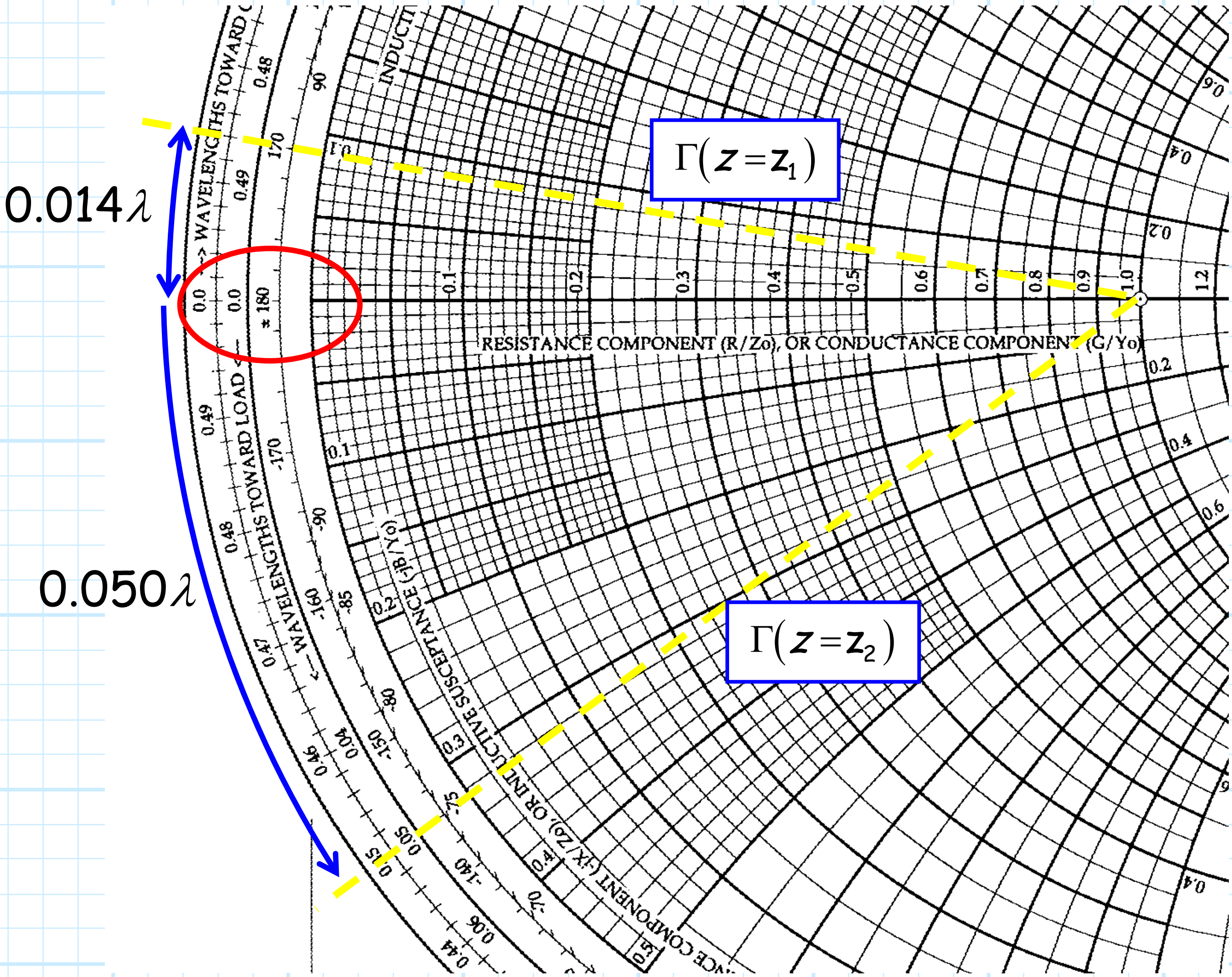
And the distance from the **beginning of the scale** to the second point is:

$$0.050\lambda - 0.000\lambda = +0.050\lambda$$

Thus the distance between the two points is:

$$0.014\lambda + 0.050\lambda = +0.064\lambda$$

The second point is just a little closer to the load than the first!



0.014λ
0.050λ
$\Gamma(z=z_1)$
$\Gamma(z=z_2)$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD LOAD