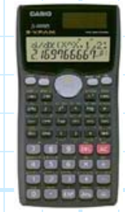
The Propagation Series

Q: *You earlier stated that signal flow graphs are helpful in (count em') **three** ways. I now understand the **first** way:*

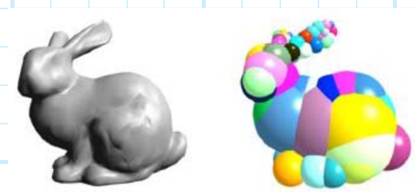
"**Way 1** - Signal flow graphs provide us with a **graphical** means of **solving** large systems of simultaneous equations."



But what about ways 2 and 3 ??

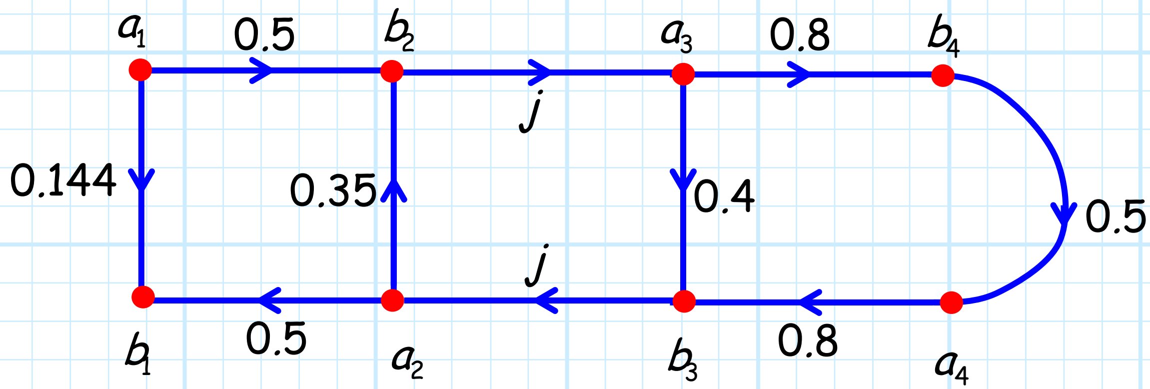
"**Way 2** – We'll see the a signal flow graph can provide us with a **road map** of the wave **propagation paths** throughout a microwave device or network."

"**Way 3** - Signal flow graphs provide us with a quick and accurate method for **approximating** a network or device."



bunny, 64 spheres

A: Consider the *sfg* below:

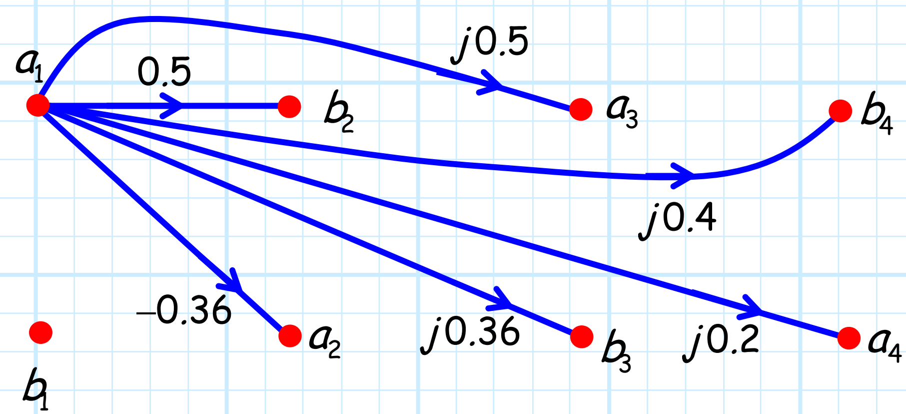


Note that node a_1 is the only **independent** node. This signal flow graph is for a rather complex **single-port** (port 1) device.

Say we wish to determine the wave amplitude **exiting port 1**. In other words, we seek:

$$b_1 = \Gamma_{in}\, a_1$$

Using our four **reduction rules**, the signal flow graph above is simplified to:



Q: *Hey, node b_1 is* ***not connected*** *to anything. What does this mean?*

A: It means that $b_1 = 0$—**regardless** of the value of incident wave a_1. I.E.,:

$$\Gamma_{in} = \frac{b_1}{a_1} = 0$$

In other words, port 1 is a **matched load!**

Q: *But look at the* ***original*** *signal flow graph; it doesn't* ***look*** *like a matched load. How can the exiting wave at port 1 be* ***zero****?*

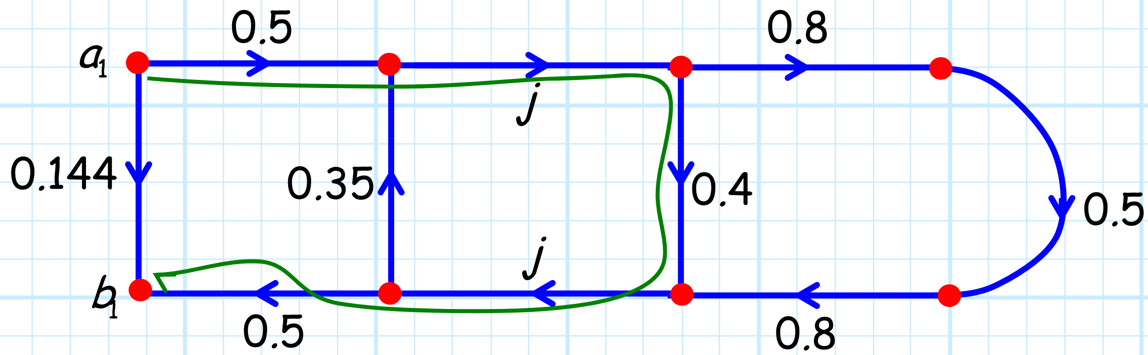
A: A signal flow graph provides a bit of a **propagation road map** through the device or network. It allows us to understand—often in a **very** physical way—the **propagation** of an incident wave once it enters a device.

We accomplish this by identifying from the *sfg* **propagation paths** from an independent node to some other node (e.g., an exiting node). These paths are simply a **sequence of branches** (pointing in the correct direction!) that lead from the independent node to this other node.

Each path has **value** that is equal to the **product** of each branch of the path.

Perhaps this is best explained with some **examples**.

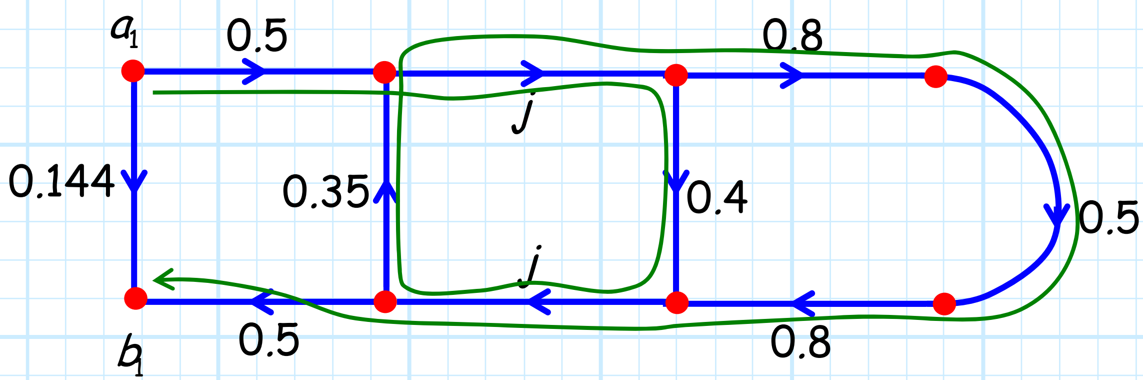
One **path** between independent (incident wave) node a_1 and (exiting wave) node b_1 is shown below:



We'll **arbitrarily** call this path 2, and its value:

$$p_2 = (0.5)\,j\,(0.4)\,j\,(0.5) = -0.1$$

Another propagation **path** (path 5, say) is:



$$
\begin{aligned}
p_5 &= (0.5)\,j\,(0.4)\,j\,(0.35)\,j\,(0.8)(0.5)(0.8)\,j\,(0.5) \\
&= j^4\,(0.35)(0.4)(0.8)^2\,(0.5)^3 \\
&= 0.0112
\end{aligned}
$$

Q: *__Why__ are we doing this?*

A: The exiting wave at port 1 (wave amplitude b_1) is simply the **superposition** of **all** the propagation paths from incident node a_1! Mathematically speaking:

$$
b_1 = a_1 \sum_n p_n \qquad \Rightarrow \qquad \Gamma_{in} = \frac{b_1}{a_1} = \sum_n p_n
$$

Q: *Won't there be an awful __lot__ of propagation paths?*

A: Yes! As a matter of fact there are an **infinite** number of paths that connect node a_1 and b_1. Therefore:

$$
b_1 = a_1 \sum_n^{\infty} p_n \qquad \Rightarrow \qquad \Gamma_{in} = \frac{b_1}{a_1} = \sum_n^{\infty} p_n
$$

Q: *Yikes! Does this infinite series __converge__?*

A: Note that the series represents a finite physical value (e.g., Γ_{in}), so that the infinite series **must** converge to the correct **finite** value.

Q: *In this example we found that $\Gamma_{in} = 0$. This means that the infinite propagation series is likewise* ***zero****:*

$$\Gamma_{in} = \sum_{n}^{\infty} p_n = 0$$

Do we conclude from this that ***all*** *propagation paths are* ***zero****:*

$$p_n = 0 \quad ?????$$

A: Absolutely **not**! Remember, we have already determined that $p_2 = -0.1$ and $p_4 = 0.0112$—definitely **not** zero-valued! In fact for this example, **none** of the propagation paths p_n are precisely equal to zero!

Q: *But then* ***why*** *is:*

$$\sum_{n}^{\infty} p_n = 0 \ ???$$

A: Remember, the path values p_n are **complex**. A **sum** of **non-zero** complex values can equal **zero** (as it apparently does in this case!).

Thus, a **perfectly rational** way of viewing this network is to conclude that there are an **infinite number of non-zero waves exiting port 1**:

$$\Gamma_{in} = \sum_{n}^{\infty} p_n \qquad \text{where } p_n \neq 0$$

It just so happens that these waves **coherently add** together to **zero**:

$$\Gamma_{in} = \sum_{n}^{\infty} p_n = 0$$

—they essentially **cancel each other out** !

Q: *So, I now appreciate the fact that signal flow graphs: 1) provides a **graphical method** for solving linear equations and 2) also provides a method for **physically evaluating** the wave **propagation paths** through a network/device.*

*But what about helpful **Way 3**:*

"Way 3 - Signal flow graphs provide us with a quick and accurate method for **approximating** a network or device." ??

bunny, 64 spheres

A: The propagation series of a microwave network is very analogous to a **Taylor Series** expansion:

$$f(x) = \sum_{n=0}^{\infty} \left. \frac{d^n f(x)}{d x^n} \right|_{x=a} (x-a)^n$$

Note that there likewise is a **infinite** number of terms, yet the Taylor Series is quite helpful in engineering.

Often, we engineers simply **truncate** this infinite series, making it a finite one:

$$f(x) \approx \sum_{n=0}^{N} \left. \frac{d^n f(x)}{d x^n} \right|_{x=a} (x-a)^n$$

Q: *Yikes! Doesn't this result in* ***error****?*

A: Absolutely! The truncated series is an **approximation**.

We have **less** error if **more** terms are retained; more **error** if fewer **terms** are retained.

The trick is to retain the **"significant"** terms of the infinite series, and **truncate** those less important "insignificant" terms. In this way, we seek to form an **accurate** approximation, using the **fewest** number of terms.

Q: *But how do we know **which** terms are significant, and which are **not**?*

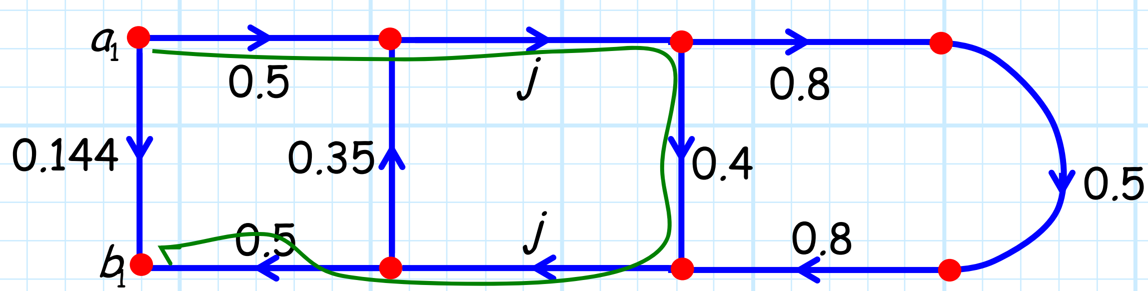
A: For a Taylor Series, we find that as the **order** ***n*** increases, the significance of the term generally (but **not** always!) decreases.

Q: *But what about our **propagation series**? How can we determine which paths are **"significant"** in the series?*

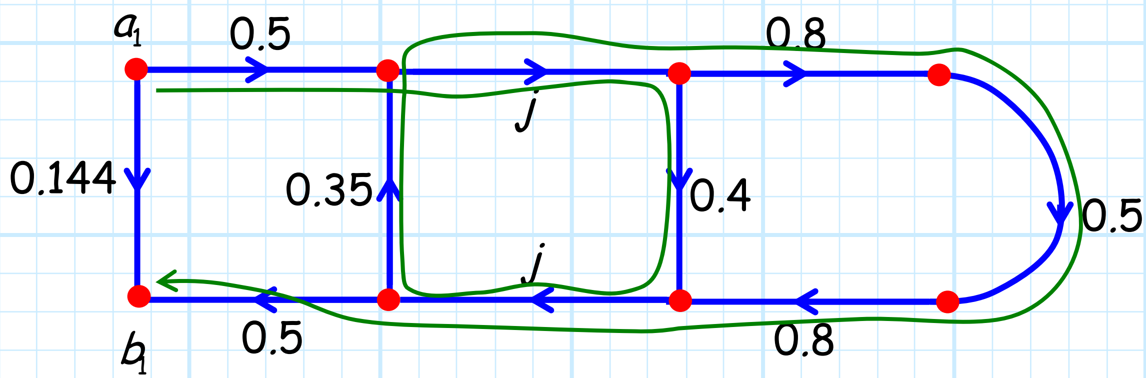
A: Almost always, the most significant paths in a propagation series are the **forward paths** of a signal flow graph.

> **forward path -** \'fȯr-wərd' päth\ *-noun*
>
> A path through a signal flow graph that passes through any given node no more than once. A path that passes through any node two times (or more) is therefore *not* a forward path.

In our example, **path 2** is a forward path. It passes through **four** nodes as it travels from node a_1 to node b_1, but it passes through each of these nodes **only once**:



Alternatively, path 5 is **not** a forward path:



We see that path 5 passes through six different nodes as it travels from node a_1 to node b_1. However, it **twice passes** through four of these nodes.

The good news about forward paths is that there are **always** a **finite** number of them. Again, these paths are typically the **most significant** in the propagation series, so we can determine an approximate value for *sfg* nodes by considering **only** these forward paths in the propagation series:

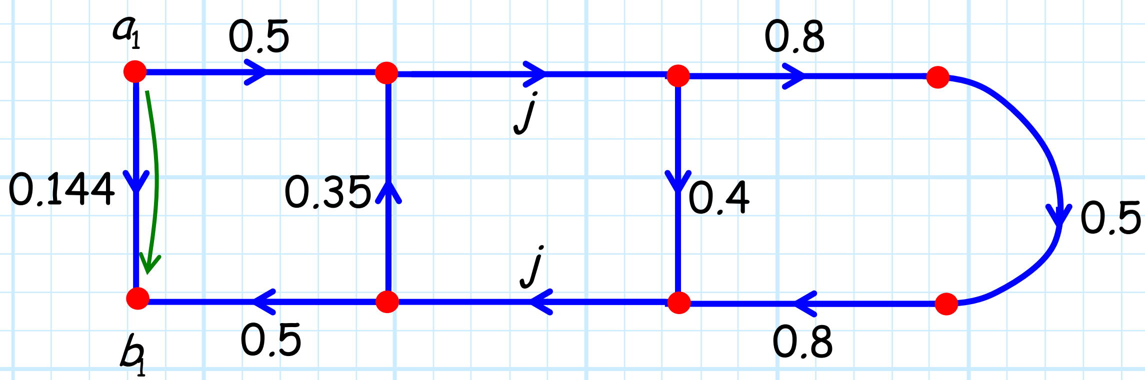
$$\sum_{n}^{\infty} p_n \approx \sum_{n=1}^{N} p_n^{fp}$$

where p_n^{fp} represents the value of one of the N forward paths.

Q: *Is path 2 the **only** forward path in our example sfg ?*

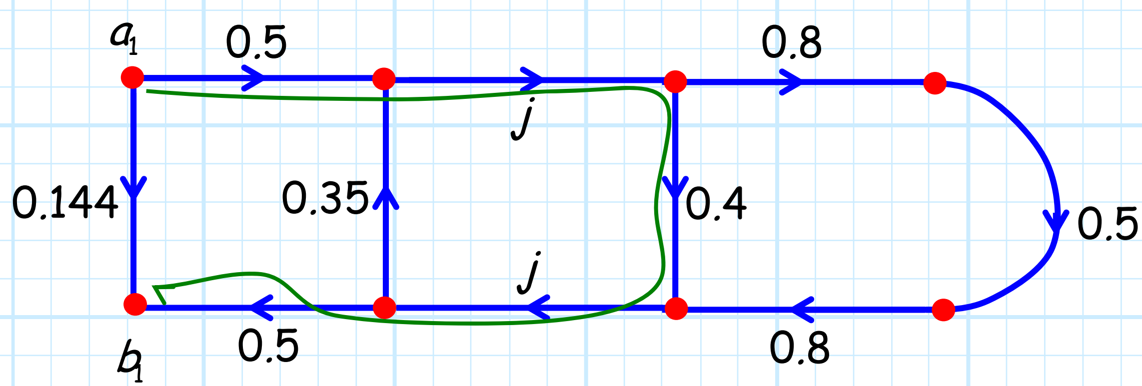
A: No, there are **three**. Path 1 is the most **direct**:

$$p_1 = 0.144$$



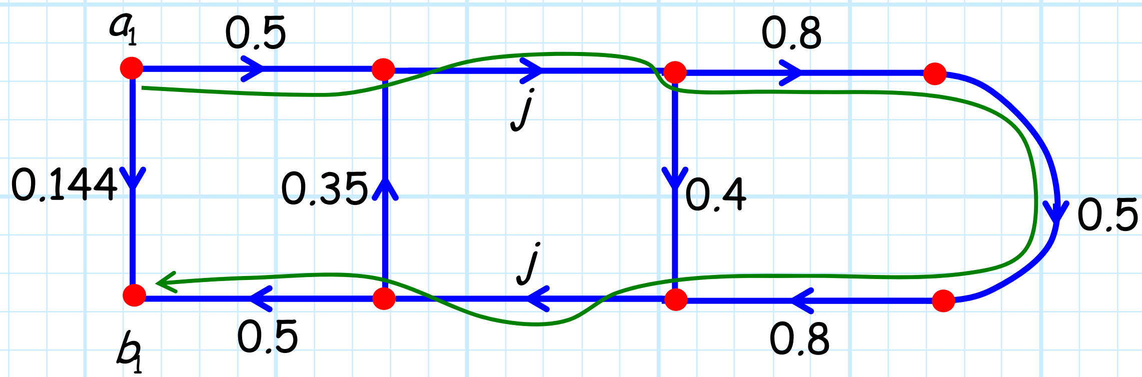
Of course we already have identified path 2:

$$p_2 = -0.1$$



And finally, path 3 is the **longest** forward path:

$$\begin{aligned} p_3 &= (0.5)\, j\, (0.8)(0.5)(0.8)\, j\, (0.5) \\ &= j^2 (0.8)^2 (0.5)^3 \\ &= -0.08 \end{aligned}$$



Thus, an **approximate** value of Γ_{in} is:

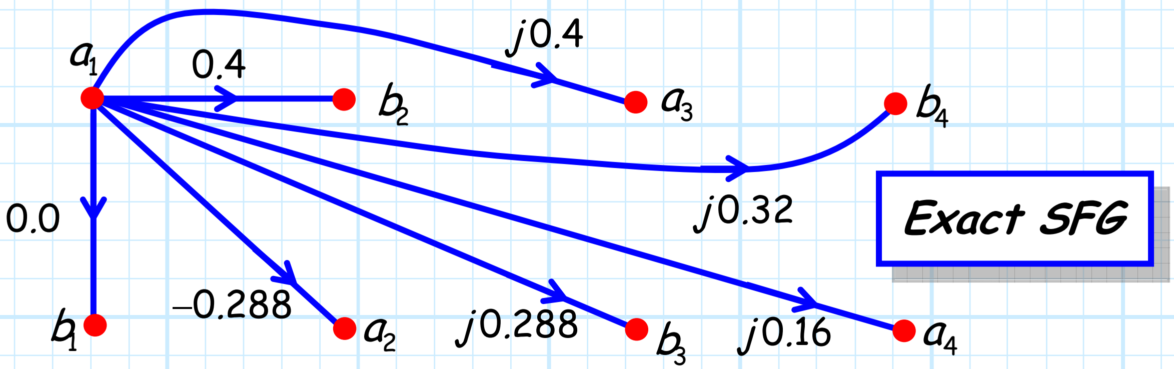
$$\begin{aligned} \Gamma_{in} &= \frac{b_1}{a_1} \\ &\approx \sum_{n=1}^{3} p_n^{fp} \\ &= 0.144 - 0.1 - 0.08 \\ &= -0.036 \end{aligned}$$

Q: *Hey wait! We determined earlier that $\Gamma_{in} = 0$, but now your saying that $\Gamma_{in} = -0.036$. **Which** is correct??*

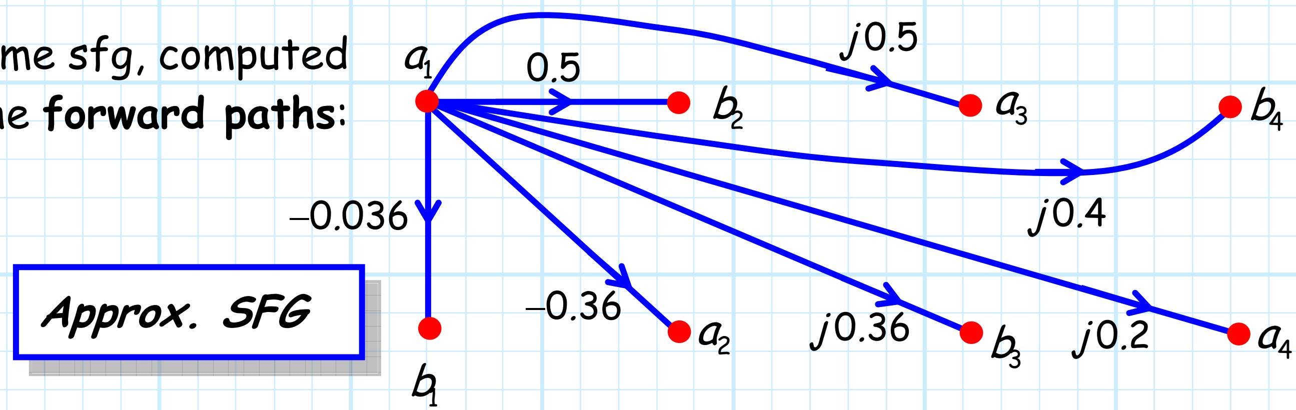
A: The correct answer is $\Gamma_{in} = 0$. It was determined using the four *sfg* reduction rules—**no approximations** were involved!

Conversely, the value $\Gamma_{in} = -0.036$ was determined using a **truncated** form of the propagation series—the series was limited to just the **three** most **significant** terms (i.e., the forward paths). The result is **easier** to obtain, but it is just an approximation (the answers will contain **error**!).

For example, consider the **reduced** signal flow graph (**no** approximation error):



Compare this to the same sfg, computed using only the **forward paths**:



No surprise, the **approximate** *sfg* (using forward paths only) is **not** the same as the **exact** *sfg* (using reduction rules).

The approximate *sfg* contains **error**, but note this error is not **too** bad. The values of the approximate *sfg* are certainly **close** to that of the exact *sfg*.

Q: *Is there any way to **improve** the accuracy of this approximation?*

A: Certainly. The error is a result of truncating the infinite propagation series. Note we **severely** truncated the series—out of an **infinite** number of terms, we retained **only three** (the forward paths). If we retain **more terms**, we will likely get a **more accurate** answer.

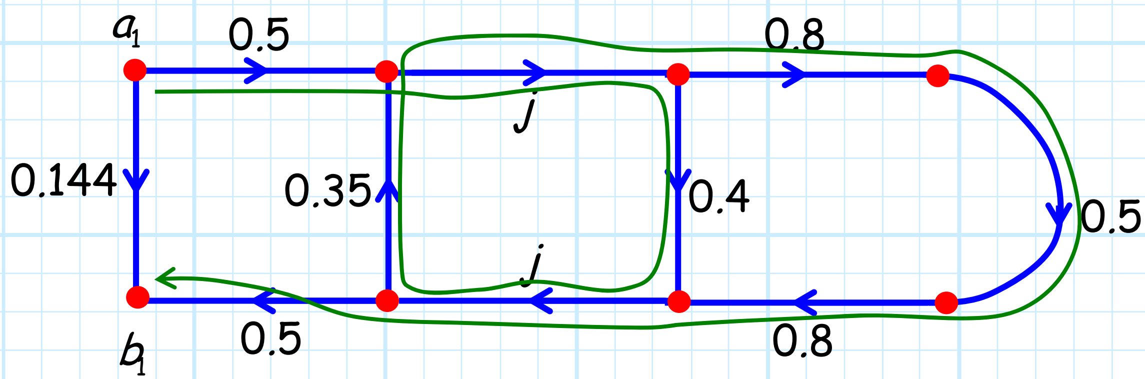
Q: *So why did these approximate answers turn out so **well**, given that we **only** used three terms?*

A: We retained the **three most significant** terms, we will find that the **forward paths** typically have the **largest magnitudes** of all propagation paths.

Q: *Any idea what the **next** most significant terms are?*

A: Yup. The **forward paths** are all those propagation paths that pass through any node no more than **one** time. The next most significant paths are almost certainly those paths that pass through any node no more than **two** times.

Path 4 is an **example** of such a path:



There are **three more** of these paths (passing through a node no more than two times)—see if **you** can find them!

After determining the values for these paths, we can add **4 more terms** to our summation (now we have **seven** terms!):

$$
\begin{aligned}
\Gamma_{in} &= \frac{b_1}{a_1} \\
&\approx \sum_{n=1}^{7} p_n \\
&= \left(p_1 + p_2 + p_3\right) + \left(p_4 + p_5 + p_6 + p_7\right) \\
&= \left(-0.036\right) + \left(0.014 + 0.0112 + 0.0112 + 0.0090\right) \\
&= 0.0094
\end{aligned}
$$

Note this value is **closer** to the correct value of **zero** than was our previous (using only **three** terms) answer of **-0.036**.

As we **add** more terms to the summation, this approximate answer will get **closer** and closer to the correct value of **zero.**
However, it will be **exactly** zero (to an **infinite** number of decimal points) **only** if we sum an **infinite** number of terms!

Q: *The **significance** of a given path seem to be inversely proportional to the **number of times** it passes through any node. Is this true? If so, then **why** is it true?*

A: It **is** true (generally speaking)! A propagation path that travels though a node **ten** times is much **less** likely to be significant to the propagation series (i.e., summation) than a path that passes through any node no more than (say) **four** times.

The reason for this is that the significance of a given term in a summation is dependent on its **magnitude** (i.e., $|p_n|$). If the magnitude of a term is **small**, it will have far **less affect** (i.e., significance) on the sum than will a term whose magnitude is large.

Q: *You seem to be saying that paths traveling through **fewer** nodes have larger **magnitudes** than those traveling through **many** nodes. Is that true? If so why?*

A: Keep in mind that a microwave *sfg* relates wave **amplitudes**. The branch values are therefore always **scattering parameters**. One important thing about scattering parameters, their magnitudes (for **passive** devices) are always **less than** or equal to one!

$$|S_{mn}| \leq 1$$

Recall the value of a path is simply the **product** of each branch that forms the path. The more branches (and thus nodes), the more terms in this product.

Since each term has a magnitude **less than one**, the magnitude of a product of **many** terms is **much smaller** than a product of a few terms. For example:

$$|-j0.7|^3 = 0.343 \qquad \text{and} \qquad |-j0.7|^{10} = 0.028$$

→ In other words, paths with **more branches** (i.e., more nodes) will typically have **smaller magnitudes** and so are **less significant** in the propagation series.

Note path 1 in our example traveled along **one** branch only:

$$p_1 = 0.144$$

Path 2 has **five** branches:

$$p_2 = -0.1$$

Path 3 **seven** branches:

$$p_3 = -0.08$$

Path 4 **nine** branches:

$$p_4 = 0.014$$

Path 5 **eleven** branches:

$$p_5 = 0.0112$$

Path 6 **eleven** branches:

$$p_6 = 0.0112$$

Path 7 **thirteen** branches:

$$p_7 = 0.009$$

Hopefully it is **evident** that the magnitude **diminishes** as the path "length" **increases**.

Q: *So, does this mean that we should* ***abandon*** *our four* ***reduction rules****, and* ***instead*** *use a truncated propagation series to* ***evaluate*** *signal flow graphs??*

A: **Absolutely not**!

Remember, truncating the propagation series always results in some **error**. This error might be sufficiently small **if** we retain enough terms, but knowing precisely **how many** terms to retain is problematic.

We find that in most cases it is simply **not worth the effort**—use the four **reduction rules** instead (it's **not** like they're particularly difficult!).

Q: *You say that in "****most cases****" it is not worth the effort. Are there some cases where this approximation is actually* ***useful****??*

A: Yes. A truncated propagation series (typically using only the **forward paths**) is used when these **three** things are true:

1. The network or device is **complex** (lots of nodes and branches).

2. We can conclude from our knowledge of the device that the **forward paths** are sufficient for an **accurate** approximation (i.e., the magnitudes of all other paths in the series are almost certainly **very** small).

3. The branch values are **not numeric**, but instead are variables that are dependent on the **physical** parameters of the device (e.g., a characteristic impedance or line length).

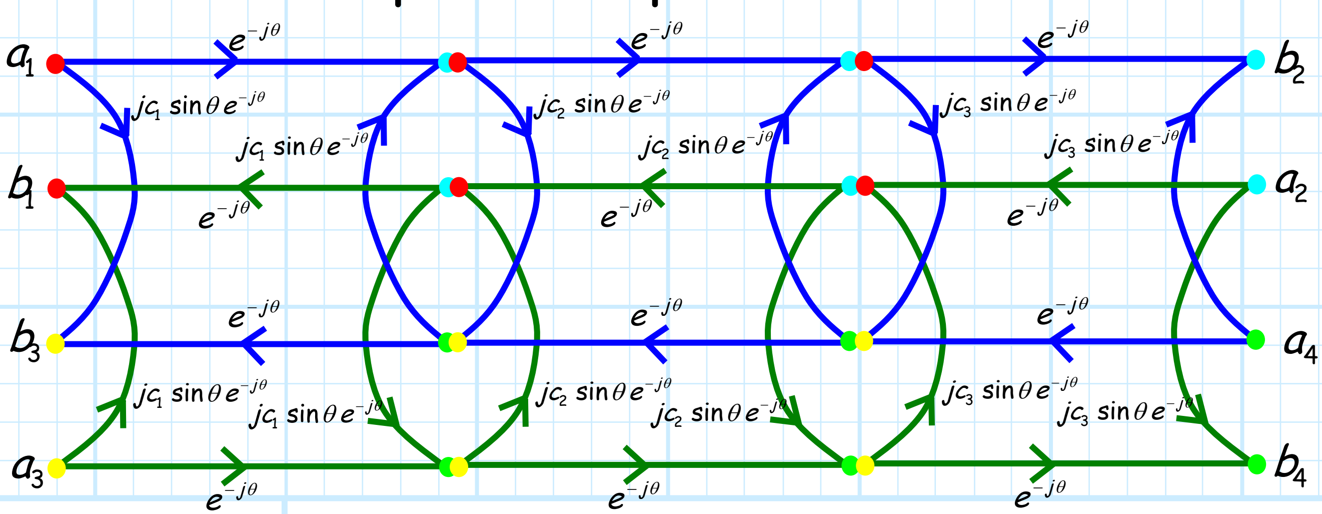
The result is typically a **tractable** mathematical equation that relates the **design variables** (e.g., Z_0 or ℓ) of a complex device to a specific **device parameter**.

For **example**, we might use a truncated propagation series to **approximately** determine some function:

$$\Gamma_{in}\left(Z_{01},\ell_1,Z_{02},\ell_2\right)$$

If we desire a matched input (i.e., $\Gamma_{in}\left(Z_{01},\ell_1,Z_{02},\ell_2\right)=0$) we can **solve** this tractable design equation for the (nearly) proper values of $Z_{01},\ell_1,Z_{02},\ell_2$.

We will use this technique to **great effect** for designing **multi-section matching networks** and **multi-section coupled line couplers**.



The signal flow graph of a three-section coupled-line coupler.