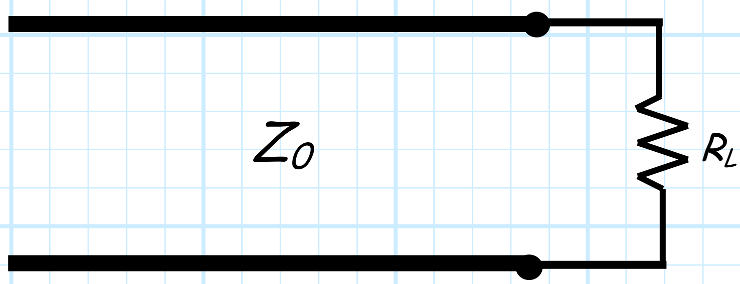
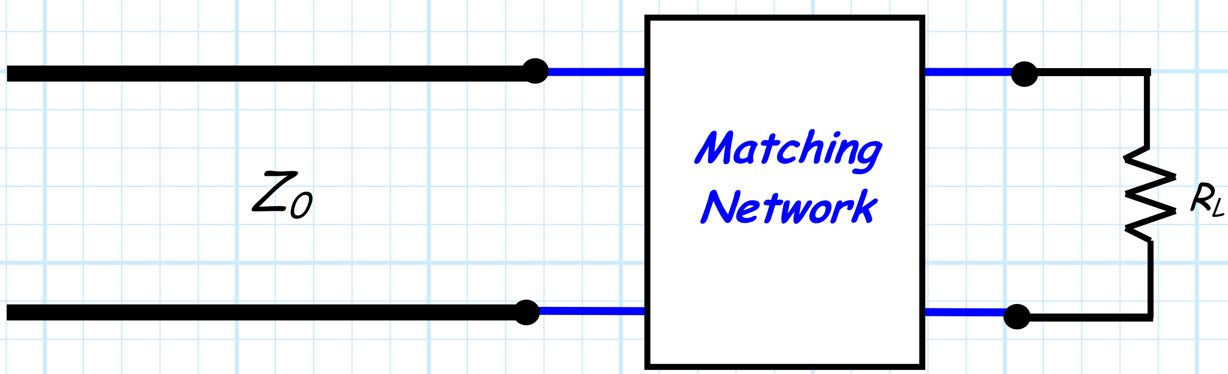
The Quarter-Wave Transformer

Say the end of a transmission line with characteristic impedance Z_0 is terminated with a **resistive** (i.e., real) load.



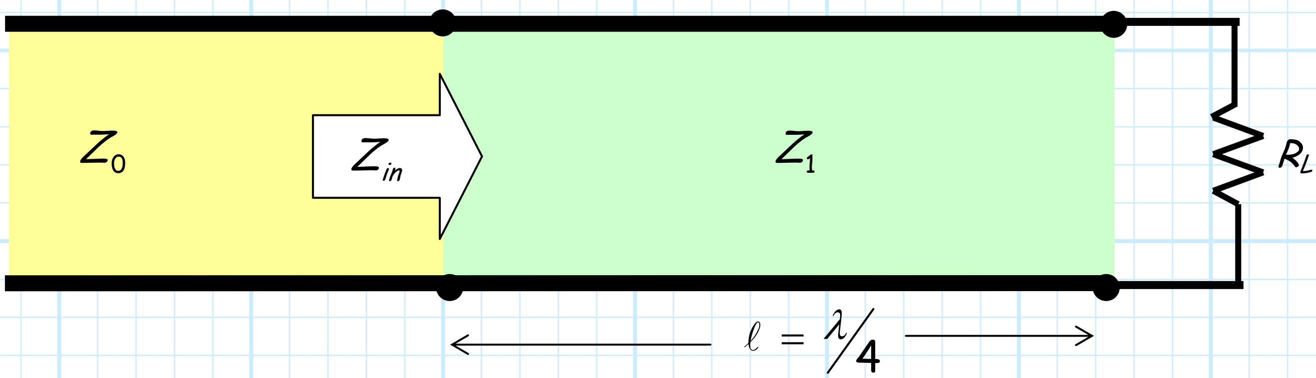
Unless $R_L = Z_0$, the resistor is **mismatched** to the line, and thus some of the incident power will be **reflected**.

We can of course correct this situation by placing a matching network between the line and the load:



In addition to the designs we have just studied (e.g., L-networks, stub tuners), one of the simplest matching network designs is the **quarter-wave transformer**.

The quarter-wave transformer is simply a transmission line with characteristic impedance Z_1 and length $\ell = \lambda/4$ (i.e., a quarter-wave line).



The $\lambda/4$ line **is** the **matching network**!

Q: *But what about the characteristic impedance Z_1; what **should** its value be??*

A: Remember, the quarter wavelength case is one of the **special** cases that we studied. We know that the **input** impedance of the quarter wavelength line is:

$$Z_{in} = \frac{(Z_1)^2}{Z_L} = \frac{(Z_1)^2}{R_L}$$

Thus, if we wish for Z_{in} to be numerically equal to Z_0, we find:

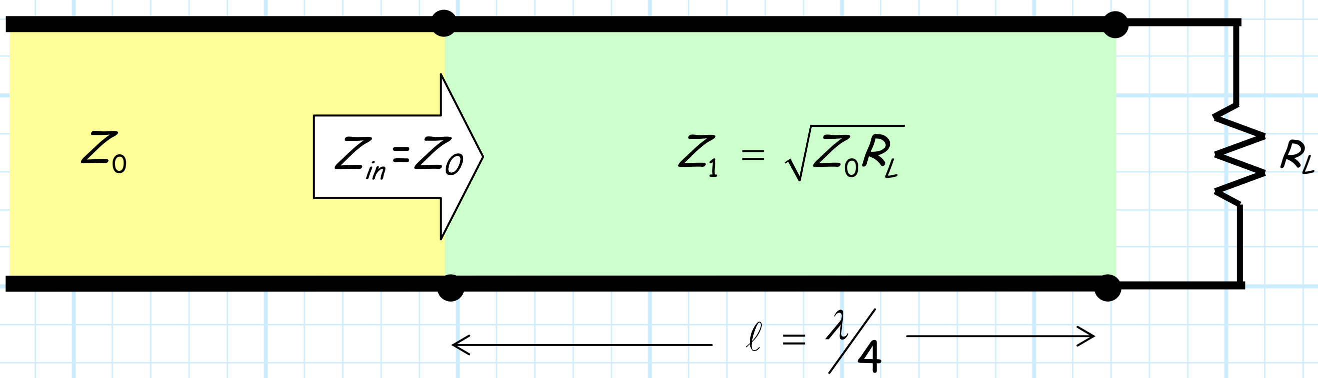
$$Z_{in} = \frac{(Z_1)^2}{R_L} = Z_0$$

Solving for Z_1, we find its **required** value to be:

$$(Z_1)^2 / R_L = Z_0$$
$$(Z_1)^2 = Z_0 R_L$$
$$Z_1 = \sqrt{Z_0 R_L}$$

In other words, the characteristic impedance of the quarter wave line is the **geometric average** of Z_0 and R_L!

Therefore, a $\lambda/4$ line with characteristic impedance $Z_1 = \sqrt{Z_0 R_L}$ will **match** a transmission line with characteristic impedance Z_0 **to** a resistive load R_L.



Thus, **all power** is delivered to load R_L !

Alas, the quarter-wave transformer (like all our designs) has a few problems!

Problem #1

The matching **bandwidth** is **narrow** !

In other words, we obtain a **perfect** match at precisely the frequency where the length of the matching transmission line is a **quarter**-wavelength.

→ But remember, this length can be a quarter-wavelength at just **one** frequency!

Remember, **wavelength** is related to **frequency** as:

$$\lambda = \frac{v_p}{f} = \frac{1}{f\sqrt{LC}}$$

where v_p is the **propagation velocity** of the wave .

For **example**, assuming that $v_p = c$ (c = the speed of light in a vacuum), one wavelength at 1 GHz is 30 cm ($\lambda = 0.3\ m$), while one wavelength at 3 GHz is 10 cm ($\lambda = 0.1\ m$). As a result, a transmission line length $\ell = 7.5\ cm$ is a quarter wavelength for a signal at 1GHz **only**.

Thus, a quarter-wave transformer provides a **perfect** match ($\Gamma_{in} = 0$) at **one** and **only** **one** signal frequency!

As the signal frequency (i.e., wavelength) changes, the **electrical** length of the matching transmission line changes. It will **no longer** be a **quarter** wavelength, and thus we **no longer** will have a **perfect** match.

We find that the **closer** R_L (R_{in}) is to characteristic impedance Z_0, the **wider** the bandwidth of the quarter wavelength transformer.

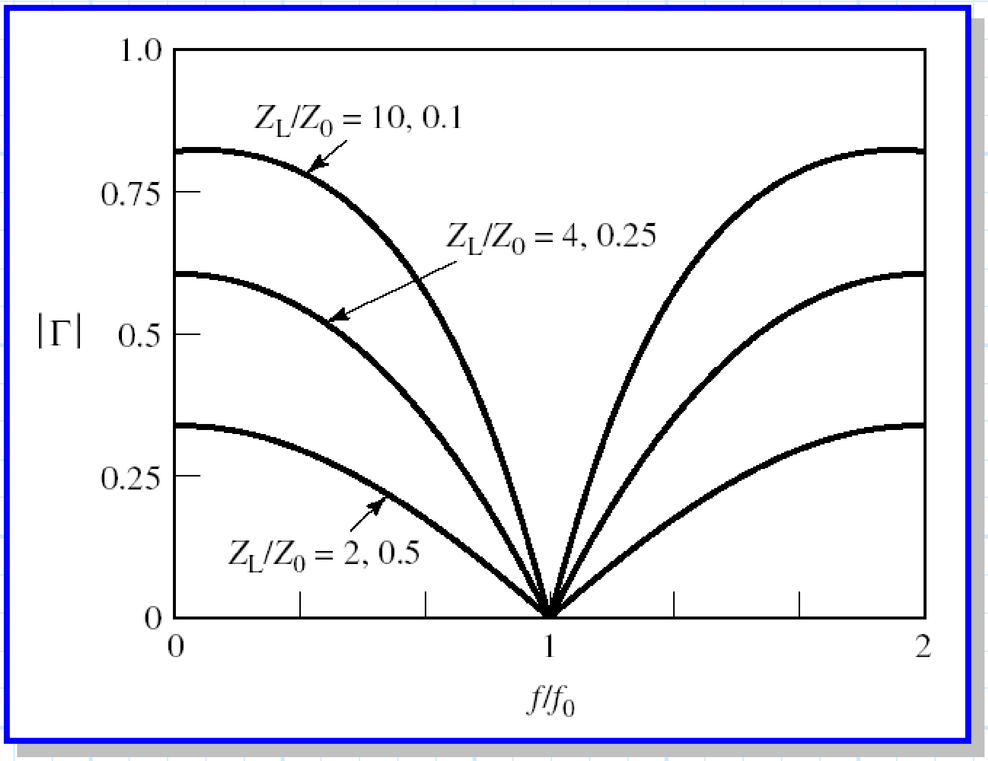


Figure 5.12 (p. 243) *Reflection coefficient magnitude versus frequency for a single-section quarter-wave matching transformer with various load mismatches.*

We will find that the bandwidth can be **increased** by adding **multiple** $\lambda/4$ sections!

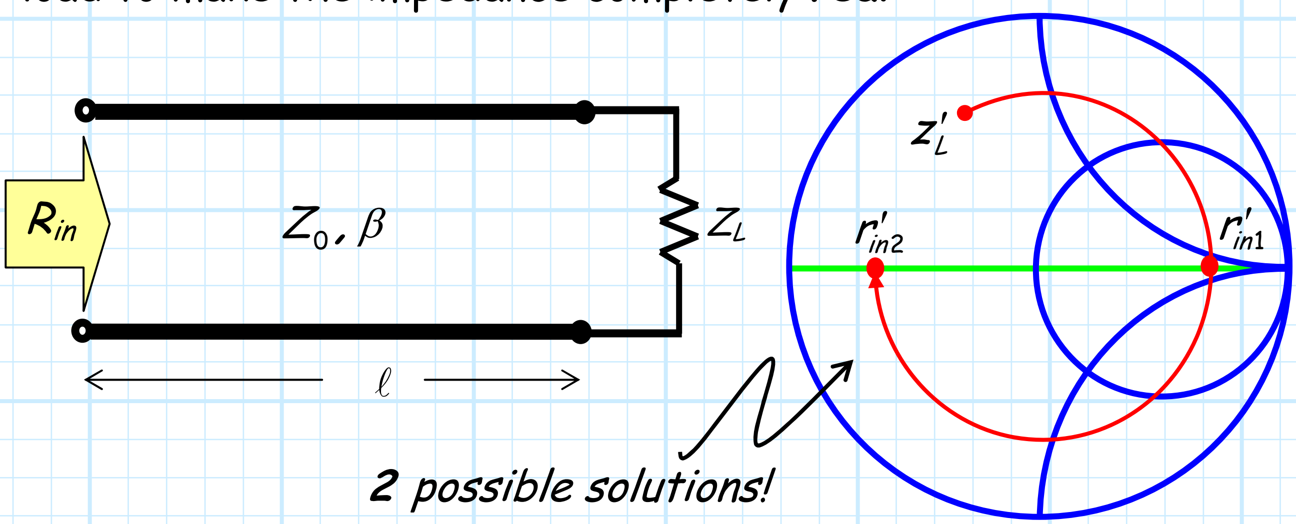
Problem #2

Recall the matching solution was limited to loads that were **purely real**! I.E.:

$$Z_L = R_L + j0$$

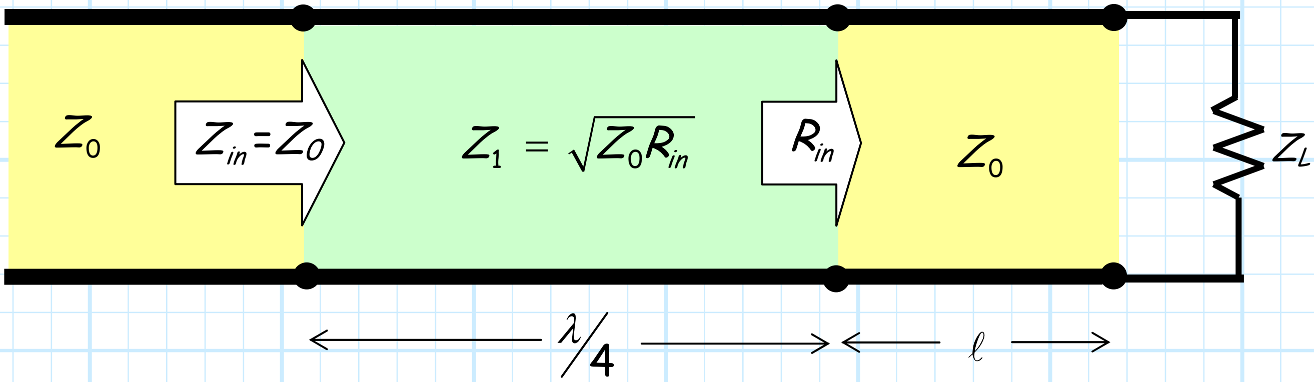
Of course, this is a BIG problem, as most loads will have a **reactive** component!

Fortunately, we have a relatively easy **solution** to this problem, as we can always add some **length** ℓ of transmission line to the load to make the impedance completely **real**:



However, remember that the input impedance will be purely real at only **one** frequency!

We can then build a quarter-wave transformer to **match** the line Z_0 to resistance R_{in}:



Again, since the transmission lines are lossless, **all** of the incident power is delivered to the **load** Z_L.