The Reflection Coefficient

So, we know that the transmission line **voltage** $V(z)$ and the transmission line **current** $I(z)$ can be related by the **line impedance** $Z(z)$:

$$V(z) = Z(z)\, I(z)$$

or equivalently:

$$I(z) = \frac{V(z)}{Z(z)}$$

Q: *Piece of cake! I fully understand the concepts of **voltage**, **current** and **impedance** from my **circuits** classes. Let's move on to something more important (or, at the very least, more **interesting**).*

Expressing the "activity" on a transmission line in terms of **voltage**, **current** and **impedance** is of course **perfectly** valid.

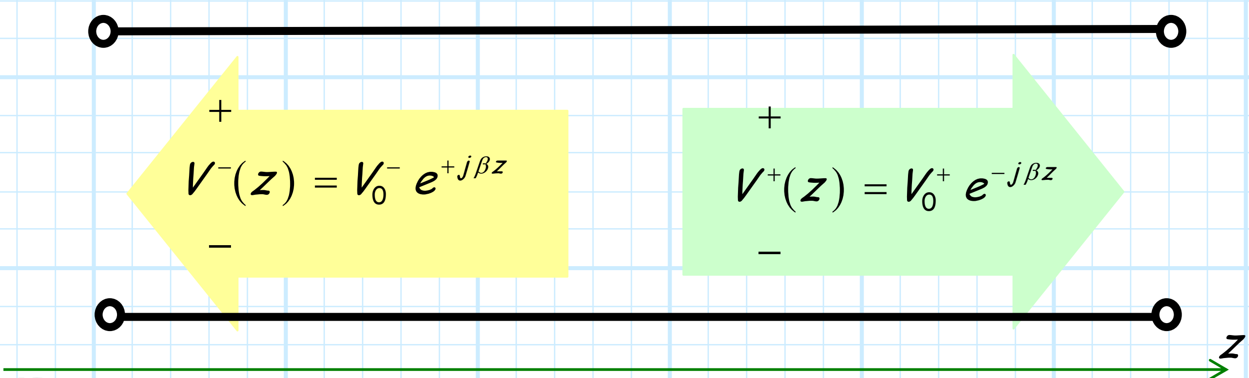
However, let us look **closer** at the expression for each of these quantities:

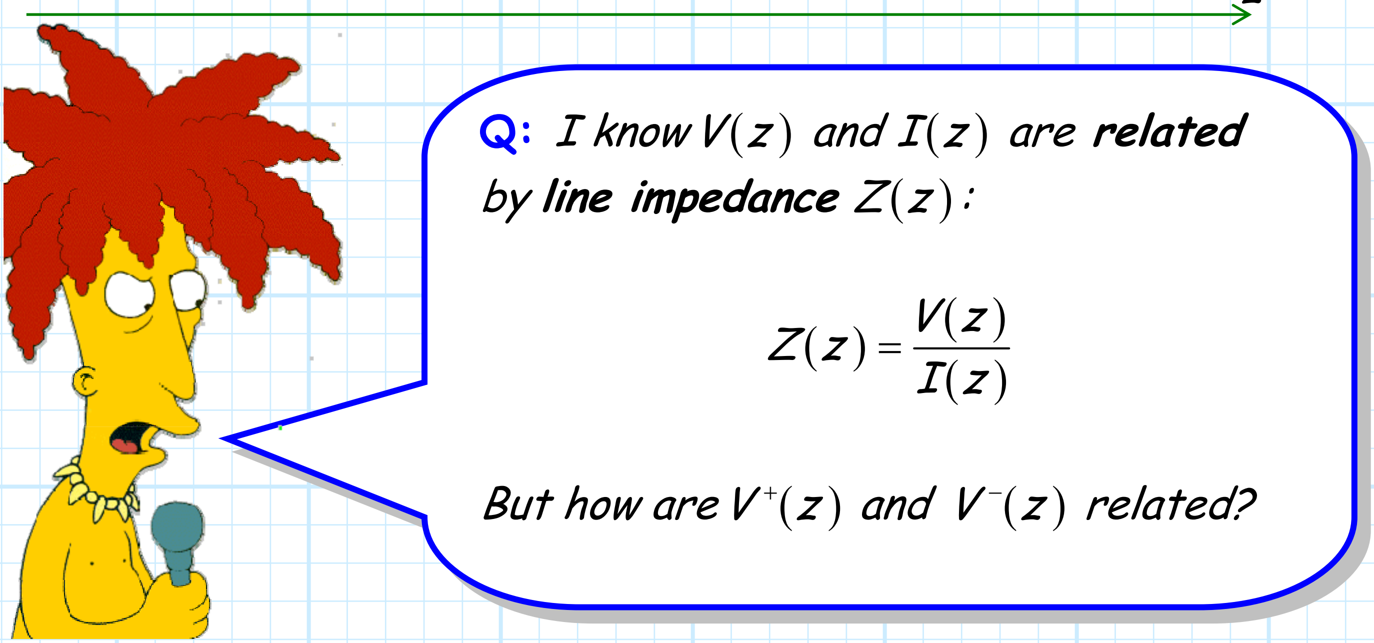
$$V(z) = V^{+}(z) + V^{-}(z)$$

$$I(z) = \frac{V^{+}(z) - V^{-}(z)}{Z_0}$$

$$Z(z) = Z_0\left(\frac{V^{+}(z) + V^{-}(z)}{V^{+}(z) - V^{-}(z)}\right)$$

It is evident that we can **alternatively** express all "activity" on the transmission line in terms of the two transmission line **waves** $V^{+}(z)$ and $V^{-}(z)$.





A: Similar to line impedance, we can define a new parameter—the **reflection coefficient** $\Gamma(z)$—as the **ratio** of the two quantities:

$$\Gamma(z) \doteq \frac{V^-(z)}{V^+(z)} \quad \Rightarrow \quad V^-(z) = \Gamma(z)\, V^+(z)$$

More specifically, we can express $\Gamma(z)$ as:

$$\Gamma(z) = \frac{V_0^- \, e^{+j\beta z}}{V_0^+ \, e^{-j\beta z}} = \frac{V_0^-}{V_0^+} \; e^{+j2\beta z}$$

Note then, the value of the reflection coefficient at z=0 is:

$$\Gamma(z=0) = \frac{V^-(z=0)}{V_0^+(z=0)} \; e^{+j2\beta(0)} = \frac{V_0^-}{V_0^+}$$

We define this value as Γ_0, where:

$$\Gamma_0 \doteq \Gamma(z=0) = \frac{V_0^-}{V_0^+}$$

Note then that we can alternatively write $\Gamma(z)$ as:

$$\Gamma(z) = \Gamma_0 \; e^{+j2\beta z}$$

So now we have **two different** but equivalent ways to describe transmission line activity!

We can use (total) **voltage** and **current**, related by **line impedance**:

$$Z(z) = \frac{V(z)}{I(z)} \quad \therefore \quad V(z) = Z(z)\,I(z)$$

Or, we can use the two propagating **voltage waves**, related by the **reflection coefficient**:

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} \quad \therefore \quad V^-(z) = \Gamma(z)\;V^+(z)$$

These are **equivalent** relationships—we can use **either** when describing a transmission line.