The Resistive Divider

Consider the resistive power divider:

This symmetric power divider will be matched at port 1 if $R$ is selected as:

\[
Z_0 = R + (R + Z_0) \parallel (R + Z_0)
\]

\[
= R + \frac{R + Z_0}{2}
\]

\[
= 1.5R + \frac{Z_0}{2}
\]

Solving this equation, we find that port 1 is matched if:

\[
R = \frac{Z_0}{3}
\]
From the symmetry of the circuit, we find that all the other ports will be matched as well (i.e., $S_{11} = S_{22} = S_{33} = 0$). Moreover, it can be shown that:

$$S_{12} = S_{21} = S_{31} = S_{32} = S_{23} = \frac{1}{2}$$

So:

$$S = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Note the magnitude of each column is less than one. E.G.,:

$$|S_{21}|^2 + |S_{31}|^2 = \frac{1}{2} < 1$$

Therefore this power divider is lossy!

In fact, we find that the power out of each port is just one-quarter of the input power:

$$P_2^- = P_3^+ = \frac{P_1^+}{4}$$

In other words, half the input power is absorbed by the divider!