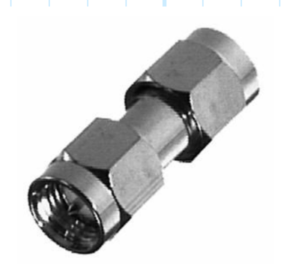
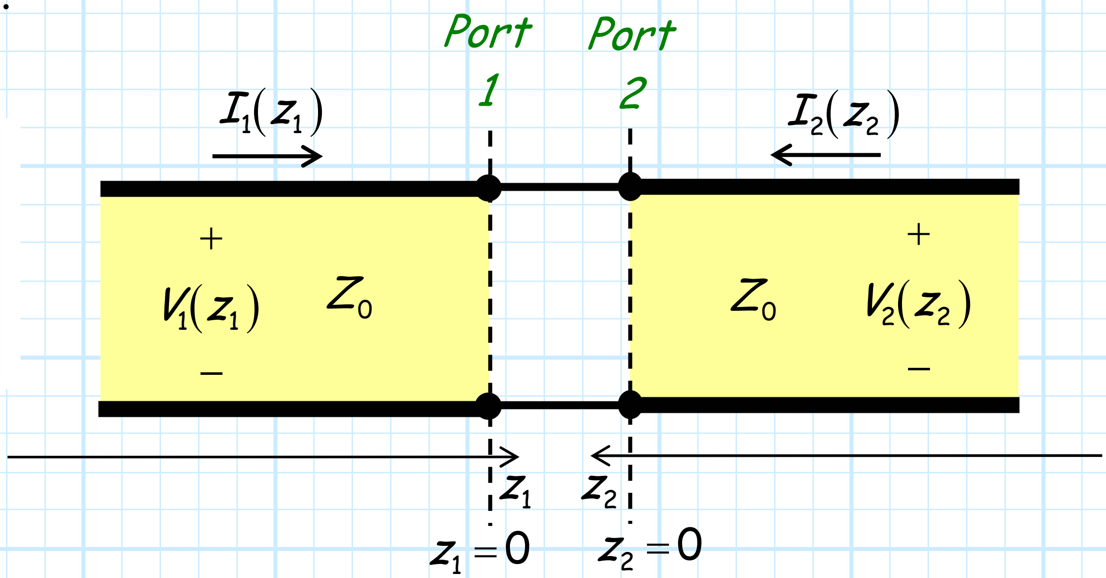
Example: The Scattering Matrix of a Connector

First, let's consider the scattering matrix of a **perfect connector**—an electrically **very small** two-port device that allows us to connect the ends of different transmission lines together.





If the connector is ideal, then it will exhibit **no** series inductance **nor** shunt capacitance, and thus from KVL and KCL:

$$V_1(z_1=0) = V_2(z_2=0) \qquad\qquad I_1(z_1=0) = -I_2(z_2=0)$$

Terminating **port 2 in a matched load**, and then analyzing the resulting circuit, we find that (not surprisingly!):

$$V_{01}^{-} = 0 \quad \text{and} \quad V_{02}^{-} = V_{01}^{+}$$

From this we conclude that (since $V_{02}^{+} = 0$):

$$S_{11} = \frac{V_{01}^{-}}{V_{01}^{+}} = \frac{0}{V_{01}^{+}} = 0.0 \qquad\qquad S_{21} = \frac{V_{02}^{-}}{V_{01}^{+}} = \frac{V_{01}^{+}}{V_{01}^{+}} = 1.0$$

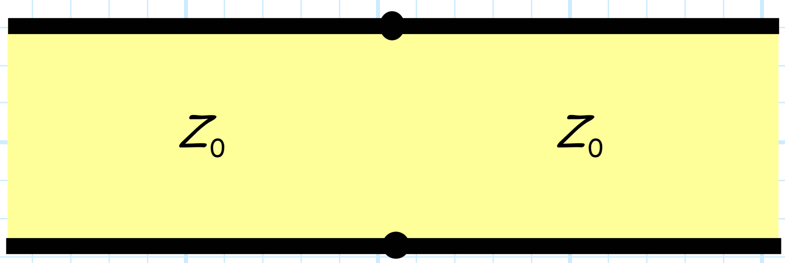
This two-port device has D_2 symmetry (a plane of bilateral symmetry), meaning:

$$S_{22} = S_{11} = 0.0 \qquad \text{and} \qquad S_{21} = S_{12} = 1.0$$

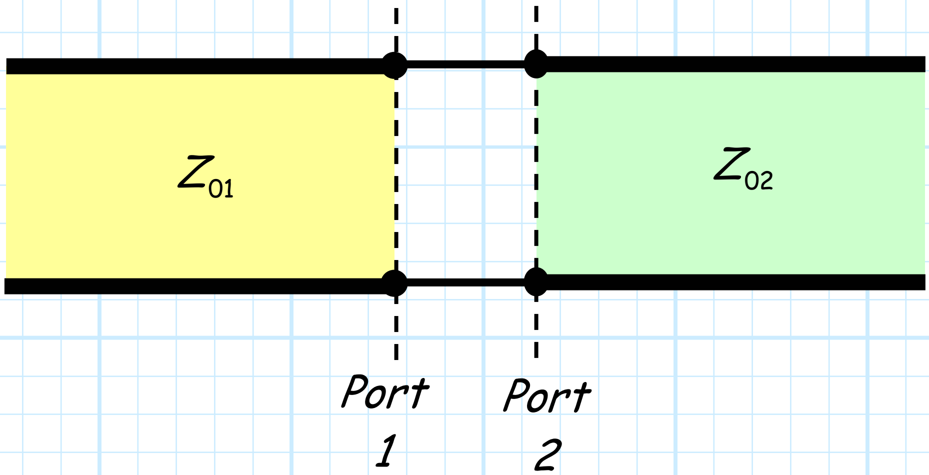
The **scattering matrix** for such this ideal connector is therefore:

$$\mathcal{S} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

As a result, the perfect connector allows two transmission lines of **identical characteristic impedance** to be connected together into **one "seamless" transmission line**.



Now, however, consider the case where the transmission lines connected together have **dissimilar** characteristic impedances (i.e., $Z_0 \neq Z_1$):



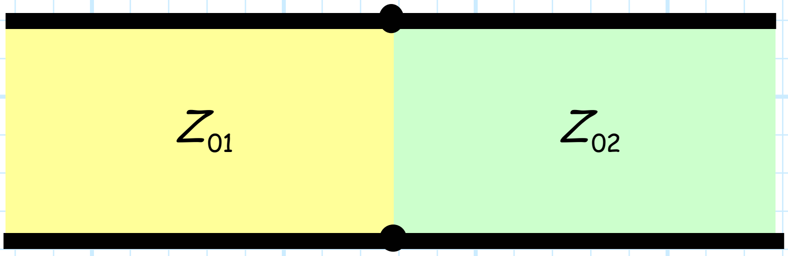
Q: *Won't the scattering matrix of this ideal connector remain the **same**? After all, the **device itself** has not changed!*

A: The impedance, admittance, and transmission matrix **will** remained unchanged—these matrix quantities **do not** depend on the characteristics of the transmission lines connected to the device.

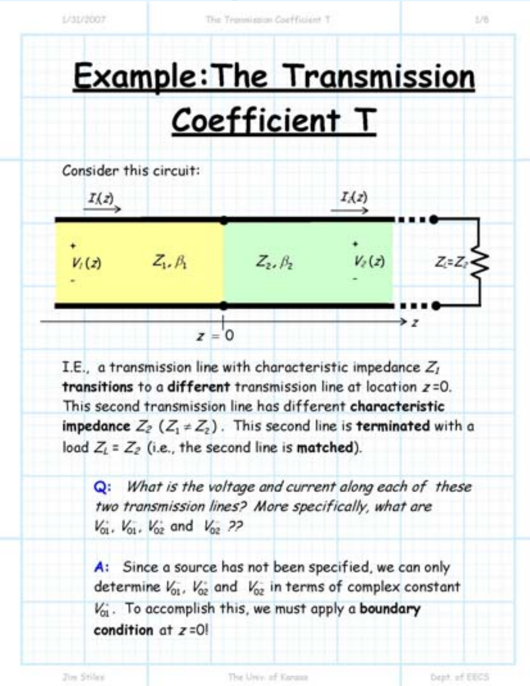
But remember, the **scattering matrix** depends on **both** the device **and** the characteristic impedance of the transmission lines attached to it.

After all, the **incident** and **exiting** waves are traveling on these transmission lines!

The ideal connector in this case establishes a "seamless" **interface** between two **dissimilar** transmission lines.



Remember, this is the **same** structure that we evaluated in an **earlier** handout!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Example: The Transmission Coefficient T

Consider this circuit:

I.E., a transmission line with characteristic impedance Z_1 **transitions** to a **different** transmission line at location $z=0$. This second transmission line has different **characteristic impedance** Z_2 $(Z_1 \neq Z_2)$. This second line is **terminated** with a load $Z_L = Z_2$ (i.e., the second line is **matched**).

Q: *What is the voltage and current along each of these two transmission lines? More specifically, what are* V_{01}^+, V_{01}^-, V_{02}^+ and V_{02}^- *??*

A: Since a source has not been specified, we can only determine V_{01}^-, V_{02}^+ and V_{02}^- in terms of complex constant V_{01}^+. To accomplish this, we must apply a **boundary condition** at $z=0$!

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In that analysis we found that—when $V_{02}^+ = 0$:

$$\frac{V_{01}^-}{V_{01}^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \qquad \text{and} \qquad \frac{V_{02}^-}{V_{01}^+} = \frac{2Z_{02}}{Z_{02} + Z_{01}}$$

And so the (generalized) scattering parameters S_{11} and S_{21} are:

$$S_{11} = \frac{V_{01}^-}{V_{01}^+}\frac{\sqrt{Z_{01}}}{\sqrt{Z_{01}}} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \qquad \text{and} \qquad S_{21} = \frac{V_{02}^-}{V_{01}^+}\frac{\sqrt{Z_{01}}}{\sqrt{Z_{02}}} = \frac{2\sqrt{Z_{01}Z_{02}}}{Z_{02} + Z_{01}}$$

As a result we can conclude that the **scattering matrix** of the ideal connector (when connecting dissimilar transmission lines) is:

$$\mathcal{S} = \begin{bmatrix} \dfrac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} & \dfrac{2\sqrt{Z_{01}Z_{02}}}{Z_{01} + Z_{02}} \\[2ex] \dfrac{2\sqrt{Z_{01}Z_{02}}}{Z_{01} + Z_{02}} & \dfrac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}} \end{bmatrix}$$