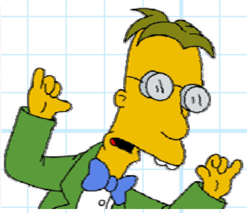
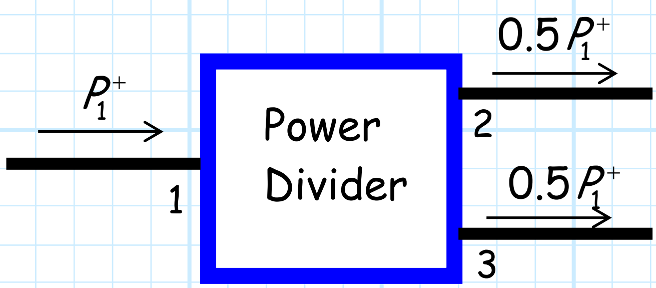
The (nearly) Ideal T-Junction Power Divider



Recall that we **cannot** build a matched, lossless reciprocal **three**-port device.

So, let's **mathematically** try and determine the scattering matrix of the best possible T-junction 3 dB **power divider**.



To **efficiently** divide the power **incident** on the input port, the port (port 1) must first be **matched** (i.e., all incident power should be delivered to port 1):

$$S_{11} = 0$$

Likewise, this delivered power to port 1 must be divided efficiently (i.e., **without loss**) between ports 2 and 3.

Mathematically, this means that the first column of the scattering matrix must have **magnitude of 1.0:**

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$$

Since we have already determined that $S_{11} = 0$, this simply means :

$$|S_{21}|^2 + |S_{31}|^2 = 1$$

Provided that we wish to evenly divide the input power, we can conclude from the expression above that:

$$|S_{21}|^2 = |S_{31}|^2 = 1/2 \qquad \therefore \; |S_{21}| = |S_{31}| = 1/\sqrt{2}$$

Note that **this** device would take the power into port 1 and divide into **two equal parts**—half exiting **port 2**, and half exiting **port3** (provided ports 2 and 3 are terminated in matched loads!).

$$P_2^- = |S_{21}|^2 P_1^+ = 0.5 P_1^+ \qquad P_3^- = |S_{31}|^2 P_1^+ = 0.5 P_1^+$$

In addition, it is **desirable** that ports 2 and 3 be **matched** (the whole device is thus matched):

$$S_{22} = S_{33} = 0$$

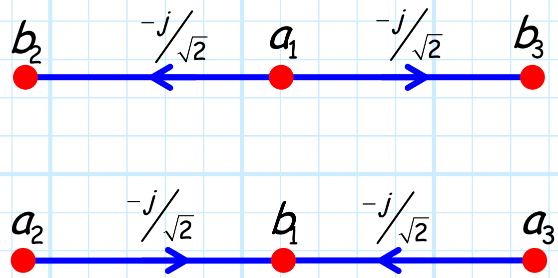
And also **desirable** that ports 2 and 3 be **isolated**:

$$S_{23} = S_{32} = 0$$

This last requirement ensures that no signal incident on port 2 (e.g., reflected from a load) will **"leak"** into port 3—and vice versa.

This ideal 3 dB power divider **could** therefore have the form:

$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}/_{\sqrt{2}} & {}^{-j}/_{\sqrt{2}} \\ {}^{-j}/_{\sqrt{2}} & 0 & 0 \\ {}^{-j}/_{\sqrt{2}} & 0 & 0 \end{bmatrix}$$



Since we can describe this ideal power divider **mathematically**, we can potentially build it **physically**!

Q: *Huh!? I thought you said that a matched, lossless, reciprocal three-port device is **impossible**?*

A: It is! This divider **is** clearly a **lossy** device. The magnitudes of both column 2 and 3 are less than one:

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = \left|{}^{-j}/_{\sqrt{2}}\right|^2 + 0 + 0 = 0.5 < 1.0$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = \left|{}^{-j}/_{\sqrt{2}}\right|^2 + 0 + 0 = 0.5 < 1.0$$

Note then that **half** the power incident on port 2 (or port 3) of this device would **exit** port 1 (i.e., reciprocity), but no power would exit port 3 (port2), since ports 2 and 3 are **isolated**. I.E.,:

$$P_1^- = |S_{12}|^2 P_2^+ = 0.5 P_2^+ \qquad P_3^- = |S_{32}|^2 P_2^+ = 0$$

$$P_1^- = |S_{13}|^2 P_3^+ = 0.5 P_3^+ \qquad P_2^- = |S_{23}|^2 P_3^+ = 0$$

Q: *Any ideas on how to* ***build*** *this thing?*

A: Note that the **first column** of the scattering matrix is precisely the same as that of the **lossless 3 dB divider**.

Also note that since the device is **lossy**, the design must include some **resistors**.

Lossless Divider + resistors = The Wilkinson Power Divider

Q: *What is the Wilkinson Power Divider?*

A: It's the **subject** of our next section!