The Reflection Coefficient

So, we know that the transmission line **voltage** $V(z)$ and the transmission line **current** $I(z)$ can be related by the **line impedance** $Z(z)$:

$$V(z) = Z(z)\, I(z)$$

or equivalently:

$$I(z) = \frac{V(z)}{Z(z)}$$

Expressing the "activity" on a transmission line in terms of voltage, current and impedance is of course **perfectly** valid. However, let us look **closer** at the expression for each of these quantities:

$$V(z) = V^+(z) + V^-(z)$$

$$I(z) = \frac{V^+(z) - V^-(z)}{Z_0}$$

$$Z(z) = Z_0 \left(\frac{V^+(z) + V^-(z)}{V^+(z) - V^-(z)} \right)$$

It is evident that we can **alternatively** express all "activity" on the transmission line in terms of the two transmission line **waves** $V^+(z)$ and $V^-(z)$.

In other words, we can describe transmission line activity in terms of:

$$V^+(z) \text{ and } V^-(z)$$

instead of:

$$V(z) \text{ and } I(z)$$

Q: *But $V(z)$ and $I(z)$ are related by line impedance $Z(z)$:*

$$Z(z) = \frac{V(z)}{I(z)}$$

How are $V^+(z)$ and $V^-(z)$ related?

A: Similar to line impedance, we can define a new parameter—the **reflection coefficient** $\Gamma(z)$--as the **ratio** of the two quantities:

$$\Gamma(z) \doteq \frac{V^-(z)}{V^+(z)}$$

More specifically, we can express $\Gamma(z)$ as:

$$\Gamma(z) = \frac{V_0^- \, e^{+\gamma z}}{V_0^+ \, e^{-\gamma z}} = \frac{V_0^-}{V_0^+} \, e^{+2\gamma z}$$

Note then, the value of the reflection coefficient at z=0 is:

$$\Gamma(z=0) = \frac{V_0^-}{V_0^+} \, e^{+2\gamma(0)}$$

$$= \frac{V_0^-}{V_0^+}$$

We define this value as Γ_0, where:

$$\Gamma_0 \doteq \Gamma(z=0) = \frac{V_0^-}{V_0^+}$$

Note then that we can alternatively write $\Gamma(z)$ as:

$$\Gamma(z) = \Gamma_0 \, e^{+2\gamma z}$$

Thus, we now know:

$$V^-(z) = \Gamma(z)\, V^+(z)$$

and therefore we can express line current and voltage as:

$$V(z) = V^+(z)\big(1 + \Gamma(z)\big)$$

$$I(z) = \frac{V^+(z)}{Z_0}\big(1 - \Gamma(z)\big)$$

Or, more explicitly, since $V_0^- = \Gamma_0\, V_0^+$:

$$V(z) = V_0^+\left(e^{-\gamma z} + \Gamma_0 e^{+\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_0 e^{+\gamma z}\right)$$

More importantly, we find that **line impedance** $Z(z) = V(z)/I(z)$ is:

$$Z(z) = Z_0\left(\frac{1+\Gamma(z)}{1-\Gamma(z)}\right)$$

Look what happened—the line impedance can be **completely** and explicitly expressed in terms of **reflection coefficient** $\Gamma(z)$!

Or, rearranging, we find that the reflection coefficient $\Gamma(z)$ can **likewise** be written in terms of line impedance:

$$\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

Thus, the values $\Gamma(z)$ and $Z(z)$ are **equivalent** parameters—if we know **one**, then we can determine the **other**!

Likewise, the relationships:

$$V(z) = Z(z)\, I(z)$$

and:

$$V^-(z) = \Gamma(z)\, V^+(z)$$

are **equivalent** relationships—we can use **either** when describing an transmission line.