Turning a Gain Element into an Amplifier

Say the design criteria for our amplifier is to maximize the power delivered to the load (i.e., maximize P_L). This power is maximized when:

1. The available power from the **source** is entirely delivered to the **input** of the gain element $P_{in} = P_{avs}$.

2. The available power from the **output** of the gain element is entirely delivered to the **load** $P_L = P_{avn}$.

Recall this happy occurrence results when $\Gamma_{in} = \Gamma_s^*$ and $\Gamma_L = \Gamma_{out}^*$.

Q: *But what if this is* ***not*** *the case? What if our gain element is not matched to our source, or to our load? Must we simply accept inferior power transfer?*

A: Nope! Remember, we can always build **lossless matching networks** to efficiently transfer power between mismatched sources and loads.

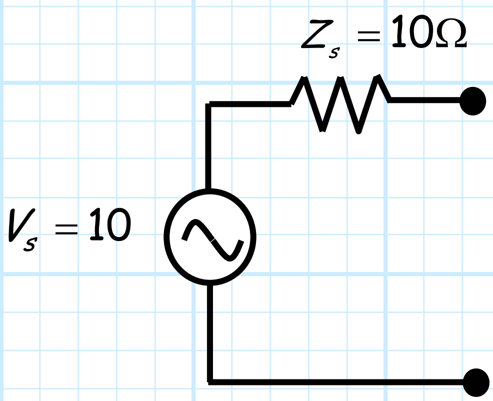
Q: *I see! We need to* ***modify*** *the source impedance Z_s and modify the output impedance Z_{out} such that $Z_s = Z_{in}^*$ and $Z_{out} = Z_L^*$. Right?*

A: Not exactly.

Remember, it is true that a lossless matching network can change the source impedance to match a specific load. But the lossless matching network likewise alters the source voltage V_s such that the available power is preserved!

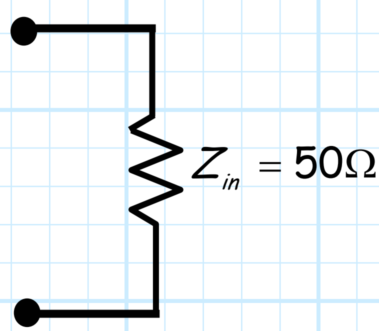
Messing around **directly** with the source impedance will undoubtedly **reduce** the available power of the source (this is bad!).

For **example,** consider this simple problem. Say we have this source, with a robust **available power** of 1.25 W:

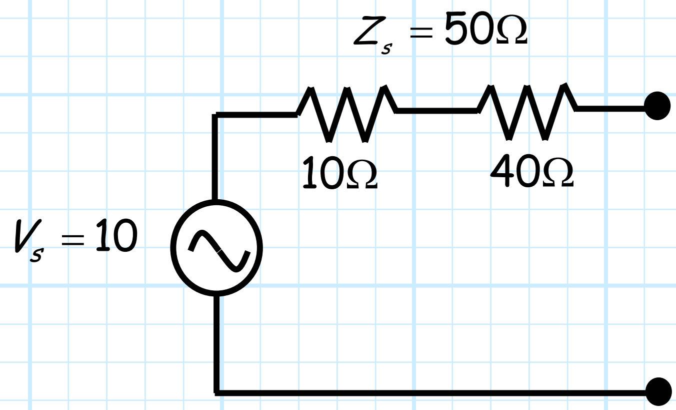


$$P_{avs} = \frac{|V_s|^2}{8\,Re\{Z_s\}} = \frac{10^2}{8(10)} = 1.25 \quad W$$

and wish to deliver this power to an impedance of $Z_{in} = 50\Omega$:



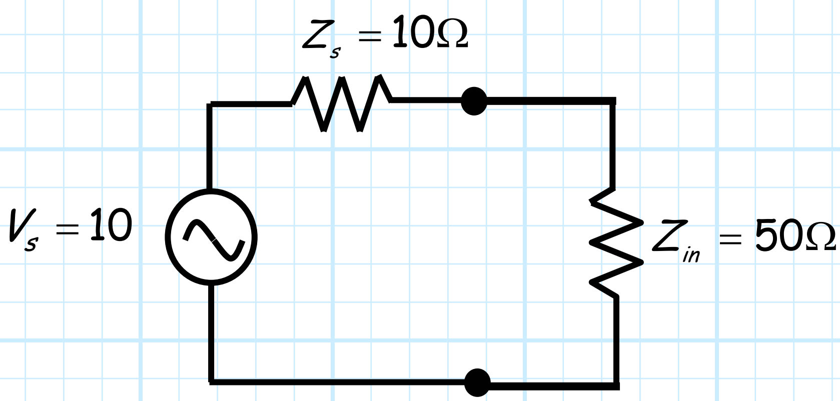
Although increasing the source impedance by 40 Ω would result in a **conjugate match**, it would likewise **reduce** the available power to a **measly 0.25 Watts**.



$$
\begin{aligned}
P_{avs} &= \frac{|V_s|^2}{8\,Re\{Z_s\}} \\
&= \frac{10^2}{8(50)} \\
&= 0.25 \quad W
\end{aligned}
$$

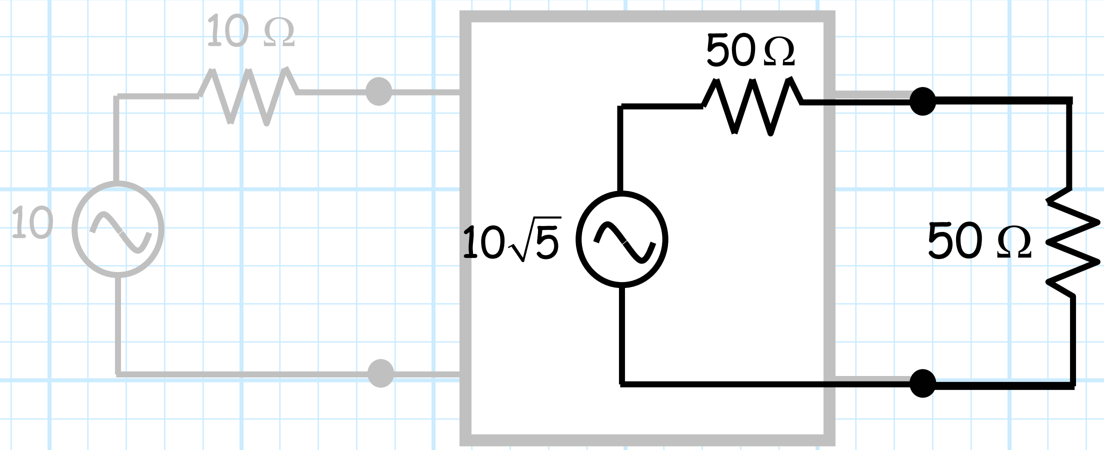
Thus, although finagling the source impedance does result in extracting **all** the available power from the resulting source, it likewise **decreases** this available power by 80% !

Moreover, we find that the delivered power to would be greater if we simply left the darn thing **alone**!



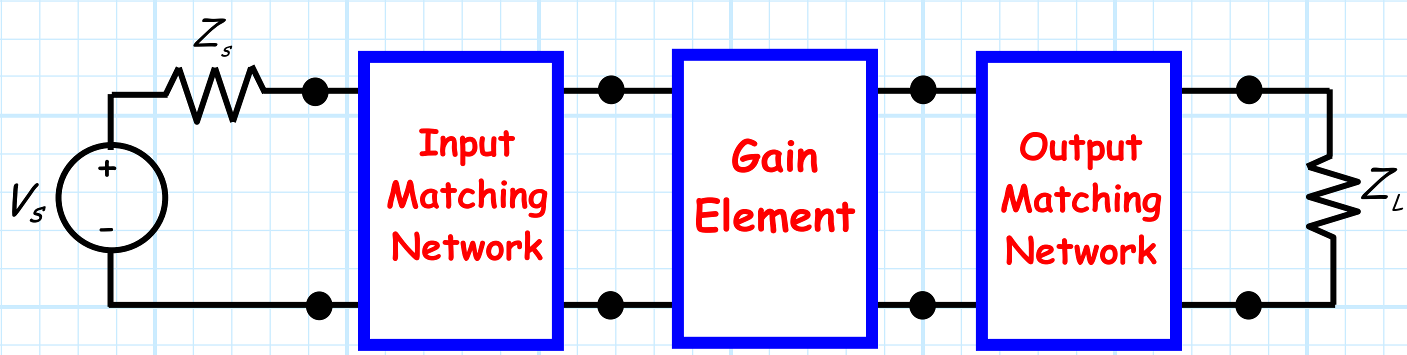
$$
\begin{aligned}
P_L &= \frac{1}{2}|V_g|^2 \frac{Re\{Z_L\}}{|Z_g + Z_L|^2} \\
&= \frac{10^2}{2}\frac{50}{(20+50)^2} \\
&= \frac{50^2}{70^2} \\
&= 0.51 \quad W
\end{aligned}
$$

In contrast, a properly designed matching network will transform the source impedance to a matched value of 50 Ω, but it **likewise** transforms the source **voltage** such that the absorbed power remains the **same**—1.25 Watts is delivered to the 50 Ω load!

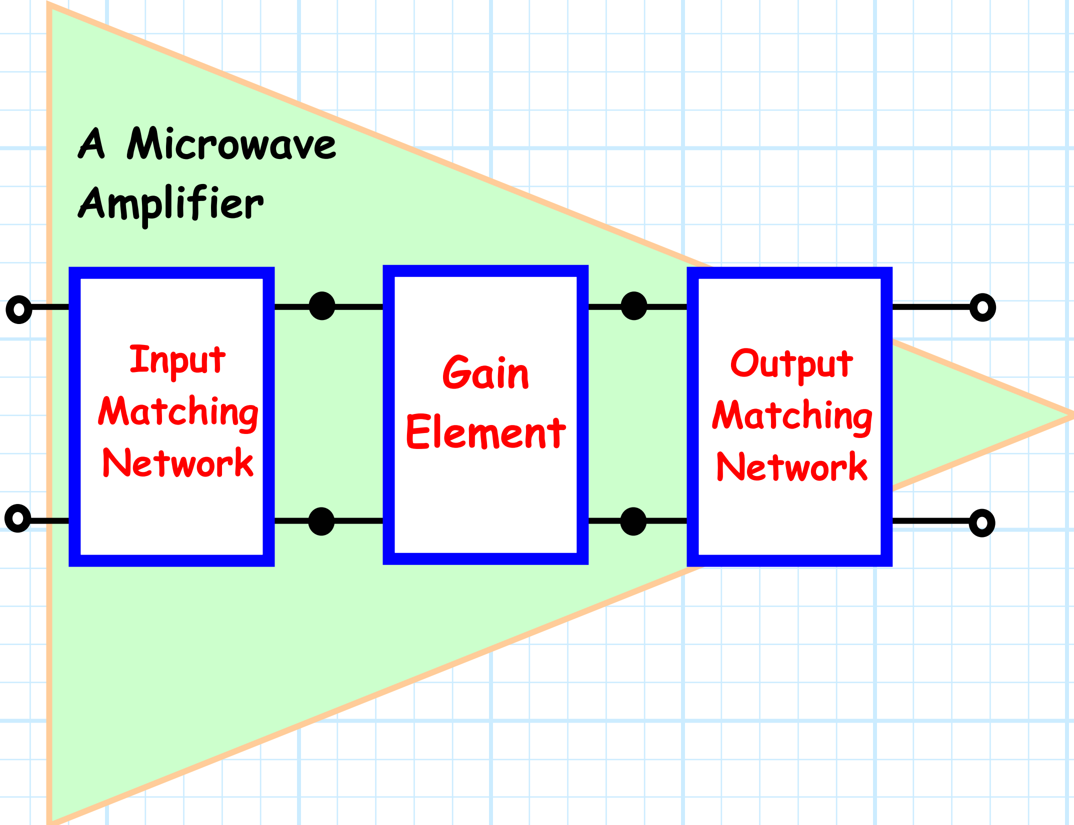


We **have** our cake. We **eat** it too.

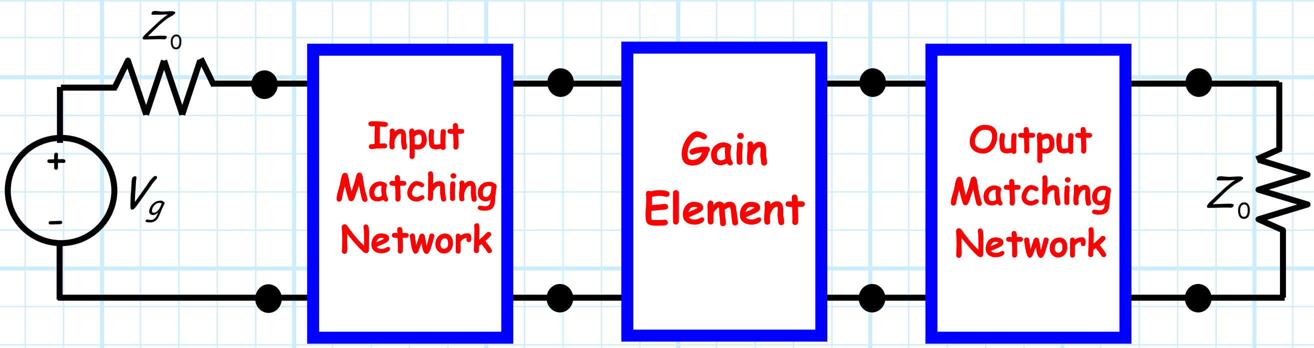
So, to maximize the power delivered to a load, we need to insert **lossless matching networks** between the source and gain element, and between the gain element and the load:



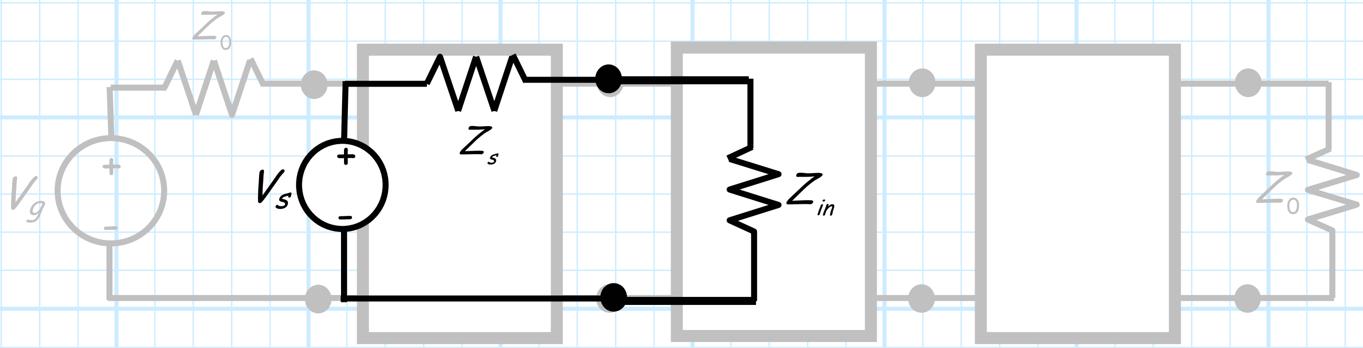
The **three stages** together—input matching network, gain element, and output matching network—form a **microwave amplifier**!



Of course, the impedance of both the source and the load connected to this amp will most certainly be that of transmission line **characteristic impedance** Z_0. Thus, our amplifier circuit is typically:



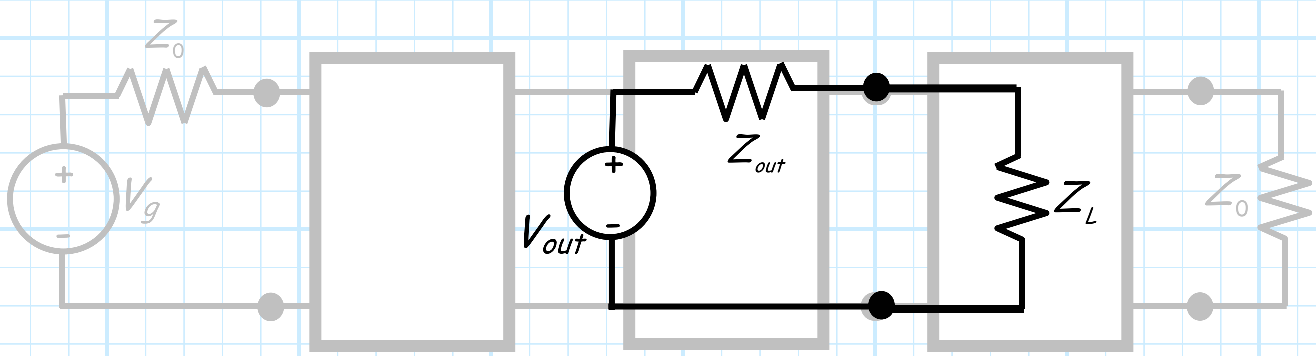
The **input network** is thus required to match Z_0 to the gain element input impedance Z_{in}. For the purposes of amplifier design, we view the input matching network as one that transforms the source impedance Z_0 into a new source impedance Z_s, one that is conjugate matched to the gain element input impedance Z_{in}:



If our input matching network is properly designed, we then find:

$$Z_s = Z_{in}^* \qquad \text{and so} \qquad \Gamma_s = \Gamma_{in}^*$$

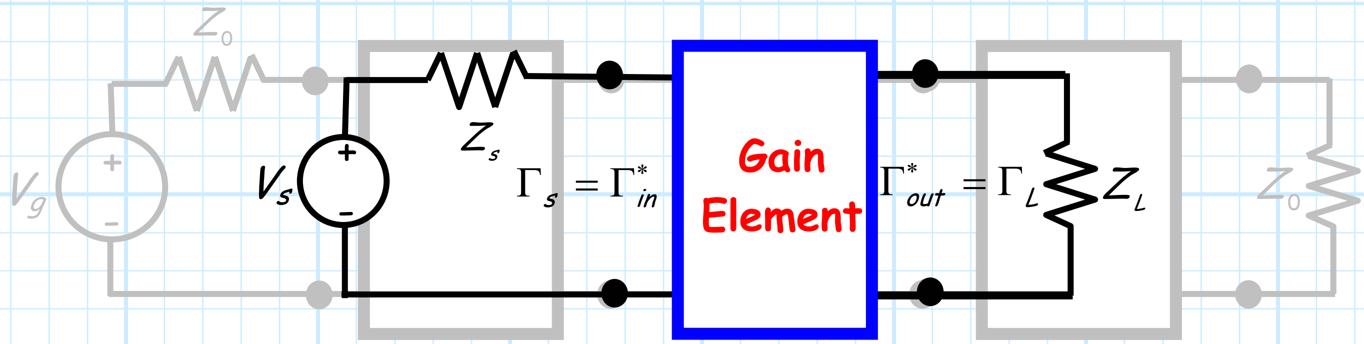
Likewise, the **output matching network** is used to match Z_0 to the gain element output impedance Z_{out}. For the purposes of amplifier design, we view the output matching network as one that transforms the load impedance Z_0 into a new load impedance Z_L, one that is conjugate matched to the gain element output impedance Z_{out}:



Thus, if our input matching network is properly designed, then we find:

$$Z_L = Z_{out}^* \qquad \text{and so} \qquad \Gamma_L = \Gamma_{out}^*$$

And so, our amplifier design problem can be described as:

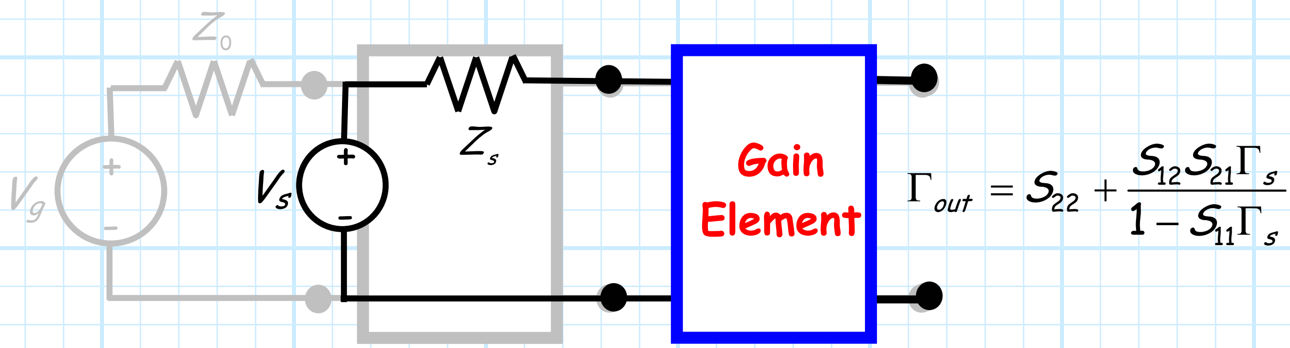


where **the values of Γ_s and Γ_L depend on the input and output matching networks.**

Q: *Alright, we get it. We **know** how to make matching networks. Can't we move on to something else?*

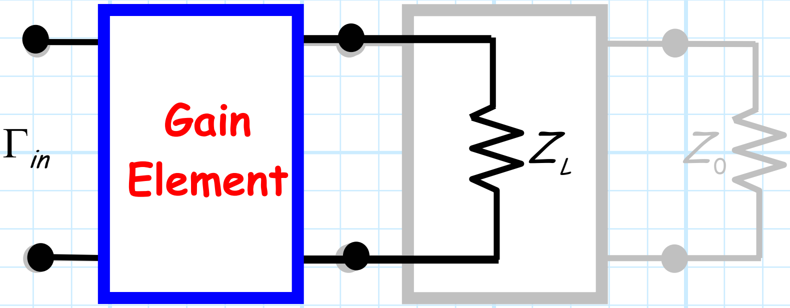
A: Not so fast! There's one little **problem** that makes this procedure more difficult than it otherwise might appear.

Note that the value of Γ_{out} depends on the value of Z_s (i.e., depends on Γ_s).



Likewise, the value of Γ_{in} depends on the value of Z_L (i.e., depends on Γ_L).

$$S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \Gamma_{in}$$



It's a classic **chicken and egg**!

1. We can't design the input matching network until we determine Γ_{in}.

2. We can't determine Γ_{in} until we design the output matching network.

3. We can't determine the output matching network until we determine Γ_{out}.

4. We can't determine Γ_{out} until we design the input matching network.

5. But we can't design the input matching network until we determine Γ_{in}!

Our matching network design problems are thus **coupled**. The solution to this coupled problem is provided in your textbook on page 550, and provides simultaneous solutions for $\Gamma_s = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$.

Now for some **good news**!

Recall that for many gain elements, the value of S_{12} is exceedingly small. Often it is so small that we can approximate as zero.

Q: *So?*

A: Look at what this does to the value of Γ_{in} and Γ_{out}!

$$\Gamma_{out}\Big|_{S_{12}=0} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\Bigg|_{S_{12}=0} = S_{22}$$

$$\Gamma_{in}\Big|_{S_{12}=0} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\Bigg|_{S_{12}=0} = S_{11}$$

Thus, for this **unilateral** gain element, the matching network design problem decouples, and our matching network design simplifies to these two independent equations:

$$\Gamma_s = \Gamma_{in}^* \qquad \text{and} \qquad \Gamma_L = \Gamma_{out}^*$$