Two-Port Power Gains

There are **three** standard ways of defining amplifier gain:

1. Power Gain

Power gain is defined as:

$$G \doteq \frac{P_L}{P_{in}}$$

Thus, it describes the increase in **delivered** (i.e., absorbed) power from input to output. From our power definitions, we find that:

$$
\begin{aligned}
G &= \frac{P_L}{P_{in}} \\
&= \frac{|S_{21}|^2}{|1-\Gamma_s S_{11}|^2} \frac{|1-\Gamma_s|^2}{|1-\Gamma_{out}\Gamma_L|^2}\left(1-|\Gamma_L|^2\right) \frac{|1-\Gamma_s \Gamma_{in}|^2}{|1-\Gamma_s|^2} \frac{1}{1-|\Gamma_{in}|^2} \\
&= \frac{|S_{21}|^2}{1-|\Gamma_{in}|^2} \frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2} \frac{1}{|1-\Gamma_s S_{11}|^2} |1-\Gamma_s \Gamma_{in}|^2 \\
&= \frac{|S_{21}|^2}{1-|\Gamma_{in}|^2} \frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2} \frac{1}{|1-\Gamma_s S_{11}|^2} \frac{|1-\Gamma_s S_{11}|^2 |1-\Gamma_{out}\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \\
&= \frac{|S_{21}|^2}{1-|\Gamma_{in}|^2} \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}
\end{aligned}
$$

Where we have used the fact (trust me!) that:

$$\left|1-\Gamma_s\,\Gamma_{in}\right|^2 = \frac{\left|1-\Gamma_s S_{11}\right|^2 \left|1-\Gamma_{out}\,\Gamma_L\right|^2}{\left|1-S_{22}\,\Gamma_L\right|^2}$$

2. Available Gain

Available gain is defined as:

$$G_A \doteq \frac{P_{avn}}{P_{avs}}$$

Thus, it describes the increase in **available** power from input to output. From our power definitions, we find that:

$$\begin{aligned} G_A &= \frac{P_{avn}}{P_{avs}} \\ &= \frac{\left|S_{21}\right|^2}{\left|1-\Gamma_s S_{11}\right|^2}\,\frac{\left|1-\Gamma_s\right|^2}{1-\left|\Gamma_{out}\right|^2}\;\frac{1-\left|\Gamma_s\right|^2}{\left|1-\Gamma_s\right|^2} \\ &= \frac{\left|S_{21}\right|^2}{\left|1-\Gamma_s S_{11}\right|^2}\,\frac{1-\left|\Gamma_s\right|^2}{1-\left|\Gamma_{out}\right|^2} \end{aligned}$$

3. Transducer Gain

Transducer gain is defined as:

$$G_T \doteq \frac{P_L}{P_{avs}}$$

Thus, it relates the power available from the source to the power delivered to the load. It in effect describes how **effectual** the amplifier was in extracting the available power from the source, increasing this power, and then delivering the power to the load.

$$
\begin{aligned}
G_T &= \frac{P_L}{P_{avs}} \\
&= \frac{|S_{21}|^2}{|1-\Gamma_s S_{11}|^2} \frac{|1-\Gamma_s|^2}{|1-\Gamma_{out}\Gamma_L|^2}\left(1-|\Gamma_L|^2\right) \frac{1-|\Gamma_s|^2}{|1-\Gamma_s|^2} \\
&= \frac{|S_{21}|^2 \, 1-|\Gamma_s|^2 \left(1-|\Gamma_L|^2\right)}{1} \frac{1}{|1-\Gamma_{out}\Gamma_L|^2 \, |1-\Gamma_s S_{11}|^2} \\
&= \frac{|S_{21}|^2 \, 1-|\Gamma_s|^2 \left(1-|\Gamma_L|^2\right)}{1} \frac{1}{|1-\Gamma_s \Gamma_{in}|^2 \, |1-\Gamma_L S_{22}|^2} \\
&= \frac{|S_{21}|^2 \left(1-|\Gamma_s|^2\right) 1-|\Gamma_L|^2}{|1-\Gamma_s \Gamma_{in}|^2 \, |1-\Gamma_L S_{22}|^2}
\end{aligned}
$$

There are likewise a few **special cases** that we need to be aware of. If both the source and the load impedance are Z_0, then we find $\Gamma_s = \Gamma_L = 0$, and then not surprisingly:

$$G_T = |S_{21}|^2$$

Additionally, we often find that $S_{12} = 0$ (or least approximately so), and as a result $\Gamma_{in} = S_{11}$, so:

$$G_T = \frac{|S_{21}|^2 \left(1-|\Gamma_s|^2\right) 1-|\Gamma_L|^2}{|1-\Gamma_s S_{11}|^2 \, |1-\Gamma_L S_{22}|^2} \doteq G_{TU}$$

We call this gain the **unilateral transducer power gain** G_{TU}

Q: *I'm so confused! **Which** gain definitions should I use when specifying an amp? **Which** gain definition do amplifier vendors use to specify their performance?*

A: We find that for a **well-designed** amplifier, the three gain values generally do **not** provide significantly differing values. Your book (on page 539-540) provides a typically example, where G=5.58, G_A=5.85, and G_T=5.49.

Most often then, microwave amplifier vendors do **not** explicitly specify the three values (for an assumed Z_0 source and load impedance). Instead, they provide a somewhat ambiguous value that they simply call **gain***.

* If you are inclined to be mischievous, ask an amplifier vendor if their gain spec. is actually **available** gain or **transducer** gain.