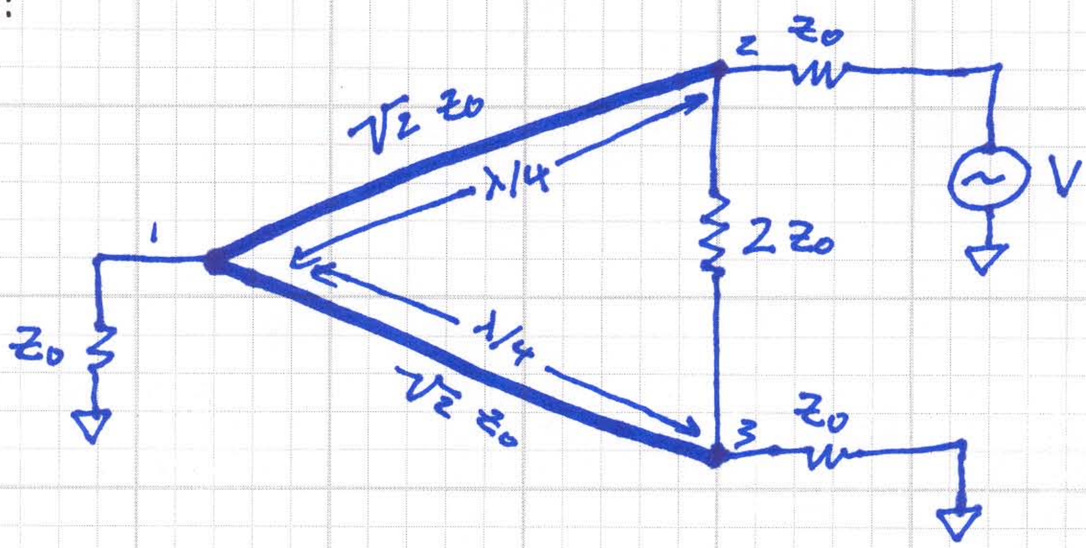
Even and Odd Mode Analysis

Consider a matched **Wilkinson power divider**, with a source at port 2:



Q: *How do we **analyze** this circuit ?*

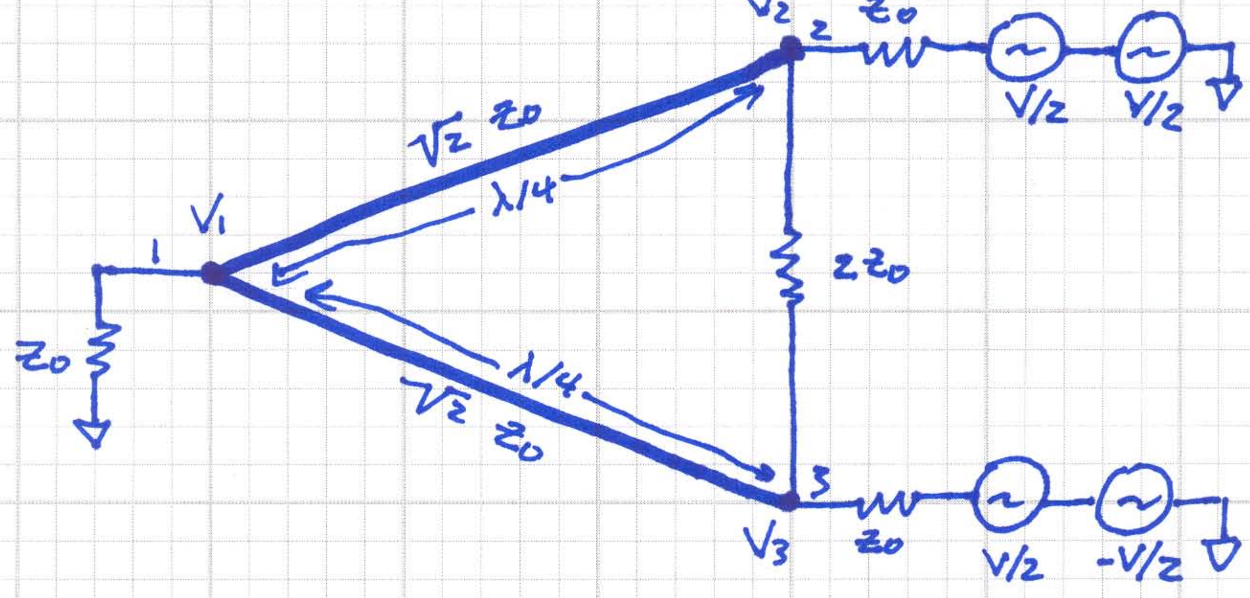
A: Use **Even-Odd mode** analysis!

Even-Odd mode analysis uses **two** important principles:

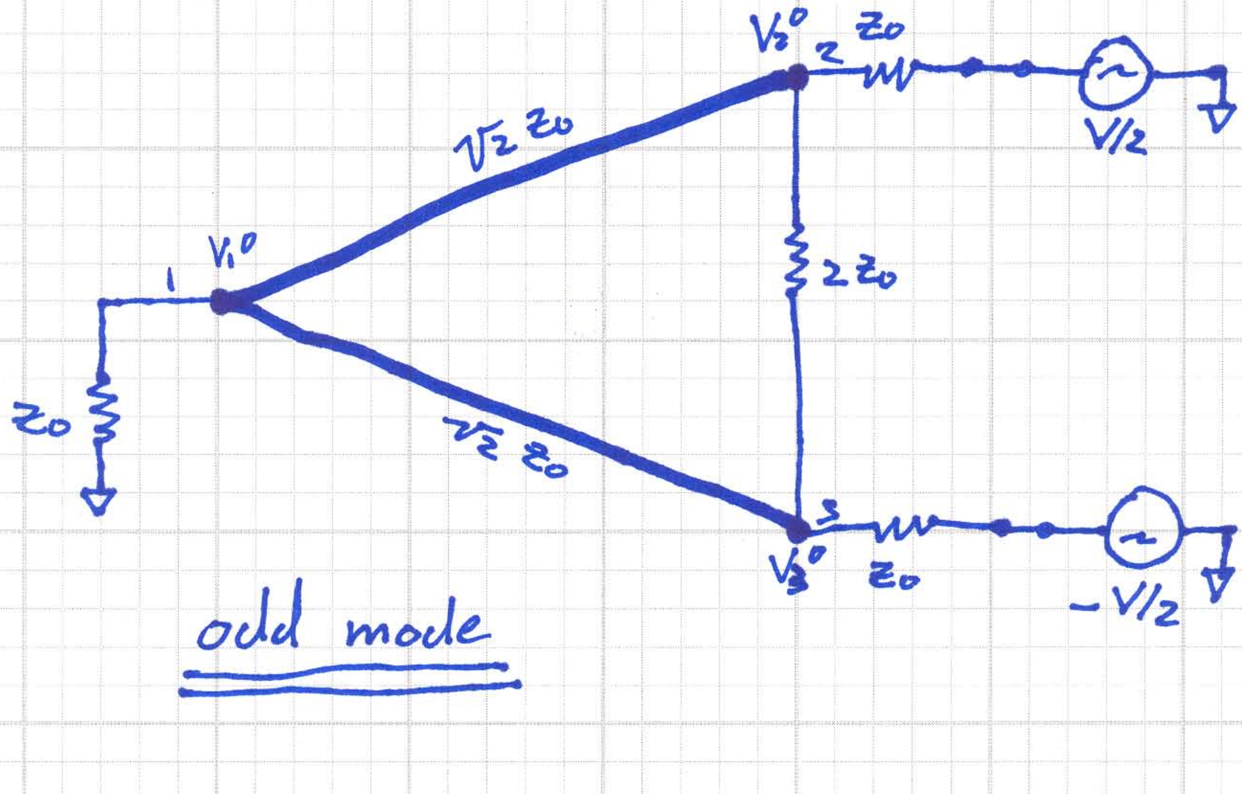
a) **superposition**

b) circuit **symmetry**

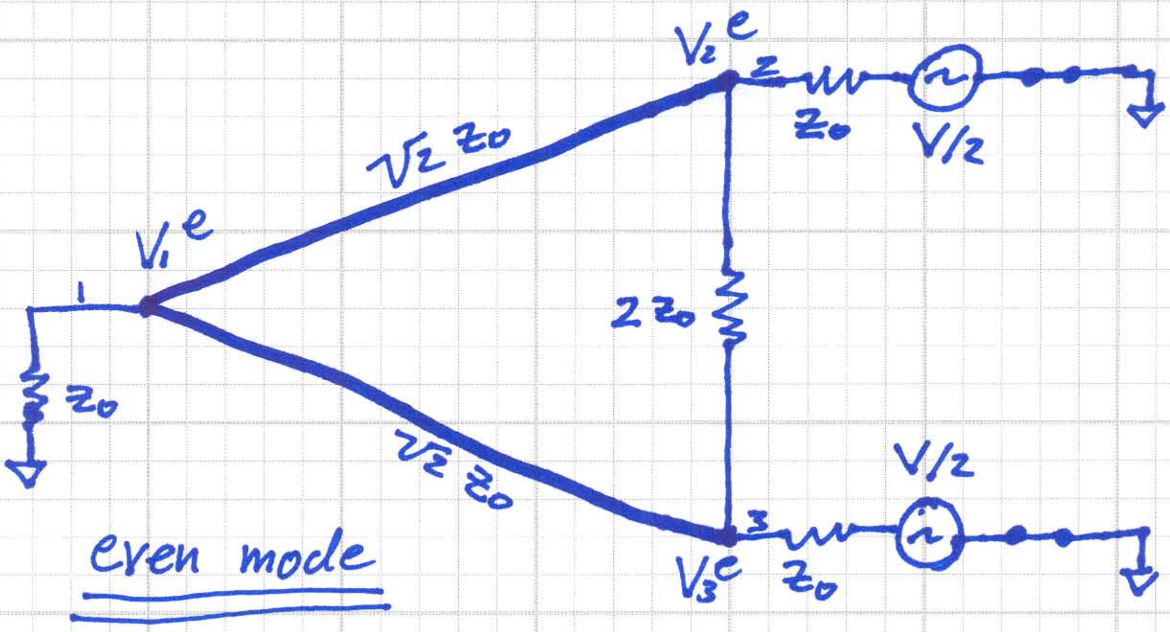
To see how we apply these principles, let's first rewrite the circuit with **four** voltage sources:



We can thus use superposition to determine all the currents and voltages within this circuit. We can first determine all **odd mode** currents and voltages:



And then determine all **even mode** voltages and currents:



And thus the voltages and currents within the circuit is simply the **sum** of the two modes:

$$V_1 = V_1^o + V_1^e$$

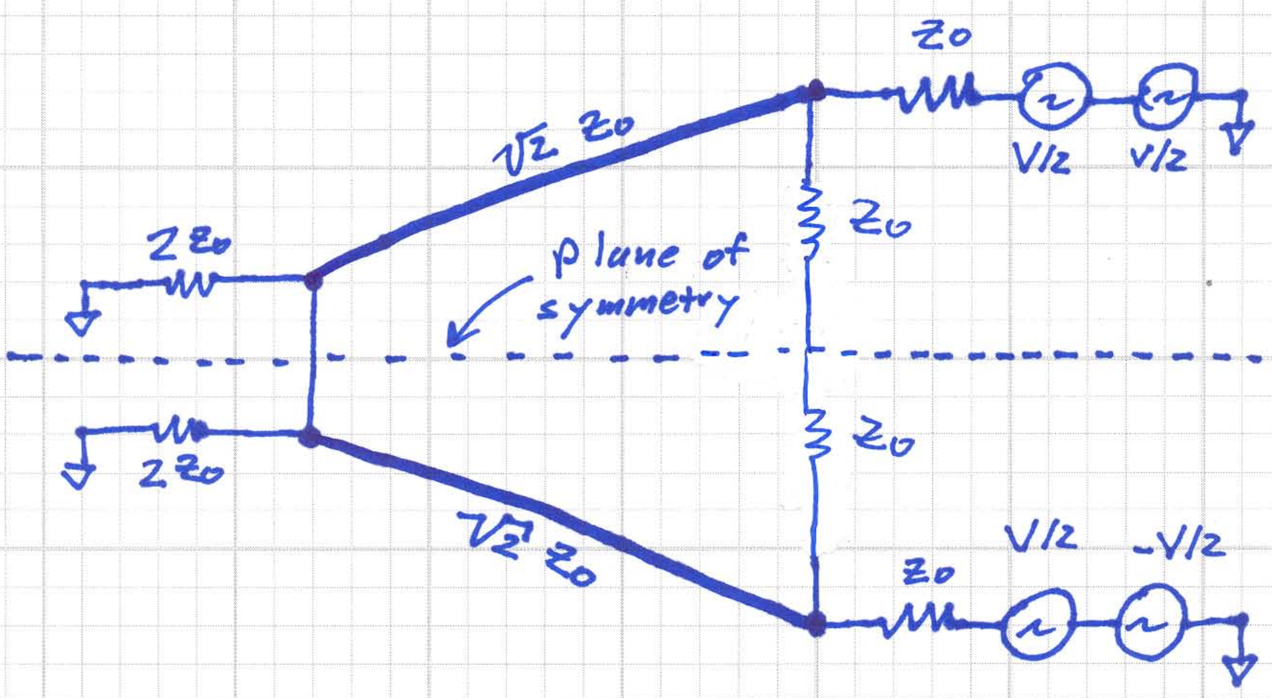
$$V_2 = V_2^o + V_2^e$$

$$V_3 = V_3^o + V_3^e$$

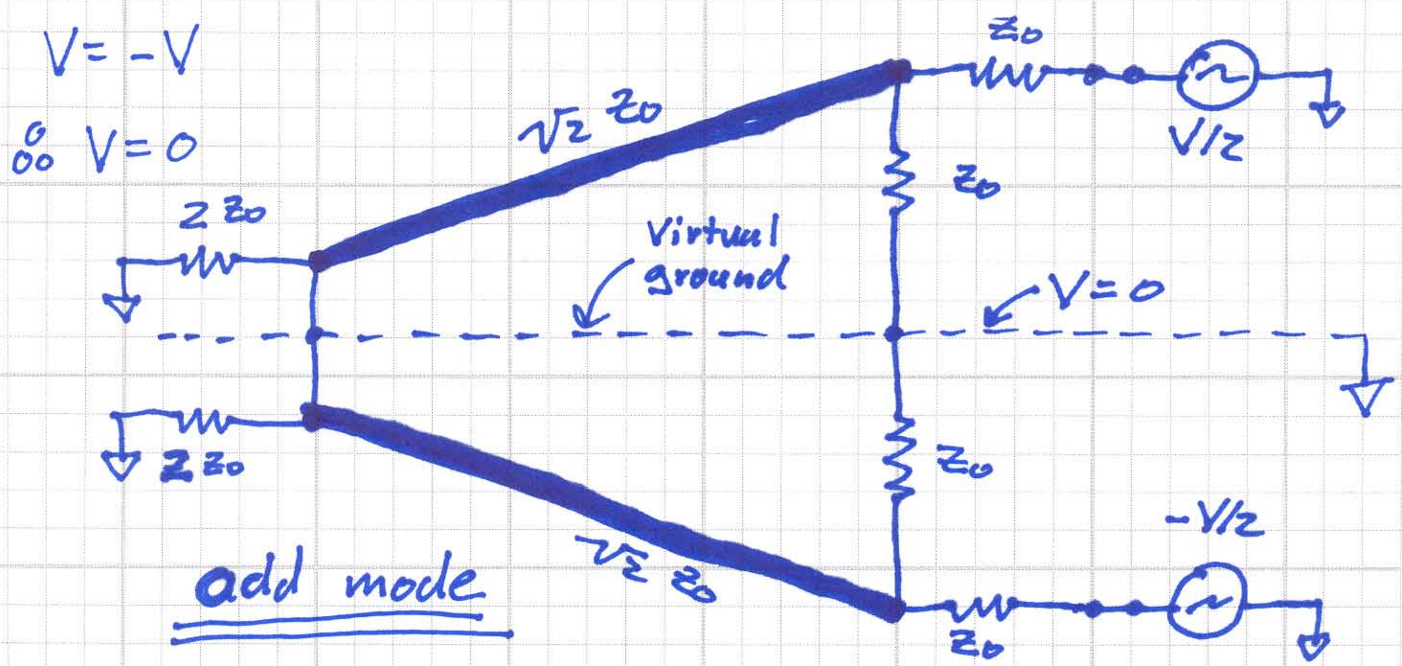
Q: *Yikes!* ***Why*** *in the world would we want to analyze a circuit* ***this*** *way? This "odd-even mode" analysis seems to just make things much* ***harder****!*

A: Be patient! We have yet to apply our second principle—circuit **symmetry**.

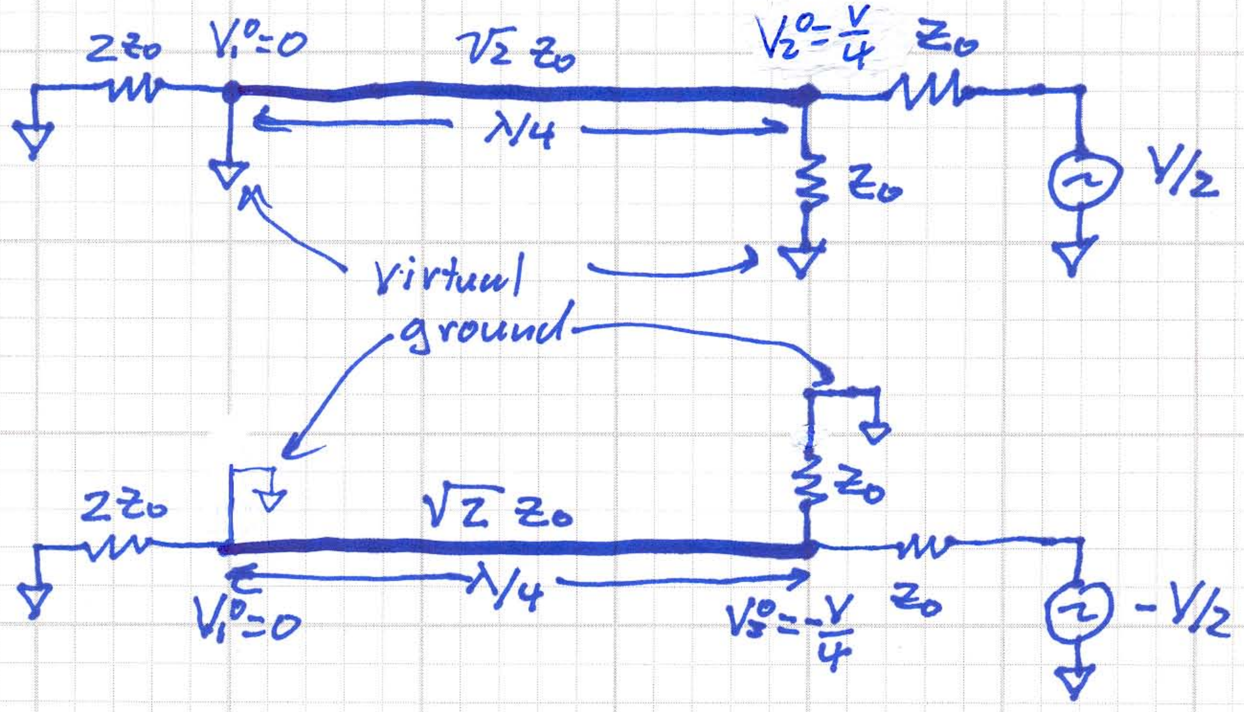
Note that we can rewrite the circuit with a perfect plane of circuit **symmetry**:



Consider now this symmetry with respect to the **odd** mode. It is evident that at **every** point along this symmetry plane, the **voltage** must be **zero**!

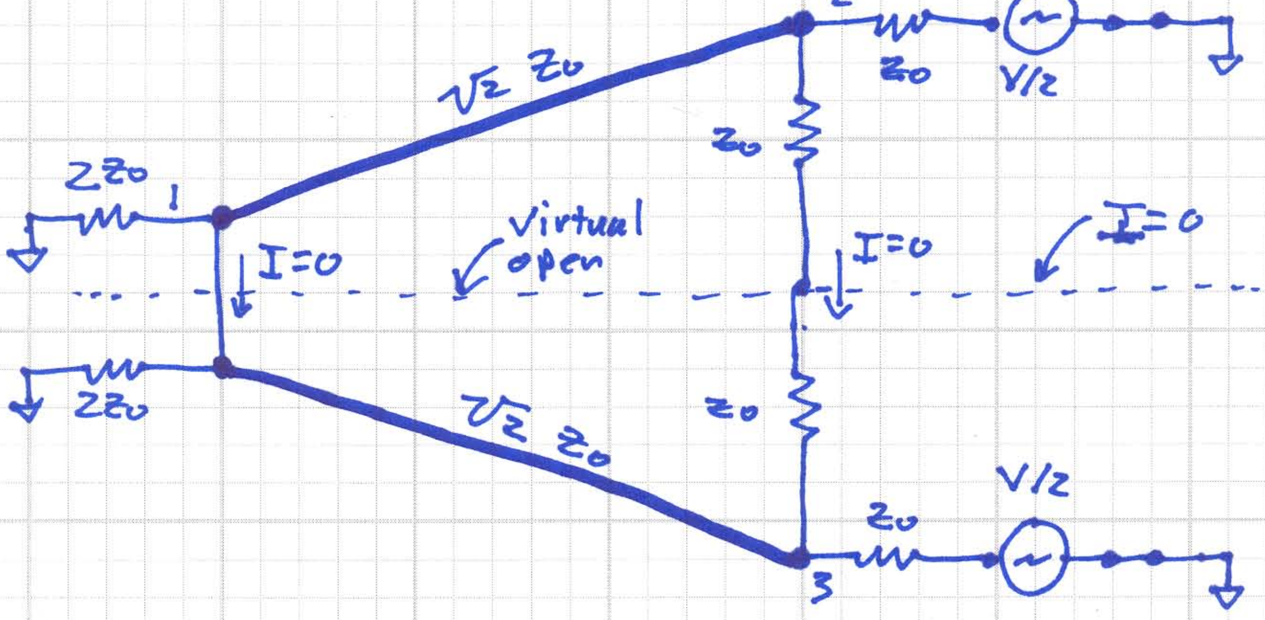


We call this odd mode symmetry plane a **virtual short**, as the voltage along this plane **must** be zero. As a result, we can divide the odd mode circuit into **two** pieces:

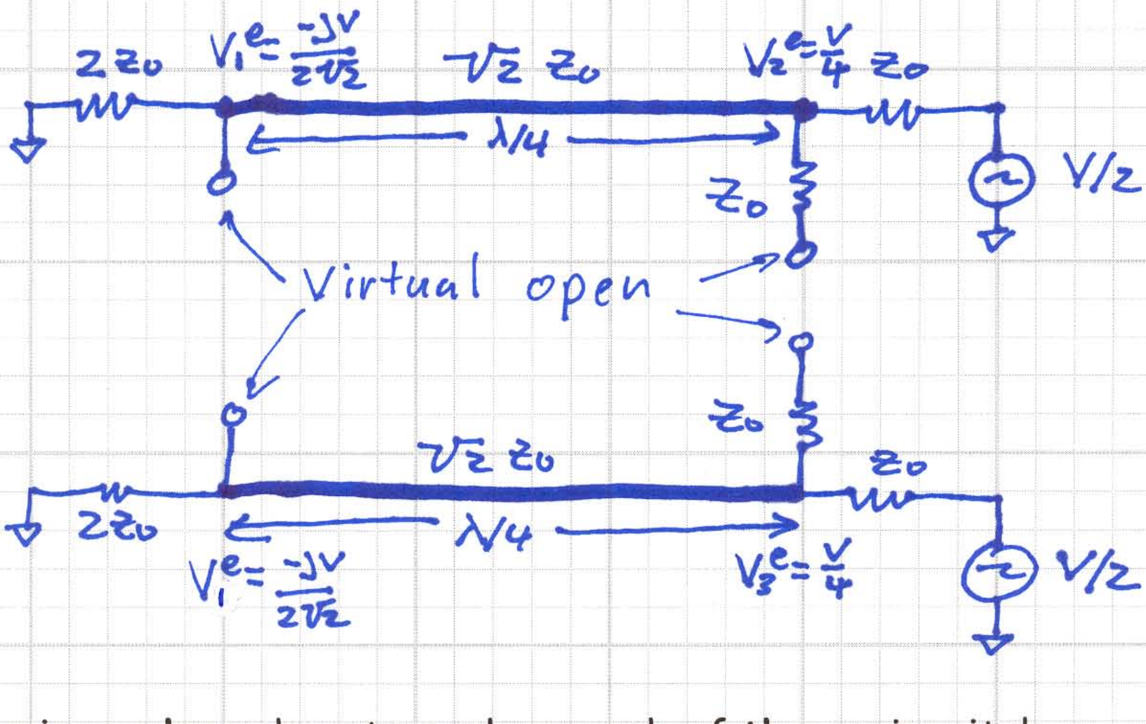


We **can** analyze these circuits!

Likewise, when we apply the **even mode** analysis, we find that at **every** point along this symmetry plane, the **current** crossing it must be **zero**!



We call this even mode symmetry plane a **virtual open**, as the **current** along this plane **must** be zero. As a result, we can divide the **even** mode circuit into **two** pieces:



Again, we **know** how to analyze each of **these** circuits!

Applying **superposition**, we can find the **total** voltages and currents by simply **adding** the results of each mode:

$$V_1 = V_1^o + V_1^e$$
$$= 0 + \frac{-jV}{2\sqrt{2}} = \frac{-jV}{2\sqrt{2}}$$

$$V_2 = V_2^o + V_2^e$$
$$= \frac{V}{4} + \frac{V}{4} = \frac{V}{2}$$

$$V_3 = V_3^o + V_3^e$$
$$= \frac{-V}{4} + \frac{V}{4} = 0$$

And from these results we can determine the following **scattering parameters**:

$$S_{22} = 0$$

$$S_{12} = \frac{-j}{\sqrt{2}}$$

$$S_{32} = 0$$

where $V_2^+(z_2=0) = \frac{V}{2}$, $V_1^-(z_1=0) = \frac{-j}{2\sqrt{2}}$

$V_3^-(z_3=0) = 0$, $V_2^-(z_2=0) = 0$

And from circuit **symmetry** and **reciprocity**, we can conclude:

$$S_{33} = 0$$

$$S_{21} = \frac{-j}{\sqrt{2}}$$

$$S_{23} = 0$$

$$S_{13} = \frac{-j}{\sqrt{2}}$$

$$S_{31} = \frac{-j}{\sqrt{2}}$$