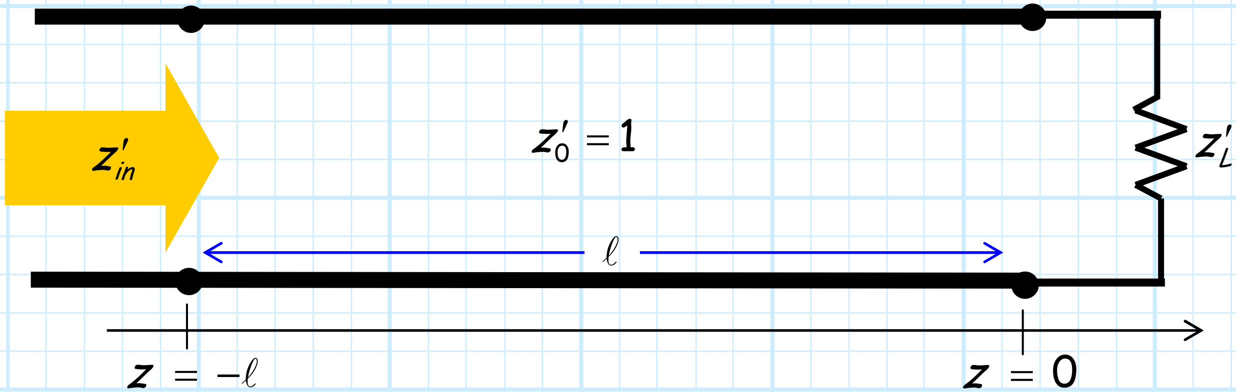
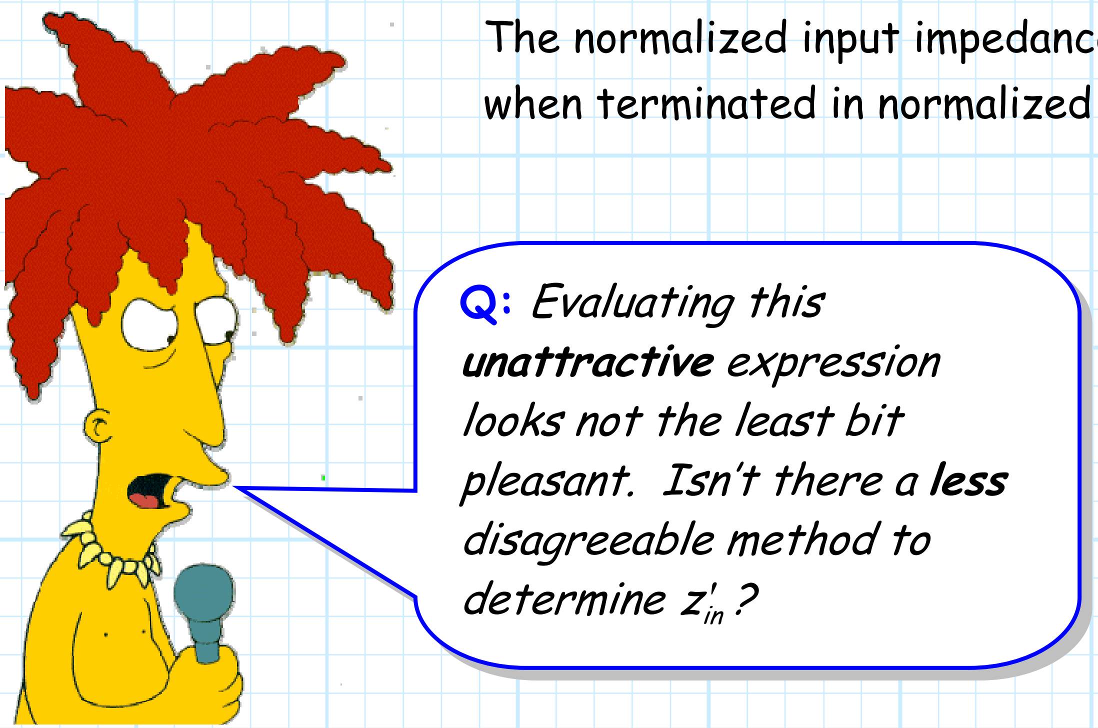
Zin Calculations using the Smith Chart



The normalized input impedance z'_{in} of a transmission line **length** ℓ, when terminated in normalized **load** z'_L, **can** be determined as:

$$\begin{aligned} z'_{in} &= \frac{Z_{in}}{Z_0} \\ &= \frac{1}{Z_0} Z_0 \left(\frac{Z_L + j\, Z_0 \tan\beta\ell}{Z_0 + j\, Z_L \tan\beta\ell} \right) \\ &= \frac{Z_L/Z_0 + j \tan\beta\ell}{1 + j\, Z_L/Z_0 \tan\beta\ell} \\ &= \frac{z'_L + j \tan\beta\ell}{1 + j\, z'_L \tan\beta\ell} \end{aligned}$$



A: **Yes** there is! Instead, we could determine this normalized input impedance by following these **three** steps:

1. Convert z'_L to Γ_L, using the equation:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z'_L - 1}{z'_L + 1}$$

2. Convert Γ_L to Γ_{in}, using the equation:

$$\Gamma_{in} = \Gamma_L \, e^{-j2\beta\ell}$$

3. Convert Γ_{in} to z'_{in}, using the equation:

$$z'_{in} = \frac{Z_{in}}{Z_0} = \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

Q: *But performing these **three** calculations would be even **more** difficult than the **single** step you described earlier. What short of dimwit would ever use (or recommend) this approach?*

A: The benefit in this last approach is that **each** of the three steps can be executed using a **Smith Chart—no** complex calculations are required!

1. **Convert z'_L to Γ_L**

Find the point z'_L from the impedance mappings on your Smith Chart. **Place you pencil at that point—you have now located the correct Γ_L on your complex Γ plane!**

For **example**, say $z'_L = 0.6 - j1.4$. We find on the Smith Chart the circle for r=0.6 and the circle for x=-1.4. The **intersection** of these two circles is the point on the complex Γ plane corresponding to normalized impedance $z'_L = 0.6 - j1.4$.

This point is a **distance** of 0.685 units from the origin, and is located at **angle** of –65 degrees. Thus the value of Γ_L is:

$$\Gamma_L = 0.685\, e^{-j65^\circ}$$

2. Convert Γ_L to Γ_{in}

Since we have correctly located the point Γ_L on the complex Γ plane, we merely need to **rotate** that point **clockwise** around a circle ($|\Gamma| = 0.685$) by an angle $2\beta\ell$.

When we **stop**, we are located at the point on the complex Γ plane where $\Gamma = \Gamma_{in}$!

For **example**, if the length of the transmission line terminated in $z'_L = 0.6 - j1.4$ is $\ell = 0.307\lambda$, we should rotate around the Smith Chart a total of $2\beta\ell = 1.228\pi$ radians, or 221°. We are now at the point on the complex Γ plane:

$$\Gamma = 0.685\, e^{+j74^\circ}$$

This is the value of Γ_{in}!

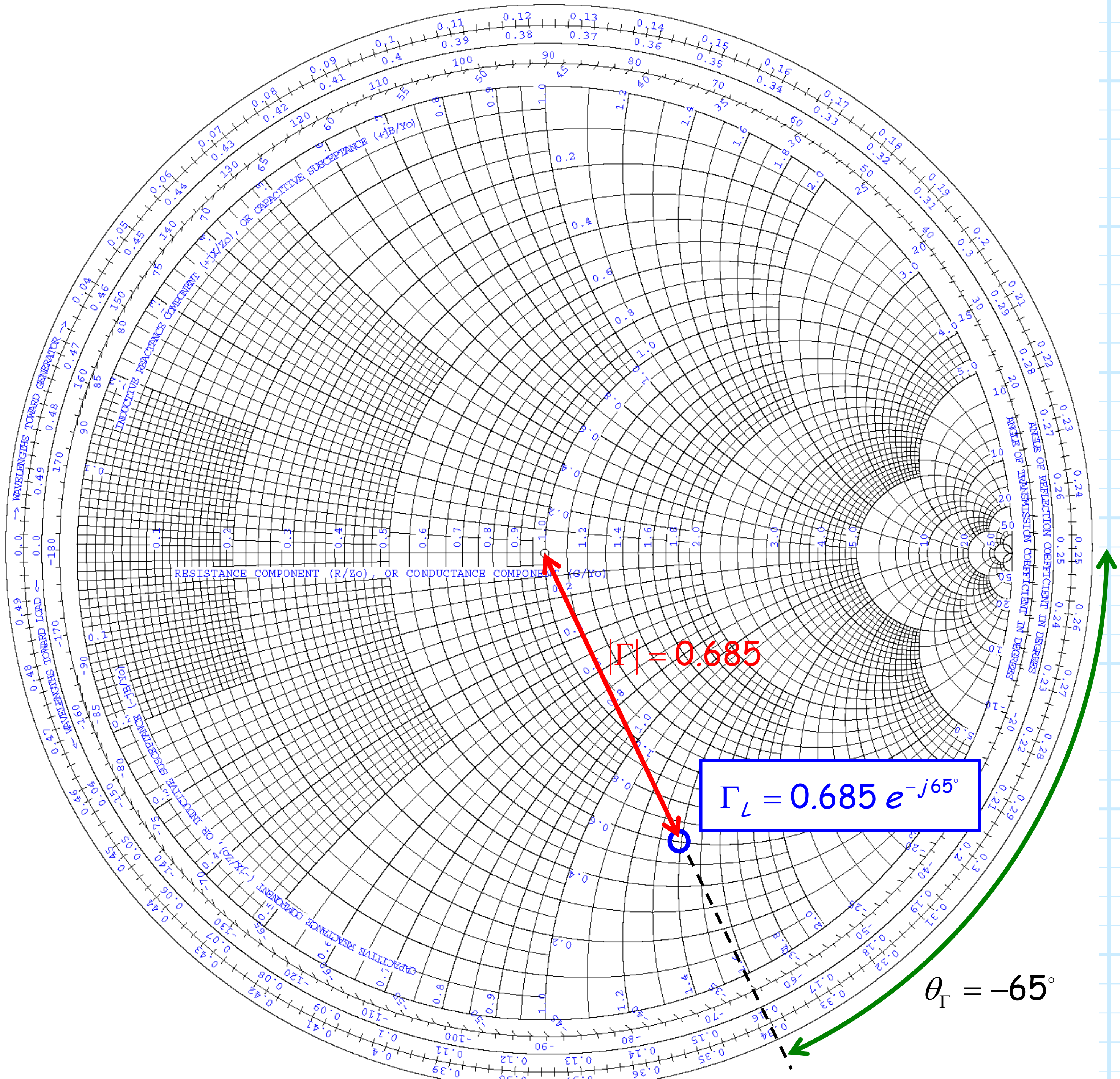
3. Convert Γ_{in} to z'_{in}

When you get finished rotating, and your pencil is located at the point $\Gamma = \Gamma_{in}$, **simply lift your pencil and determine the values *r* and *x* to which the point corresponds!**

For **example**, we can determine directly from the Smith Chart that the point $\Gamma_{in} = 0.685\, e^{+j74^\circ}$ is located at the **intersection** of circles *r* = 0.5 and *x* =1.2. In other words:

$$z'_{in} = 0.5 + j1.2$$

Step 1

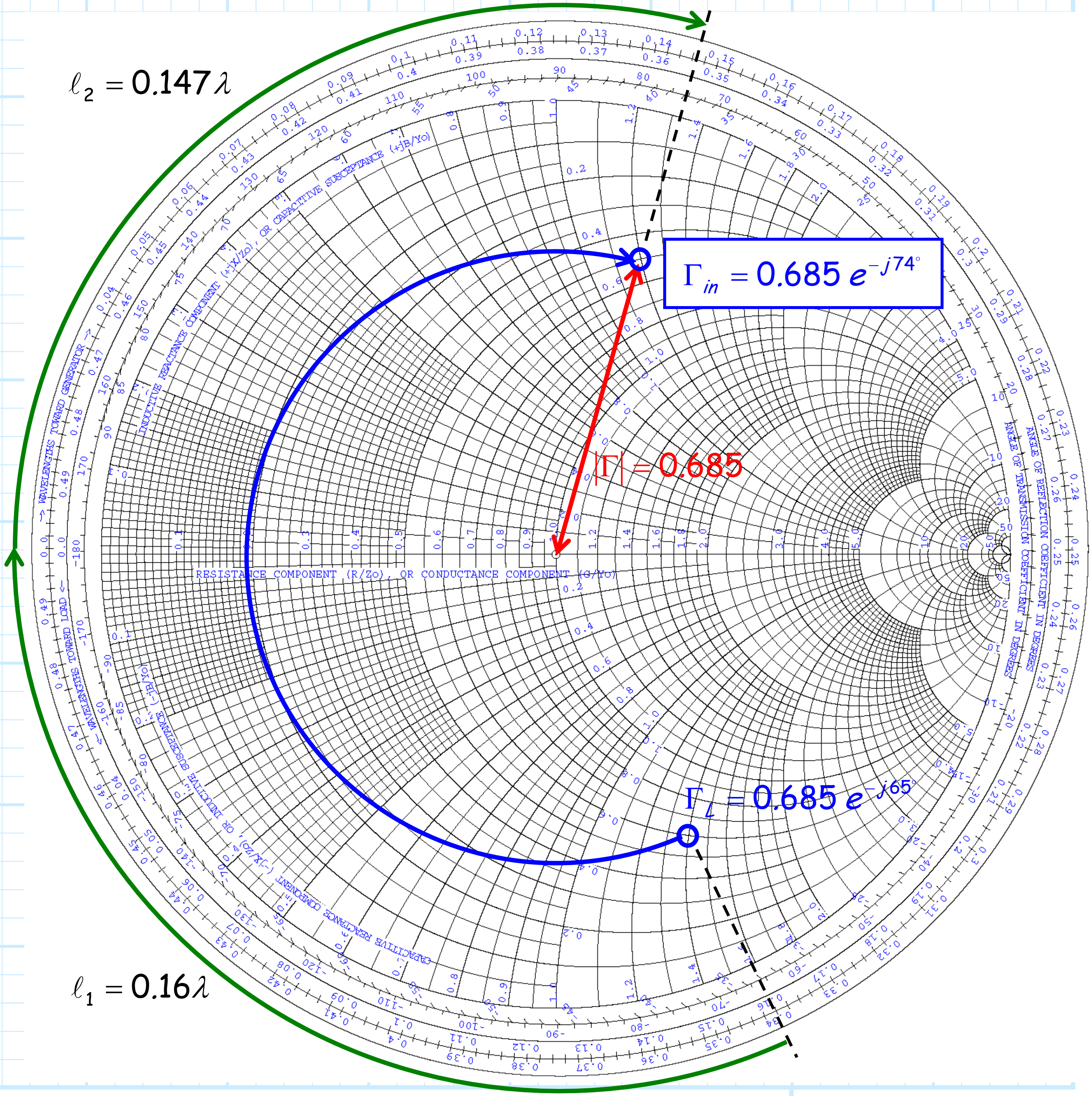


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
$|\Gamma| = 0.685$
$\Gamma_L = 0.685\, e^{-j65^\circ}$
$\theta_\Gamma = -65^\circ$

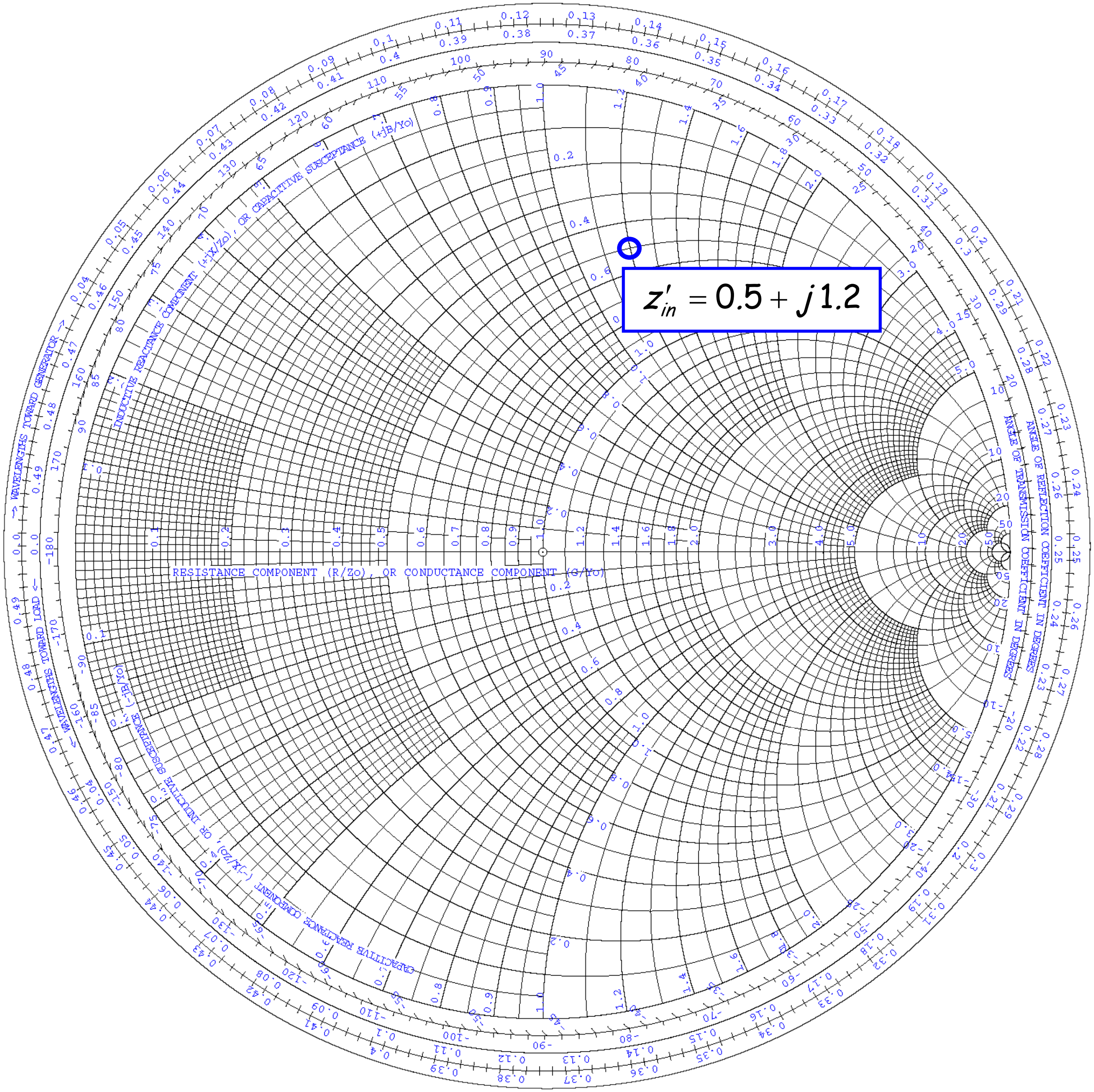
Step 2

$$\ell = \ell_1 + \ell_2$$
$$= 0.160\lambda + 0.147\lambda$$
$$= 0.307\lambda$$

$$2\beta\ell = 221^\circ$$



Step 3


$z'_{in} = 0.5 + j1.2$
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES