**Example: Input Impedance**

Consider the following circuit:

If we ignored our new μ-wave knowledge, we might erroneously conclude that the input impedance of this circuit is:

Therefore:

$$Z_{in} = \frac{-j3(2+1+j2)}{-j3+2+1+j2} = \frac{6-j9}{3-j} = 2.7-j2.1$$

Of course, this is not the correct answer!

We must use our transmission line theory to determine an accurate value. Define $Z_f$ as the input impedance of the last section:
we find that $Z_I$ is:

$$Z_I = \frac{Z_0 (Z_L \cos \beta + j Z_0 \sin \beta)}{Z_0 \cos \beta + j Z_L \sin \beta}$$

$$= 2 \left( \frac{(1+j2) \cos (\pi/4) + j 2 \sin (\pi/4)}{2 \cos (\pi/4) + j(1+j2) \sin (\pi/4)} \right)$$

$$= 2 \left( \frac{1+j4}{j} \right)$$

$$= 8 - j2$$

Therefore, our circuit now becomes:

Note the resistor is in series with impedance $Z_I$. We can combine these two into one impedance defined as $Z_2$:

$$Z_2 = 2 + Z_1 = 2 + (8 - j2) = 10 - j2$$
Now let's define the input impedance of the middle transmission line section as \( Z_3 \):

\[
Z_3 = \frac{Z_0^2}{Z_2} = \frac{Z_0^2}{10 - j2} = \frac{1.5^2}{10 - j2} = 0.21 + j0.043
\]

Thus, we can further simplify the original circuit as:

Now we find that the impedance \( Z_3 \) is parallel to the capacitor.
We can combine the two impedances and define the result as impedance \( Z_4 \):
Now we are left with this equivalent circuit:

\[
Z_4 = -j3\| (0.21 + j0.043) \\
\quad = -j3(0.21 + j0.043) \\
\quad = -j3 + 0.21 + j0.043 \\
\quad = 0.22 + j0.028
\]

Note that the remaining transmission line section is a half wavelength! This is one of the special situations we discussed in a previous handout. Recall that the input impedance in this case is simply equal to the load impedance:

\[
Z_{in} = Z_L = Z_4 = 0.22 + j0.028
\]

Whew! We are finally done. The input impedance of the original circuit is:

\[
Z_{in} = 0.22 + j0.028
\]
Note this means that this circuit:

\[ Z_m = 0.22 + j0.028 \]

and this circuit:

\[ Z_n = 0.22 + j0.028 \]

are precisely the same! They have exactly the same impedance.