The Lossless Transmission Line

If a transmission line is lossless (i.e., R=G=0), the transmission line equations are significantly simplified!

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$= \sqrt{\frac{j\omega L}{j\omega C}}$$
$$= \sqrt{\frac{L}{C}}$$

Note the characteristic impedance of a **lossless** transmission line is purely **real** (i.e., Im{Z_0} =0)!

Propagation Constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \sqrt{(j\omega L)(j\omega C)}$$
$$= \sqrt{-\omega^2 LC}$$
$$= j\omega\sqrt{LC}$$

The wave propagation constant is purely **imaginary**!

In other words, for a **lossless** transmission line:

$$\alpha = 0 \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

Voltage and Current

The **complex functions** describing the magnitude and phase of the voltage/current at every location z along a transmission line are for a **lossless** line are:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

Line Impedance

The **complex function** describing the impedance at every point along a **lossless** transmission line is:

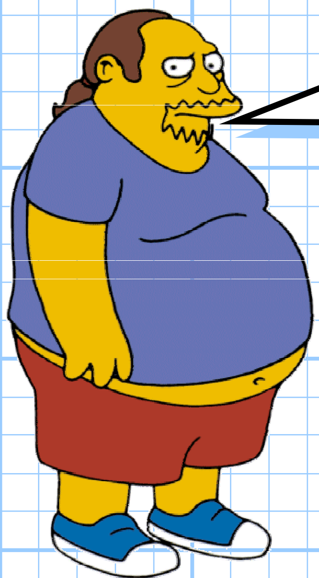
$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \frac{V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}}{V_0^+ e^{-j\beta z} - V_0^- e^{+j\beta z}}$$

Wavelength and Phase Velocity

We can now **explicitly** write the wavelength and propagation velocity of the two transmission line waves in terms of transmission line parameters L and C:

$$\lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{LC}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$



Q: *Oh **please**, continue wasting my valuable time. We both know that a **perfectly** lossless transmission line is a physical **impossibility**.*

A: True! However, a **low-loss** line **is** possible. If $R \ll \omega L$ and $G \ll \omega C$, we find that the lossless transmission line equations are excellent **approximations**!

Unless otherwise indicated, we will use the lossless equations to **approximate** the behavior of a **low-loss** transmission line.

The lone **exception** is when determining the attenuation of a **long** transmission line. For that case we will use the approximation:

$$\alpha \approx \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$

where:

$$Z_0 = \sqrt{\frac{L}{C}}$$