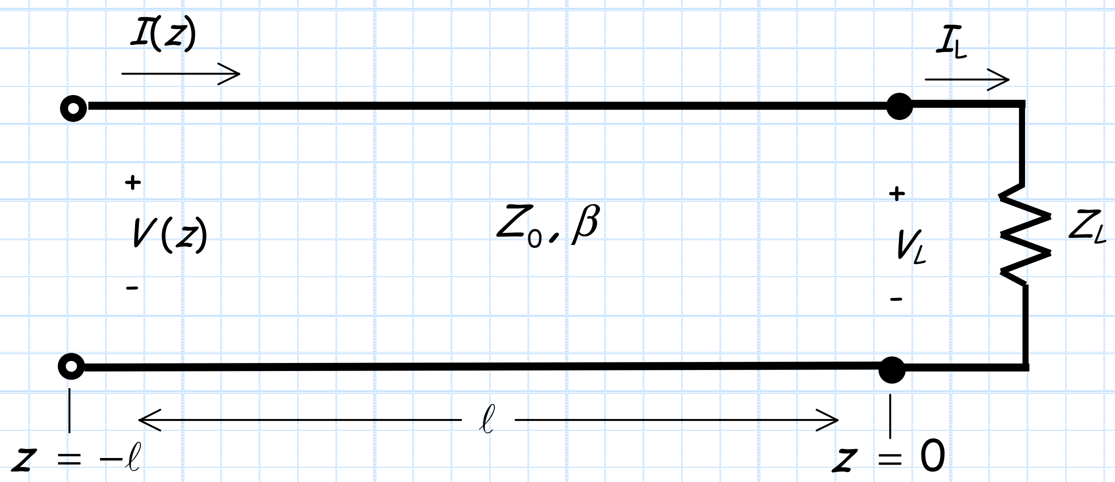
The Terminated, Lossless Transmission Line

Consider a **lossless** line, length ℓ, terminated with a **load** Z_L.



We know from the telegrapher's equations that:

$$V(z=0) = V_0^+ e^{-\beta(0)} + V_0^- e^{+\beta(0)} = V_0^+ + V_0^-$$

$$I(z=0) = \frac{V_0^+}{Z_0} e^{-\beta(0)} - \frac{V_0^-}{Z_0} e^{+\beta(0)} = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

We also know that the **load voltage** and **current** must be related by "Ohms Law":

$$\frac{V_L}{I_L} = Z_L$$

BUT, we notice that the transmission line current at z=0 **is** the current flowing into the load, while the transmission line voltage at *z=0* **is** the voltage across the load:

$$V(z=0)=V_L$$

$$I(z=0)=I_L$$

These are the **boundary conditions** of transmission line problem, and result in yet another equation that I(z) and V(z) must satisfy:

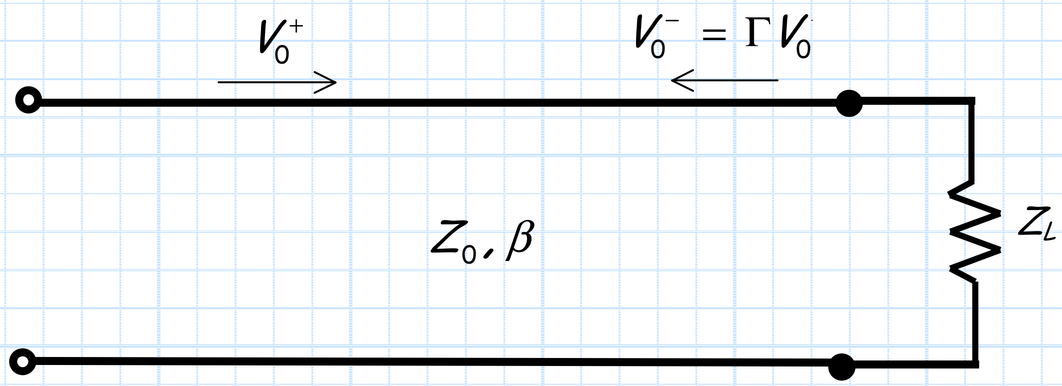
$$Z_L=\frac{V_L}{I_L}=\frac{V(z=0)}{I(z=0)}=\frac{\left(V_0^{+}+V_0^{-}\right)}{\left(\frac{V_0^{+}}{Z_0}-\frac{V_0^{-}}{Z_0}\right)}$$

Rearranging, we find that the two complex coefficients V_0^{+} and V_0^{-} are no longer **independent**, but instead must satisfy the following:

$$\frac{V_0^{-}}{V_0^{+}}=\frac{Z_L-Z_0}{Z_L+Z_0}\doteq\Gamma$$

The value Γ is a complex coefficient known as the **reflection coefficient**. It relates the magnitude and phase of the wave **incident** on the load (V_0^{+}) to the magnitude and phase of the wave emerging (i.e., **reflected** from) the load (V_0^{-}).

$$V_0^{-}=\Gamma V_0^{+}$$



Some **interesting things** to note about the reflection coeficient:

1) Since $\text{Re}\{Z_L\} > 0$, $|\Gamma| \leq 1$.

2) The current and voltage along a terminated transmission line can be written as:

$$V(z) = V_0^+ \left[e^{-j\beta z} + \Gamma e^{+j\beta z} \right]$$

$$I(z) = \frac{V_0^+}{Z_0} \left[e^{-j\beta z} - \Gamma e^{+j\beta z} \right]$$

Q: How do we determine V_0^+ ??

A: We require a **second** boundary condition to determine V_0^+. The only boundary left is at the **other end** of the transmission line. Typically, a **source** of some sort is located there. This makes physical sense, as something must generate the **incident** wave !