11.3 Single-Stage Transistor Amplifier Design

Reading Assignment: ***pp. 548-557***

We now know how to design an amplifier with maximum transducer gain. Let's look closer at the resulting device.

HO: MAXIMUM GAIN AMPLIFIERS

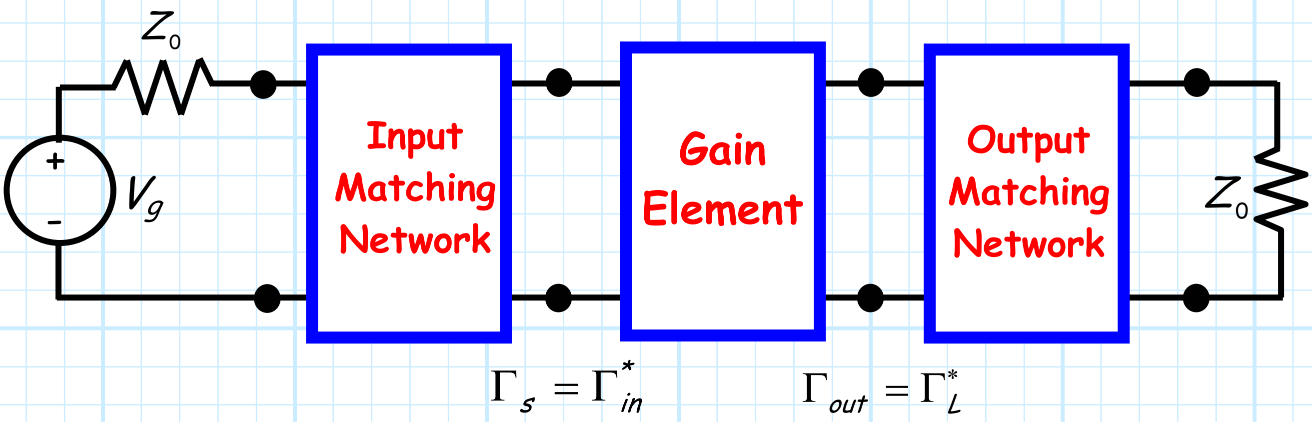
Q: *What happens if we don't like the resulting transducer gain? How can we identify a more suitable gain element?*

A: HO: THE IDEAL GAIN ELEMENT

Q: *Since we are using lossless matching networks, won't our resulting device be relatively narrow band? How can we increase the bandwidth of our design?*

A: HO: DESIGN FOR SPECIFIED GAIN

Maximum Gain Amplifiers



Q: *So if we design our amplifier such that the source is* ***matched*** *to the input of the gain element, and the output of the gain element is* ***matched*** *to the load, what* ***is*** *the resulting gain?*

A: Recall the **transducer gain** of an amplifier is:

$$G_T = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_s\Gamma_{in}|^2\,|1-\Gamma_L S_{22}|^2}$$

If the amplifier is a **unilateral** amplifier ($|S_{12}| \ll |S_{21}|$), where:

$$\Gamma_{in} = S_{11} \qquad \text{and} \qquad \Gamma_{out} = S_{22}$$

the transducer gain becomes:

$$G_{UT} = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_s S_{11}|^2\,|1-\Gamma_L S_{22}|^2}$$

Thus, inserting the **matched conditions** above, we find the transducer gain for the **matched** case is:

$$G_{T\,max} = \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

and the unilateral transducer gain for the **matched** case is:

$$\begin{aligned} G_{UT\,max} &= \frac{1}{1-|\Gamma_s|^2}|S_{21}|^2\frac{1}{1-|\Gamma_L|^2} \\ &= \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} \end{aligned}$$

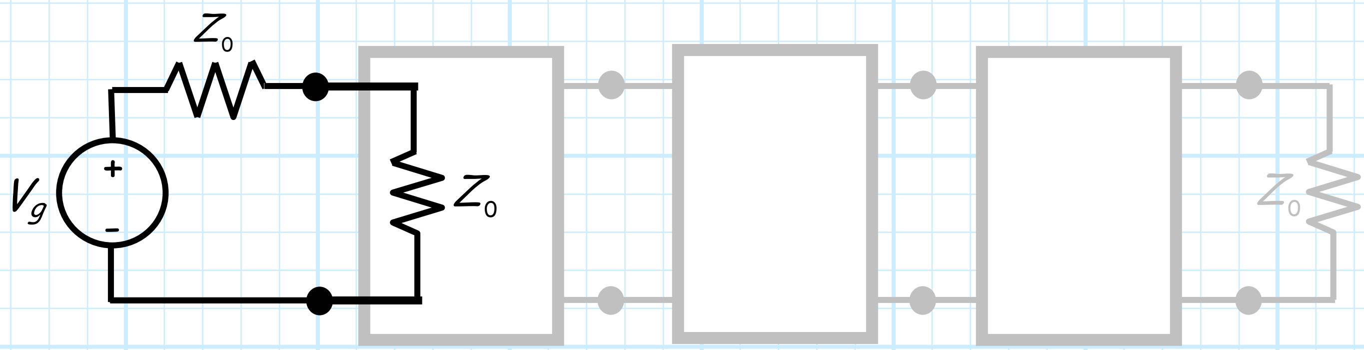
These of course are the **maximum** transducer gain possible, **given** a specific gain element, and a source and load impedance of Z_0.

Q: *What about the **scattering matrix** of the **amplifier**? Can we determine the scattering parameters of the resulting amplifier?*

A: We can certainly determine their **magnitude**!

First of all, remember that if a matching network establishes a match at its **output**, then a match is likewise present at its **input**.

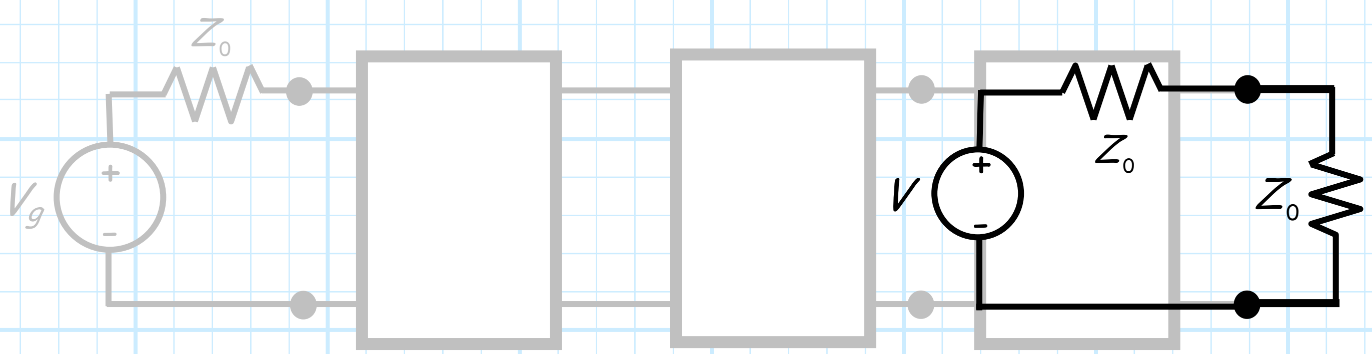
As a result, we know that the input impedance of the input matching network must be Z_0:



Meaning that the scattering parameter S_{11} of the matched amplifier is **zero**!

$$S_{11}^{amp} = 0$$

Likewise, the output impedance of the output matching network must be also be Z_0:



As a result, the scattering parameter S_{22} of the matched amplifier is also **zero**!

$$S_{22}^{amp} = 0$$

Now, since both ports of the amplifier are matched, we can determine that the magnitude of the **amplifier** scattering parameter S_{21} is simply the transducer gain $G_{T\,max}$.

$$\left|S_{21}^{amp}\right| = G_{T\,max} = \frac{1}{1-\left|\Gamma_s\right|^2}\left|S_{21}\right|^2 \frac{1-\left|\Gamma_L\right|^2}{\left|1-\Gamma_L S_{22}\right|^2}$$

Remember, the scattering parameters S_{12} and S_{21} in the expression above are those of the **gain element**.

From this result, we can likewise conclude that for the remaining scattering parameter:

$$\left|S_{12}^{amp}\right| = \frac{1}{1-\left|\Gamma_L\right|^2}\left|S_{12}\right|^2 \frac{1-\left|\Gamma_s\right|^2}{\left|1-\Gamma_s S_{11}\right|^2}$$

Note that **if** the gain element is **unilateral**, then so too will be the **amplifier**!

The Ideal Gain Element

Recall that the maximum possible transducer gain, **given a specific gain element**, and a source and load impedance of Z_0 is:

$$G_{T\,max} = \frac{1}{1-|\Gamma_s|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

By properly constructing input and output matching networks, we can maximize the transducer gain—it's the largest value that we can get for **that particular gain element**.

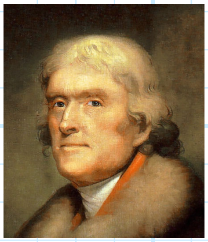
→But what if this gain is **insufficient**?

In that case we must **change the gain element**, but what should we change the gain element to? What are the characteristics of an **ideal gain element**?

The answers to these questions are best determined by examining the maximum **unilateral** transducer gain:

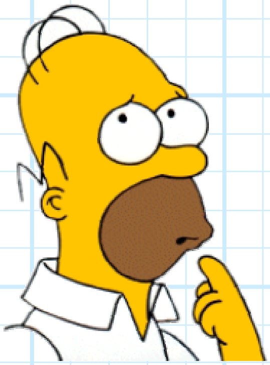
$$G_{UT\,max} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

Recall that for most gain elements, $|S_{12}|$ **is** small (i.e., approximately unilateral), and in fact $S_{12}=0$ is **one** ideal characteristic of an **ideal gain element**.



From the maximum unilateral gain expression, we can determine the remaining **ideal characteristics** of a gain element. Some of these results are rather **self-evident**, but others are a bit **surprising**!

For example, it is clear that **gain** is increased as $|S_{21}|$ is **maximized**—no surprise here. What might catch you off guard are the conclusions we reach when we observe the **denominator** of $G_{UT\,max}$:



$$G_{UT\,max} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

It appears that the gain will go to **infinity** if $|S_{11}| = 1$ and/or $|S_{22}| = 1$!

Q: *But that would mean the input and/or output impedance of the gain element is **purely reactive** (e.g. and open or a short). Is this conclusion **accurate**?*

A: Yes and no.

Remember, this maximum gain is achieved when we establish a **conjugate match**. The equation above says that this maximum gain will increase to infinity if we match to a **reactive** input/output impedance.

And **that's** the catch.

→ It is **impossible** to match Z_0 to load that is purely reactive!

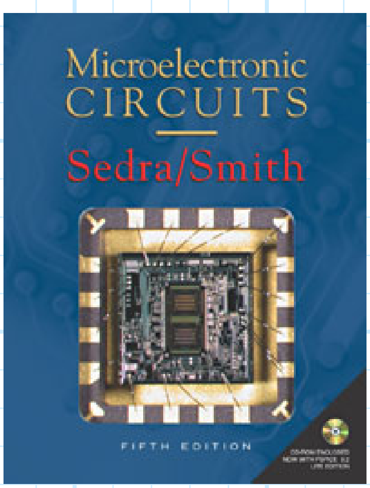
We can only match to an impedance that has a **non-zero resistive component** (i.e., $|\Gamma| < 1$); otherwise, there's no way for the available power can be **absorbed!**

Still, it is quite evident that—all other things being equal—a gain element with **larger values** of $|S_{11}|$ and $|S_{22}|$ will produce **more gain** than gain elements with smaller values of $|S_{11}|$ and $|S_{22}|$.

Q: *This seems very **counter intuitive**; I would think that an inherently **better-matched** gain element (e.g., $|S_{11}| \approx 0$ and $|S_{22}| \approx 0$) would provide **more gain**.*

A: It does doesn't it?

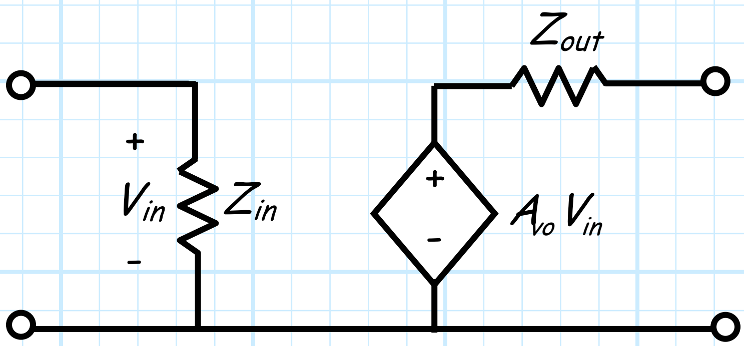
But remember back to your initial academic discussion of amplifiers (probably **way** back in an undergraduate **electronics course**).



Recall you studied **four types** of amplifier (gain element) models: voltage gain, current gain, trans-impedance, and trans-conductance. Each of these amplifiers was likewise characterized in terms of its **input impedance** and its **output impedance**.

Recall also that for each of these models, the **ideal** values of input/output impedance was **always** either zero (a short) or infinity (an open)

In other words, **ideal** amplifiers (**gain elements**) always have $S_{11} = S_{22} = 0$!



Ideally:

$|Z_{in}|$ very small

$|Z_{out}|$ very small

$|A_{vo}|$ very small

For example, an **ideal voltage amplifier** has a **high input** impedance ($|S_{11}| \approx 1$) and a **low output** impedance ($|S_{22}| \approx 1$). If we construct matching networks on either side of this ideal gain element, the result is an amplifier with **very** high transducer gain !

Q: *So how do we "**change**" a gain element to a **more ideal** one?*

A: Of course we could always select a **different** transistor, but we also could simply change the **DC bias** of the transistor we are using!

Recall the **small-signal parameters** (and thus the scattering parameters) of a transistor change as we modify the **DC bias** values. We can select our DC bias such that the value of G_{TUmax} **is maximized.**

Q: *Is there any **downside** to this approach?*

A: Absolutely! Recall that we can theoretically match to a very low or very high resistance—at precisely **one frequency**! But we found that the resulting match will typically be **extremely narrowband** for these cases.

Thus, we might consider **reducing** the amplifier gain (i.e., reducing the values $|S_{11}|$ and $|S_{22}|$), in return for achieving a more moderate gain over a **wider frequency bandwidth**!

Additionally, DC bias likewise affects **other** amplifier characteristics, including compression points and noise figure!

Design for Specified Gain

The **conjugate matched** design of course **maximizes** the transducer gain of an amplifier. But there are times when wish to design an amplifier with **less** than this maximum possible gain!

Q: *Why on Earth would we want to design such a **sub-optimal** amplifier?*

A: A general characteristic about amplifiers is that we can always trade **gain** for **bandwidth** (the gain-bandwidth product is an approximate **constant**!). Thus, if we desire a **wider** bandwidth, we must **decrease** the amplifier gain.

Q: *Just **how** do we go about doing this?*

A: We simply design a "matching" network that is actually **mismatched** to the gain element. We know that the **maximum** transducer gain will be achieved if we design a matching network such that:

$$\Gamma_s = \Gamma_{in}^* \qquad \text{and} \qquad \Gamma_L = \Gamma_{out}^*$$

Thus, a **reduced gain** (and so wider bandwidth) amplifier must have the characteristic that:

$$\Gamma_s \neq \Gamma_{in}^* \qquad \text{and} \qquad \Gamma_L \neq \Gamma_{out}^*$$

Specifically, we should select Γ_s and Γ_L (and then design the matching network) to provide the **desired** transducer gain G_T:

$$G_T = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_s\Gamma_{in}|^2\,|1-\Gamma_L S_{22}|^2} < G_{T\,max}$$

We find that there are **many values** of Γ_s and Γ_L that will provide this sub-optimal gain.

Q: *So **which** of these values do we choose?*

A: We choose the values of Γ_s and Γ_L that satisfies the above equation, **and** has the **smallest** of all possible magnitudes of $|\Gamma_s|$ and $|\Gamma_L|$.

→ Remember—**smaller** $|\Gamma|$ leads to **wider** bandwidth!

This design process is much easier if the gain element is **unilateral**. Recall for that case we find that the transducer gain is:

$$G_{UT} = \frac{\left(1-|\Gamma_s|^2\right)|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_s S_{11}|^2\,|1-\Gamma_L S_{22}|^2}$$

We can **rewrite** this gain as a product of **three terms**:

$$G_{UT} = G_S\, G_0\, G_L$$

where:

$$G_S = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2}$$

$$G_0 = |S_{21}|^2$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

Notice that the value of Γ_s affects G_S **only**, and the value of Γ_L affects G_L **only**. Therefore, the unilateral case again decouples into two **independent** problems.

We can compare the values above with their **maximum** values (when $\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$):

$$G_{S\,max} = \frac{1}{1-|S_{11}|^2}$$

$$G_0 = |S_{21}|^2$$

$$G_{L\,max} = \frac{1}{1-|S_{22}|^2}$$

Thus, to increase the bandwidth of an amplifier, we **select** values of G_S and G_L that are **less** (typically by a few dB) than the maximum (i.e., matched) values $G_{S\,max}$ and $G_{L\,max}$.

Unlike the values G_{Smax} and G_{Lmax}—where there is precisely **one** solution for each ($\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$)—there are an **infinite** number of $\Gamma_s(\Gamma_L)$ solutions for a specific value of $G_S(G_L)$.

Q: *So **which** do we choose?*

A: We choose the solutions that have the **smallest magnitude**! This will maximize our amplifier **bandwidth**.

Q: *How do we **determine** what these values are?*

A: We can solve these equations to determine all Γ_s and Γ_L solutions for **specified** design values of G_S and G_L.

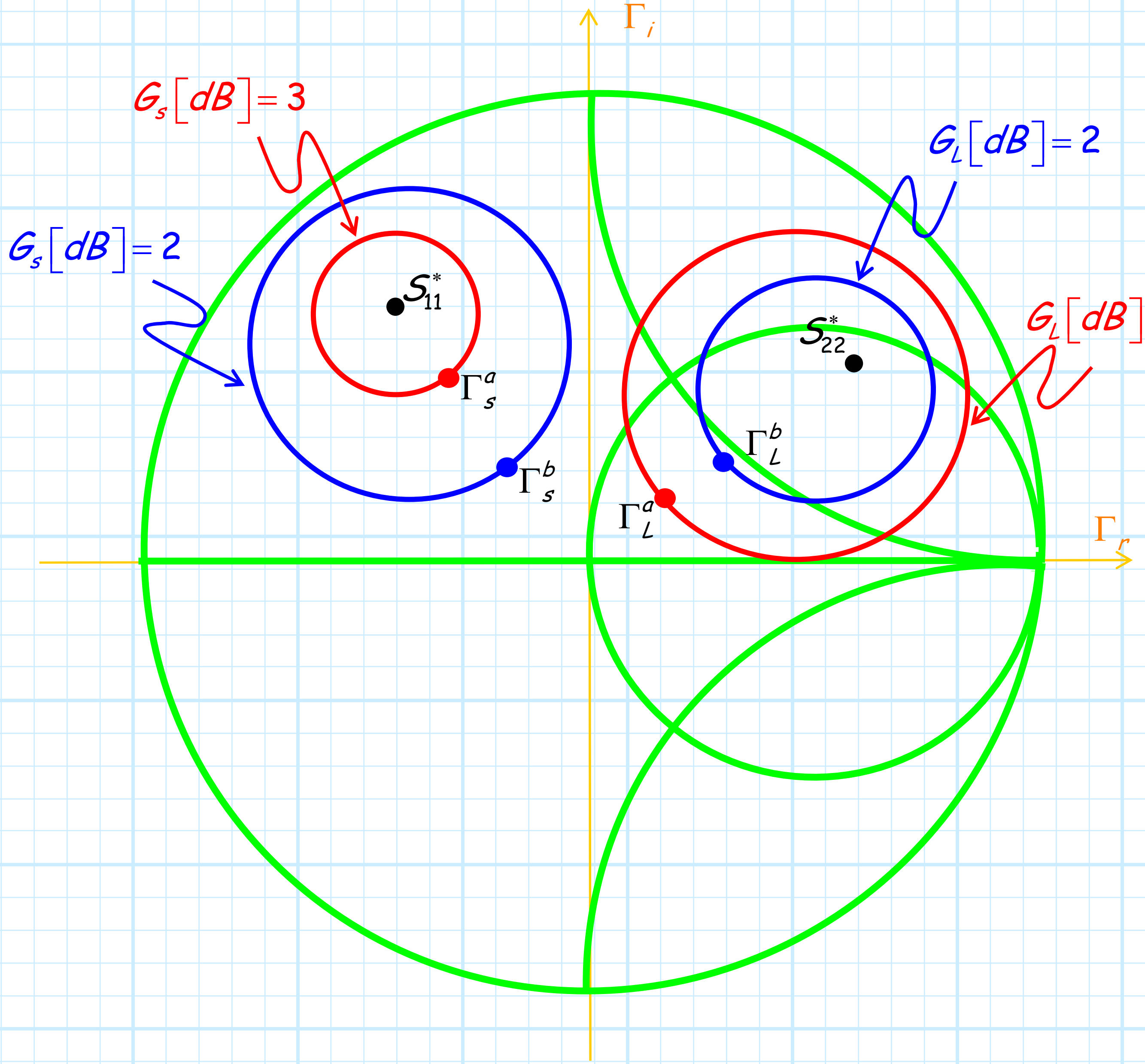
$$G_S = \frac{1-|\Gamma_s|^2}{|1-\Gamma_s S_{11}|^2} \qquad\qquad G_L = \frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2}$$

Just as with our stability solutions, the solutions to the equations above form **circles** when plotted on the **complex** Γ **plane**.

These circles are known as **constant gain circles**, and are defined by two values: a **complex** value $C_s(C_L)$ that denotes the **center** of the circle on the complex Γ plane, and a **real** value $R_s(R_L)$ that specifies the **radius** of that circle.

These solutions are provided on pages 554 and 555 of your text.

Any Γ point **on** (not inside!) a constant gain circle denotes a value of Γ that will provide the requisite gain. To minimize the bandwidth we should choose the point on the circle that is **closest to the center** of the complex Γ plane!



For **example**, say we have an amplifier with:

$$G_{Smax}[dB]=4.0 \qquad G_0[dB]=7.0 \qquad G_{Lmax}[dB]=3.0$$

such that its transducer gain is **14 dB** at its design frequency. To increase the **bandwidth** of this amplifier, we decide to **reduce** the gain to 11 dB.

Thus, we find that our design goal is:

$$G_S[dB] + G_L[dB] = 4.0$$

From the gain circles on the Smith Chart above (assuming they represent the gain circles for this gain element), we find there are **two solutions**; we'll call them **solution *a*** and **solution *b*.**

Solution *a*

We determine the values Γ_s^a and Γ_L^a from the gain circles:

$$G_S[dB] = 3.0 \qquad \text{and} \qquad G_L[dB] = 1.0$$

so that $G_S[dB] + G_L[dB] = 4.0$.

Solution *b*

We determine the values Γ_s^b and Γ_L^b from the gain circles:

$$G_S[dB] = 2.0 \qquad \text{and} \qquad G_L[dB] = 2.0$$

so that $G_S[dB] + G_L[dB] = 4.0$.

There are of course an **infinite** number of possible solutions, as there are an infinite number of solutions to $G_s[dB] + G_L[dB] = 4.0$. However, the two solutions provided here are fairly **representative**.

Q: *So **which** solution should we use?*

A: That choice is a bit **subjective**.

We note that the point Γ_L^a is **very close** to the center, while the point Γ_s^a pretty **far away** (i.e., $|\Gamma_L^a|$ is small and $|\Gamma_s^a|$ is large).

In contrast, both Γ_s^b and Γ_L^b are **fairly close** to the center, although neither is as close as Γ_L^a.

To get the widest bandwidth, I would choose **solution *b***, but the only way to know for sure is to design and **analyze both solutions**.

Often, the design with the widest bandwidth will depend on how you **define** bandwidth!

Q: *So we reduce the transducer gain by designing and constructing a **mismatched** matching network. Won't that result in **return loss**?*

A: Absolutely!

We find for these wideband antennas that **neither** S_{11}^{amp} **nor** S_{22}^{amp} are equal to **zero**. However, there is a bit of a **silver lining**.

A conjugate matched amplifier is not only narrow band with regard to gain, it is also **narrow band** with regard to **return loss**. **Only** at the design frequency will the amplifier ports be perfectly matched. As we move away from the design frequency, the return loss **quickly degrades!**

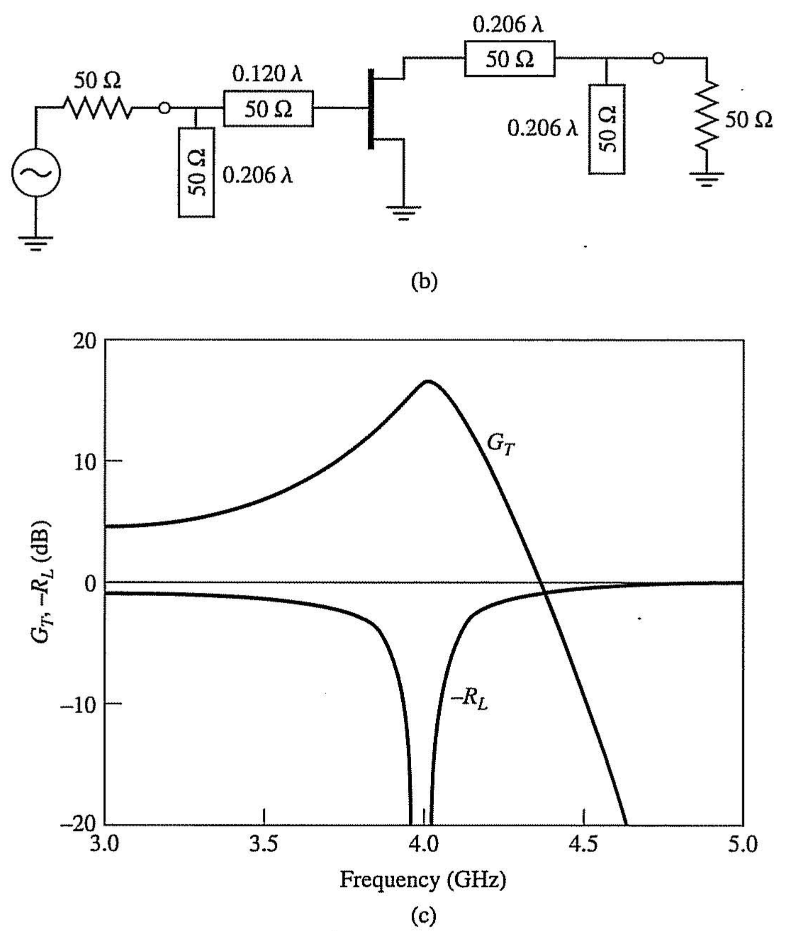


FIGURE 11.7 Continued. (b) RF circuit. (c) Frequency response.

With the "mismatched" design, we typically find that the return loss is **better** at frequencies away from the design frequency (as compared to the matched design), although at **no frequency** do we achieve a **perfect match** (unlike the matched design).

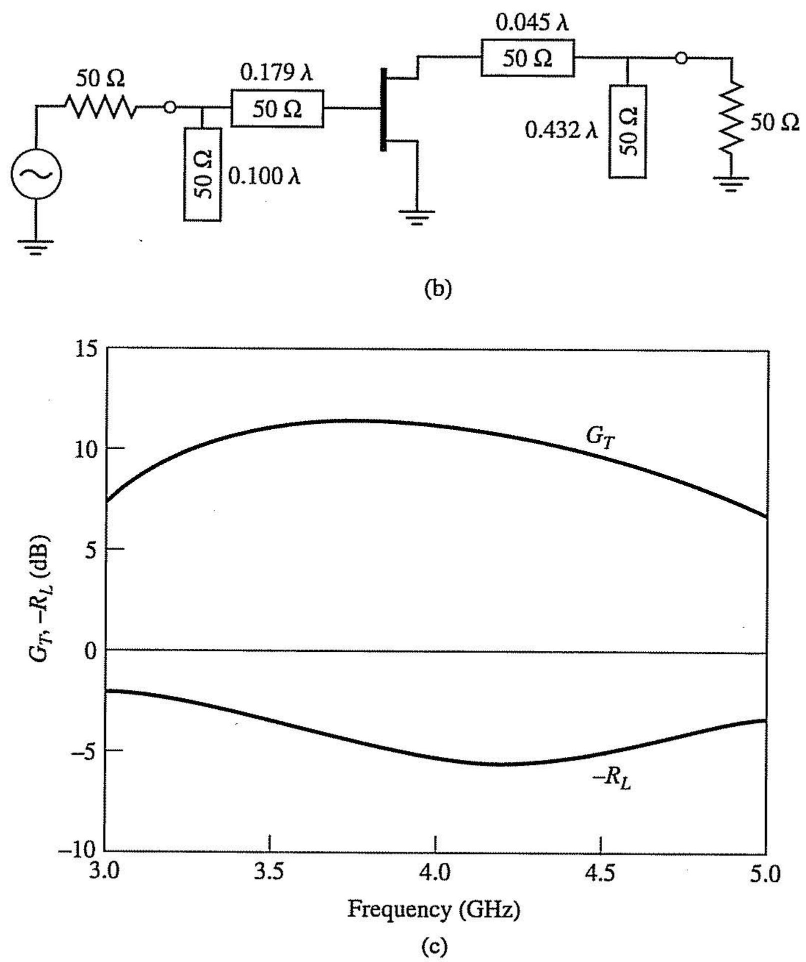


FIGURE 11.8 Continued. (b) RF circuit. (c) Transducer gain and return loss.

Generally speaking, a good (i.e., **acceptable**) return loss over a wide range of frequencies is **better** than a perfect return loss at one frequency and poor return loss everywhere else!

Q: *Won't you **ever** stop talking??*

A: Yup. I'm all done.