

4.4 - The Transmission Matrix

Reading Assignment: pp. 183-189

Let's now consider the **transmission matrix**—a matrix that can **only** be used to characterize a **two-port** device.

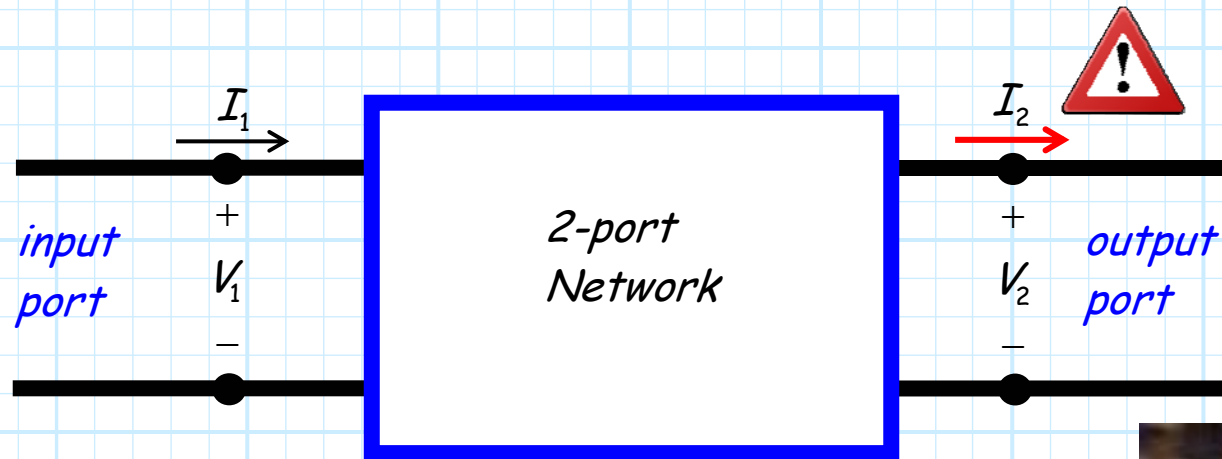
Q: *Only for 2-port devices?!? That doesn't seem particularly useful. Why bother learning about **this**?*

A: There are a **lot** of **very** important two-port devices (e.g., amplifiers, filters, attenuators).

HO: THE TRANSMISSION MATRIX

The Transmission Matrix

If a network has **two** ports, then we can **alternatively** define the voltages and currents at each port as:



Q: Say, can we somehow relate the two *input* parameters (I_1, V_1) to the two *output* parameters (I_2, V_2) ?

A: Yes we can! We can relate them with **four** parameters A, B, C, D :

$$V_1 = A V_2 + B I_2$$

$$I_1 = C V_2 + D I_2$$



Or, using linear algebra:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

The matrix is defined as the **transmission matrix** \mathcal{T} , otherwise known as the **ABCD matrix** :

$$\mathcal{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$



Q: Great. But what exactly **are** the values A , B , C , D , and how do we determine them?

A: Similar to the impedance and admittance matrices, we determine the elements of the transmission matrix using **shorts** and **opens**.

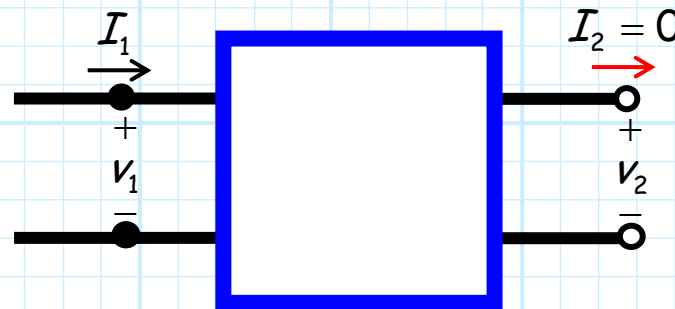
A

Note when $I_2 = 0$ then:

$$V_1 = AV_2$$

Therefore, to find value A , place an **open** on port 2, and then determine voltage V_1 in terms of V_2 . We find that:

$$A = \frac{V_1}{V_2} \quad (\text{port 2 open})$$



Note the parameter A is unitless (i.e., it is a coefficient).

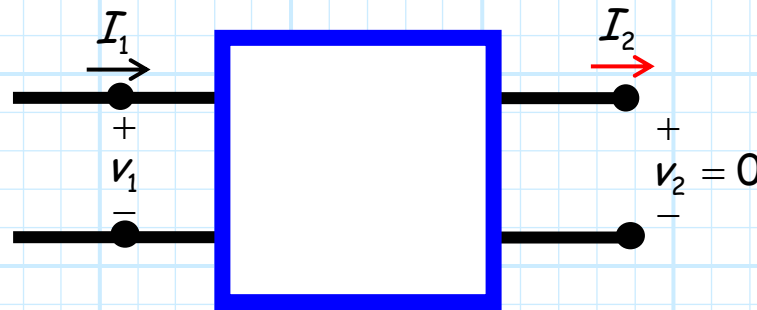
B

Note when $V_2 = 0$ then:

$$V_1 = B I_2$$

Therefore, to find value B , place a **short** on port 2, and then determine voltage V_1 in terms of I_2 . We find that:

$$B = \frac{V_1}{I_2} \quad (\text{port 2 short})$$



Note parameter B has units of **impedance** (i.e., Ohms).

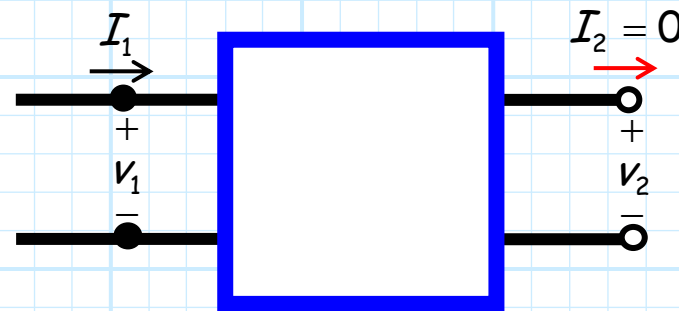
C

Note when $I_2 = 0$ then:

$$I_1 = C V_2$$

Therefore, to find value C , place an **open** on port 2, and then determine current I_1 in terms of voltage V_2 . We find that:

$$C = \frac{I_1}{V_2} \quad (\text{port 2 open})$$



Note parameter C has units of **admittance** (i.e., mhos).

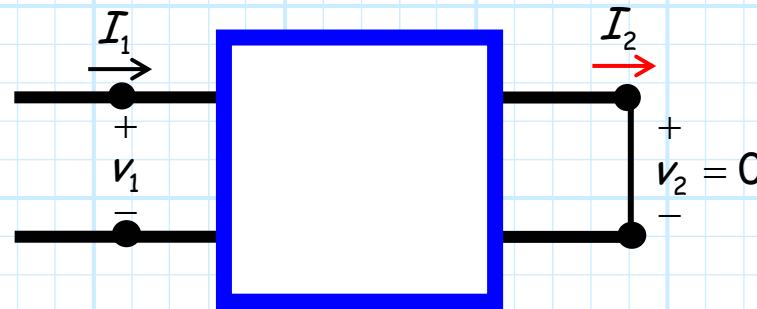
D

Note when $V_2 = 0$ then:

$$I_1 = D I_2$$

Therefore, to find value D, place a **short** on port 2, and then determine current I_1 in terms of current I_2 . We find that:

$$D = \frac{I_1}{I_2} \quad (\text{port 2 short})$$

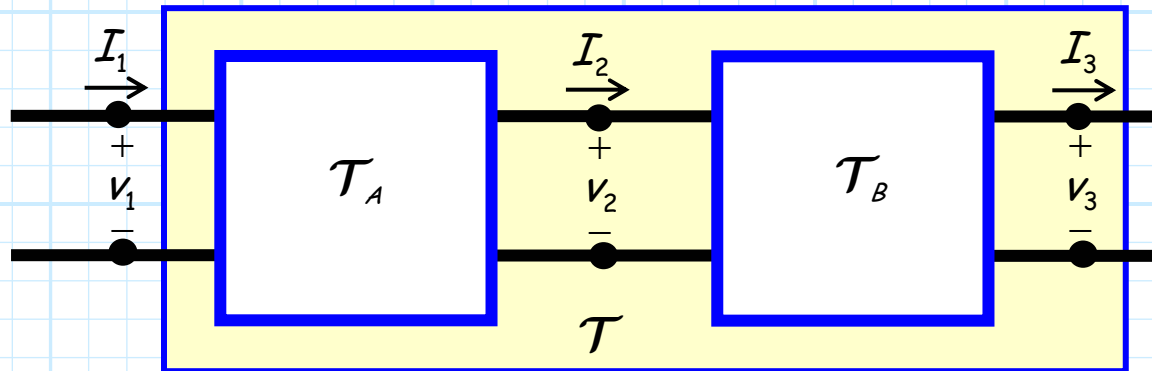


Note parameter D is unitless (another coefficient!).

Q: For cryin' out loud! We already have the impedance matrix, the scattering matrix, **and** the admittance matrix. **Why** do we need the transmission matrix **also**? Is it somehow **uniquely** useful?



A: Consider the case where a 2-port network is created by connecting (i.e., cascading) two networks:



Note

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{T}_A \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

and

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \mathcal{T}_B \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

also

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{T} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Combining the first two equations:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{T}_A \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \mathcal{T}_A \mathcal{T}_B \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

and comparing to the third:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{T}_A \mathcal{T}_B \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \mathcal{T} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

we conclude that:

$$\mathcal{T} = \mathcal{T}_A \mathcal{T}_B$$

Likewise, for N cascaded networks, the **total** transmission matrix \mathcal{T} can be determined as the product of all N networks!

$$\mathcal{T} = \mathcal{T}_1 \mathcal{T}_2 \mathcal{T}_3 \mathcal{T}_4 \cdots \mathcal{T}_N = \prod_{n=1}^N \mathcal{T}_n$$



Note this result is **only** true for the **transmission** matrix \mathcal{T} . **No** equivalent result exists for $\mathcal{S}, \mathcal{Z}, \mathcal{Y}$!

Thus, the transmission matrix can greatly **simplify** the analysis of **complex** networks constructed from **two**-port devices. We find that the \mathcal{T} matrix is particularly useful when creating design software for **CAD** applications.