5.2 - Single-Stub Tuning

Reading Assignment: pp. 228-235

Q: If we cannot use “lumped” elements like inductors or capacitors to build lossless matching networks, what can we use?

A: Recall that a section of lossless transmission line is purely reactive, thus we can build lossless matching networks using specific lengths of transmission lines.

We call these lengths of transmission line “distributed” elements.

The distributed element analogue of the lumped element L-network is the single-stub tuner.

Just like the L-network, there are two versions of this design:

HO: The Shunt-Stub Tuner

HO: The Series-Stub Tuner
Shunt Stub Tuning

Consider the follow transmission line structure, with a shunt stub:

The two design parameters of this matching network are lengths $\ell$ and $d$.

An equivalent circuit is:

where of course:
\[ Y_{in}'' = Y_0 \left( \frac{Y_L + j Y_0 \tan \beta d'}{Y_0 + j Y_L \tan \beta d'} \right) \]

and the reactance \( jB_{stub} \) of transmission line stub of length \( l \) is either:

\[
jB_{stub} = \begin{cases} 
  jY_0 \tan \beta l & \text{for an open-circuit stub} \\
  -jY_0 \cot \beta l & \text{for an short-circuit stub}
\end{cases}
\]

Therefore, for a matched circuit, we require:

\[ jB_{stub} + Y_{in}'' = Y_0 \]

Note this complex equation is actually two real equations!

i.e.,

\[ \text{Re}\{Y_{in}''\} = Y_0 \]

and

\[ \text{Im}\{jB_{stub} + Y_{in}''\} = 0 \quad \Rightarrow \quad B_{stub} = -B_{in}'' \]

where

\[ B_{in}'' = \text{Im}\{Y_{in}''\} \]
Since $Y''_i$ is dependent on $d'$ only, our design procedure is:

1) Set $d'$ such that $\text{Re}(Y''_i) = Y_0$.

2) Then set $\ell$ such that $B_{stub} = -B''_i$.

We have two choices for determining the lengths $d'$ and $\ell$. We can use the design equations (5.9, 5.10, 5.11) on p. 232,

OR

we can use the Smith Chart to determine the lengths!

1) Rotate clockwise around the Smith Chart from $y_L$ until you intersect the $g = 1$ circle. The “length” of this rotation determines the value $d'$. Recall there are two possible solutions!

2) Rotate clockwise from the short/open circuit point around the $g = 0$ circle, until $b_{stub}$ equals $-b''_i$. The “length” of this rotation determines the stub length $\ell$.

For example, your book describes the case where we want to match a load of $Z_L = 60 - j80$ (at 2 GHz) to a transmission line of $Z_0 = 50 \Omega$. 
Using **shorted** stubs, we find **two** solutions to this problem:

Whose length values $d$ and $\ell$ were determined from a Smith Chart:
Q: Two solutions! Which one do we use?

A: The one with the shortest lengths of transmission line!

Q: Oh, I see! Shorter transmission lines provide smaller and (slightly) cheaper matching networks.

A: True! But there is a more fundamental reason why we select the solution with the shortest lines—the matching bandwidth is larger!

For example, consider the frequency response of the two examples:

Clearly, solution 1 provides a wider bandwidth!
Series Stub Tuning

Consider the following transmission line structure, with a series stub:

Therefore an equivalent circuit is:

where of course:

$$Z_{in}'' = Z_0 \left( \frac{Z_L + j Z_0 \tan \beta d}{Z_0 + j Z_L \tan \beta d} \right)$$
and the reactance $jX_{stub}$ is either:

$$jX_{stub} = \begin{cases} -jZ_0\cot \beta \ell & \text{for an open-circuit stub} \\ jZ_0\tan \beta \ell & \text{for a short-circuit stub} \end{cases}$$

Therefore, for a matched circuit, we require:

$$jX_{stub} + Z_{in}'' = Z_0$$

i.e.,

$$\text{Re}\{Z_{in}''\} = Z_0$$

and

$$\text{Im}\{jX_{stub} + Z_{in}''\} = 0 \implies X_{stub} = -X_{in}''$$

where

$$X_{in}'' = \text{Im}\{Z_{in}''\}$$

Note the design parameters for this stub tuner are transmission line lengths $d$ and $\ell$. More specifically we:

1) Set $d$ such that $\text{Re}\{Z_{in}''\} = Z_0$.

2) Then set $\ell$ such that $X_{stub} = -X_{in}''$.

We have two choices for determining the lengths $d$ and $\ell$. We can use the design equations (5.14, 5.15, 5.16) on pp. 235.
OR

we can use the Smith Chart to determine the lengths!

1) Rotate clockwise around the Smith Chart from $z_L$ until you intersect the $r = 1$ circle. The “length” of this rotation determines the value $d$. Recall there are two possible solutions!

2) Rotate clockwise from the short/open circuit point around the $r = 0$ circle until $\chi_{stub}$ equals $-\chi_{in}$. The “length” of this rotation determines the stub length $\ell$.

For example, your book describes the case where we want to match a load of $Z_L = 100 + j80$ (at 2 GHz) to a transmission line of $Z_0 = 50 \Omega$.

Using open stubs, we find two solutions to this problem:

![Diagram of two solutions to the problem](image)
Whose values were determined from a Smith Chart:

Again, we should use the solution with the shortest transmission lines, although in this case that distinction is a bit ambiguous. As a result, the bandwidth of each design is about the same (depending on how you define bandwidth!).