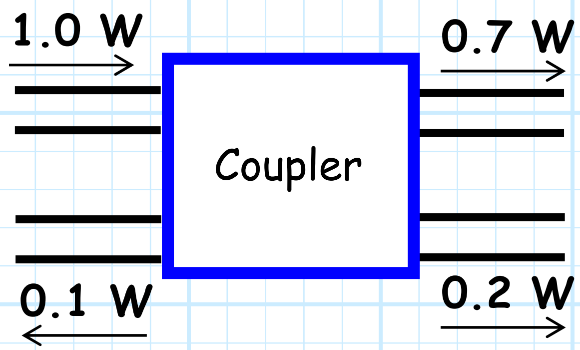
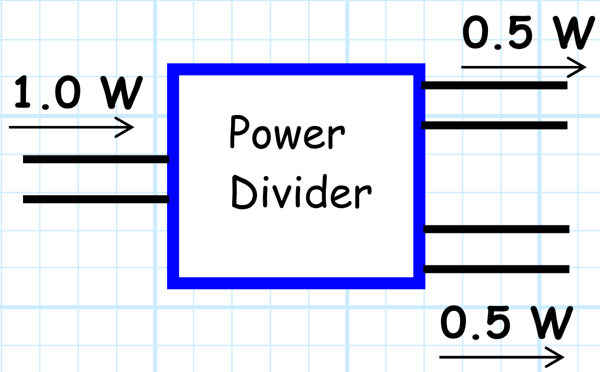
Chapter - 7 Power Dividers and Couplers

One of the most fundamental problems in microwave engineering is how to efficiently **divide** signal power.



The **simplest** microwave problem would seemingly be to **equally** divide signal power in two:



However, building these ideal **3-port** devices is more difficult than you might think!

7.1 - Basic Properties of Dividers and Couplers

Reading Assignment: *pp. 308-315*

Ideally, a coupler would be **matched**, **lossless**, and **reciprocal**. Let's see if we can build such a 3-port device.

HO: THE 3-PORT COUPLER

Q: *Is a matched, lossless, reciprocal **4-port** coupler impossible as well?*

A: HO: THE 4-PORT COUPLER

Four-port networks are called **directional couplers**, and have some fundamental values that characterize them.

HO: THE DIRECTIONAL COUPLER

The 3-Port Coupler

Say we desire a **matched** and **lossless** 3-port Coupler. Such a device would have a scattering matrix :

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Assuming the device is passive and made of simple (isotropic) materials, the device will be **reciprocal**, so that:

$$S_{21} = S_{12} \qquad S_{31} = S_{13} \qquad S_{23} = S_{32}$$

Likewise, if it is **matched**, we know that:

$$S_{11} = S_{22} = S_{33} = 0$$

As a result, a **lossless**, **reciprocal** coupler would have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{21} & 0 & S_{32} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

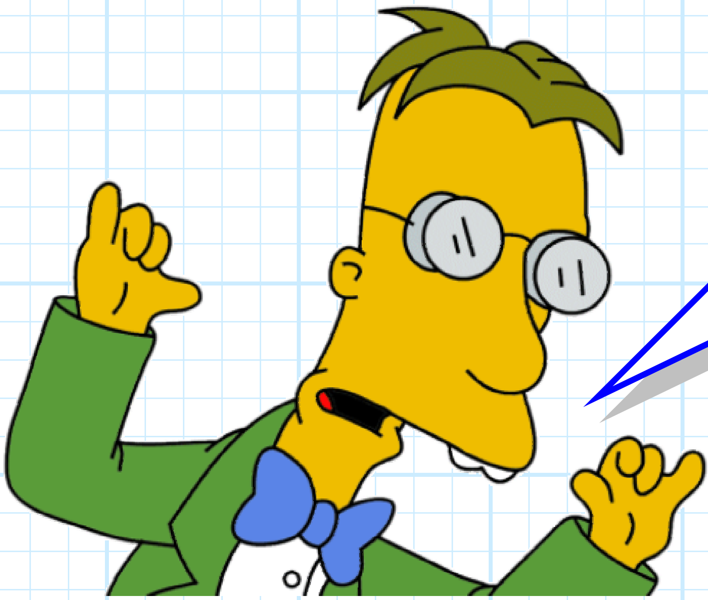
Just **3** non-zero scattering parameters define the **entire** matrix!

Likewise, if we wish for this coupler to be **lossless**, the scattering matrix must be **unitary**, and therefore:

$$|S_{21}|^2 + |S_{31}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$
$$|S_{21}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{32} = 0$$
$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{21}^* S_{31} = 0$$

Since each complex value S is represented by **two real numbers** (i.e., real and imaginary parts), the equations above result in **9** real equations. The problem is, the 3 complex values S_{21}, S_{31} and S_{32} are represented by only **6** real unknowns.

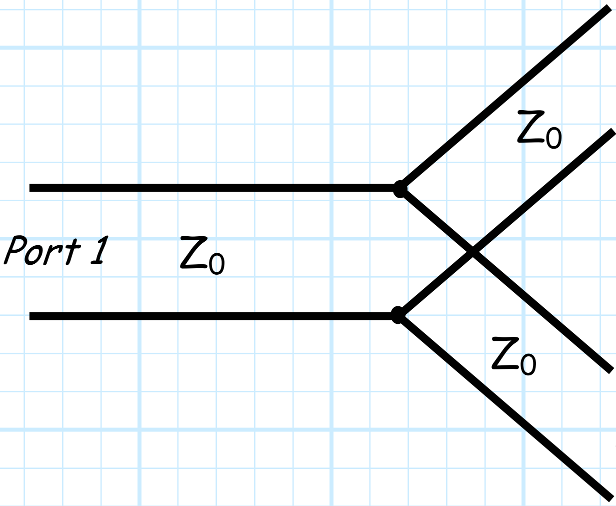
We have **over constrained** our problem ! There are **no solutions** to these equations !



*As unlikely as it might seem, this means that a matched, lossless, reciprocal **3-port** device of **any** kind is a **physical impossibility**!*

For example, the following 3 port coupler is lossless, but not matched:

$$\mathcal{S} = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix}$$



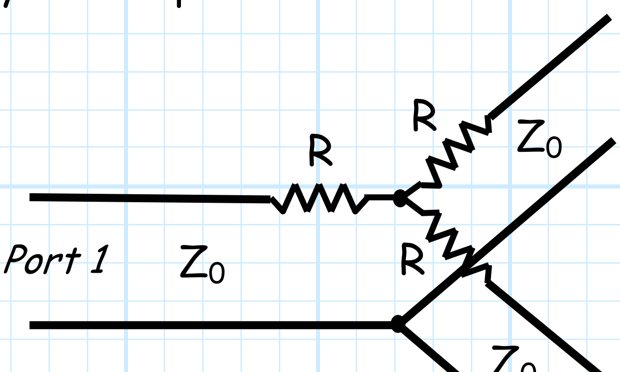
Since:

$$S_{11} = S_{22} = S_{33} = -1/3 \neq 0$$

the coupler is **not matched**! However, the matrix is unitary, and therefore this design **is lossless**.

Alternatively, we might try **this** 3-port device:

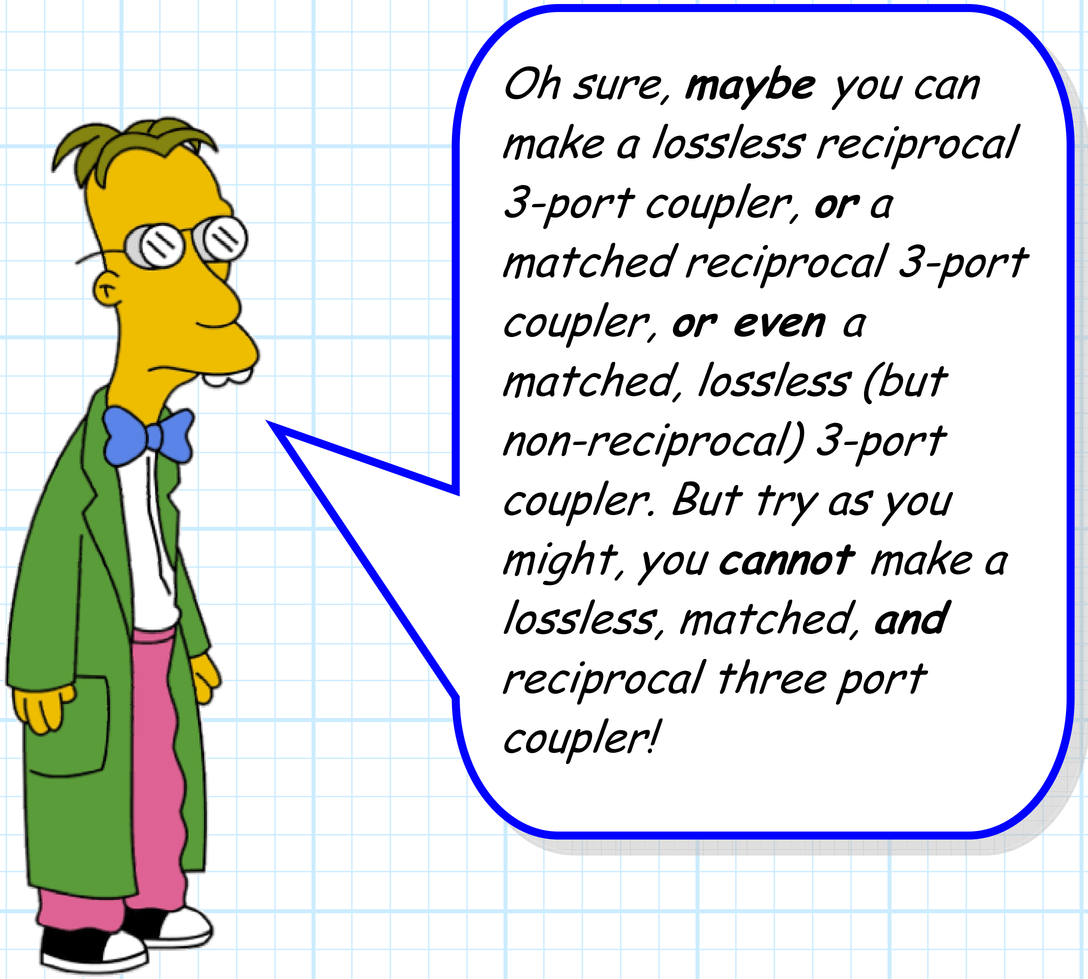
$$\mathcal{S} = \begin{bmatrix} 0 & 3/5 & 3/5 \\ 3/5 & 0 & 3/5 \\ 3/5 & 3/5 & 0 \end{bmatrix}$$



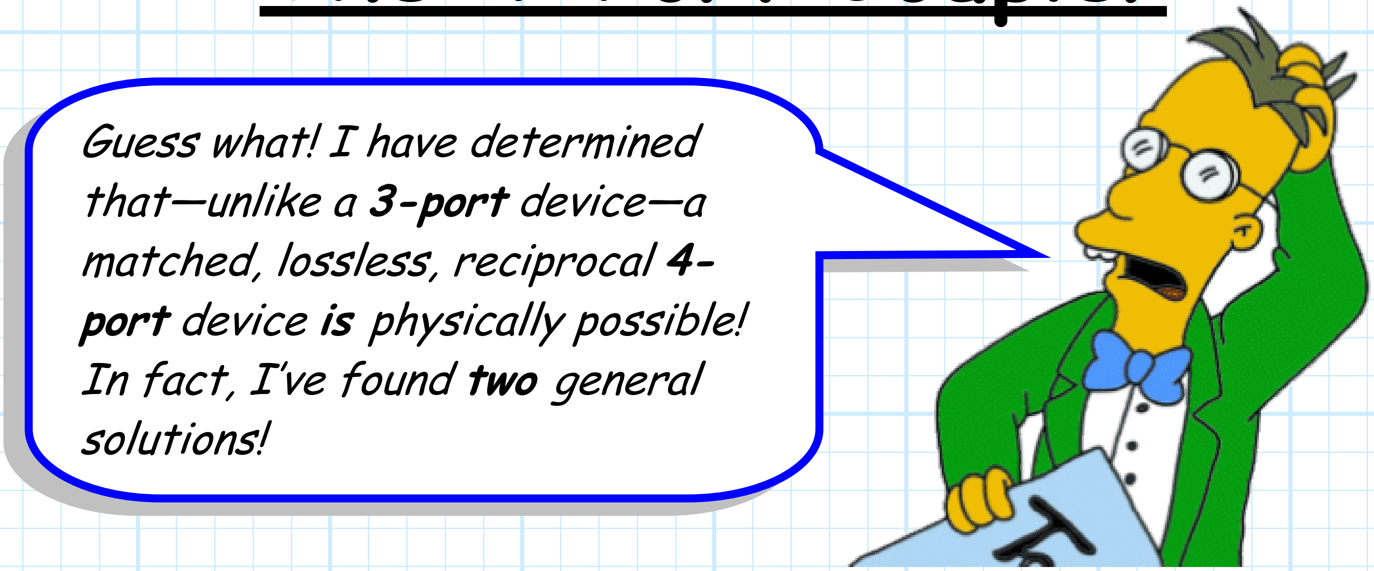
$$R = \frac{Z_0}{3}$$

For this design, the ports **are matched**! However, the resistors make the device **lossy**:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 0 + \frac{9}{25} + \frac{9}{25} = \frac{18}{25} < 1$$



The 4-Port Coupler



The first solution is referred to as the **symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Note for this symmetric solution, every row and every column of the scattering matrix has the **same** four values (i.e., α, $j\beta$, and two zeros)!

The second solution is referred to as the **anti-symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Note that for this anti-symmetric solution, **two** rows and **two** columns have the same four values (i.e., α, β, and two zeros), while the **other** two row and columns have (slightly) **different** values (α, $-\beta$, and two zeros)

It is **quite** evident that each of these solutions are **matched** and **reciprocal**. However, to ensure that the solutions are indeed **lossless**, we must place an **additional** constraint on the values of α, β. Recall that a **necessary** condition for a lossless device is:

$$\sum_{m=1}^{N} \left|S_{mn}\right|^2 = 1 \quad \text{for all } n$$

Applying this to the **symmetric** case, we find:

$$|\alpha|^2 + |\beta|^2 = 1$$

Likewise, for the **anti-symmetric** case, we also get

$$|\alpha|^2 + |\beta|^2 = 1$$

It is evident that if the scattering matrix is **unitary** (i.e., lossless), the values α and β **cannot** be independent, but must **related** as:

$$|\alpha|^2 + |\beta|^2 = 1$$

Generally speaking, we will find that $|\alpha| \geq |\beta|$. Given the constraint on these two values, we can thus conclude that:

$$0 \leq |\beta| \leq 1/\sqrt{2} \quad \text{and} \quad 1/\sqrt{2} \leq |\alpha| \leq 1$$

The Directional Coupler

A lossless, reciprocal, matched 4-port **directional** coupler will have a scattering matrix of the form:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

This ideal coupler is **completely** characterized by the **coupling coefficient** c, where we find:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$

In other words:

$$\beta = c \qquad \text{and} \qquad \alpha = \sqrt{1-\beta^2} = \sqrt{1-c^2}$$

Additionally, for a directional coupler, the coupling coefficient c will be less than $1/\sqrt{2}$ **always**. Therefore, we find that:

$$0 \le c \le \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \frac{1}{\sqrt{2}} \le \sqrt{1-c^2} \le 1$$

Lets see what this means in terms of the **physical behavior** of a directional coupler. First, consider the case where some signal is incident on **port 1**, with power P_1^+.

If all other ports are matched, we find that the power flowing out of **port 1** is:

$$P_1^- = |S_{11}|^2 P_1^+ = 0$$

while the power out of **port 2** is:

$$P_2^- = |S_{21}|^2 P_1^+ = \left(1 - c^2\right) P_1^+$$

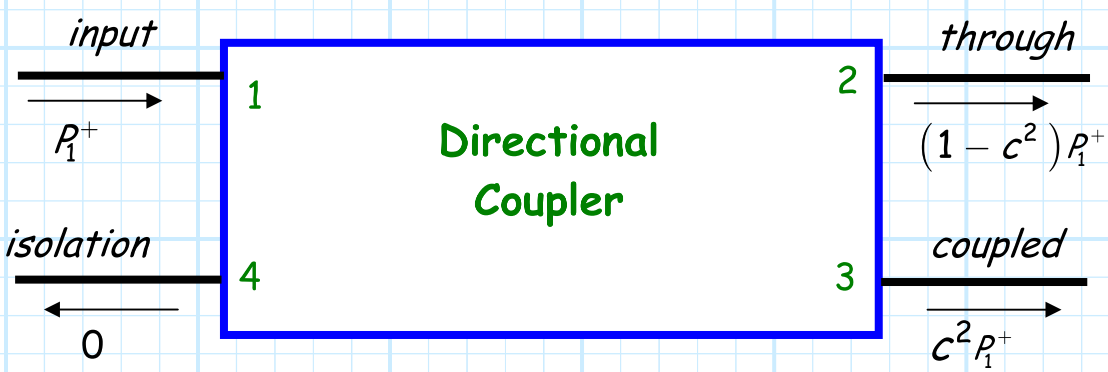
and the power out of **port 3** is:

$$P_3^- = |S_{31}|^2 P_1^+ = c^2 P_1^+$$

Finally, we find there is **no power** flowing out of **port 4**:

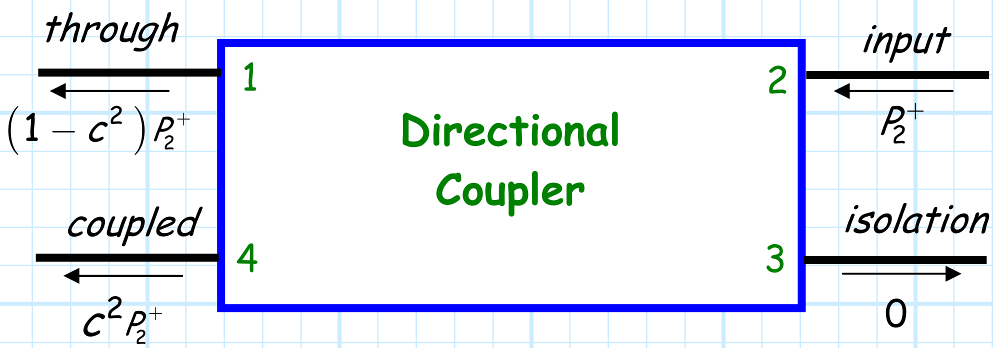
$$P_4^- = |S_{41}|^2 P_1^+ = 0$$

In the terminology of the directional coupler, we say that port 1 is the **input** port, port 2 is the **through** port, port 3 is the **coupled** port, and port 4 is the **isolation** port.



Note however, that **any** of the coupler ports can be an input, with a different through, coupled and isolation port for each case.

For example, **if** a signal is incident on **port 2**, while all other ports are matched, we find that:



Thus, from the scattering matrix of a directional coupler, we can form the following table:

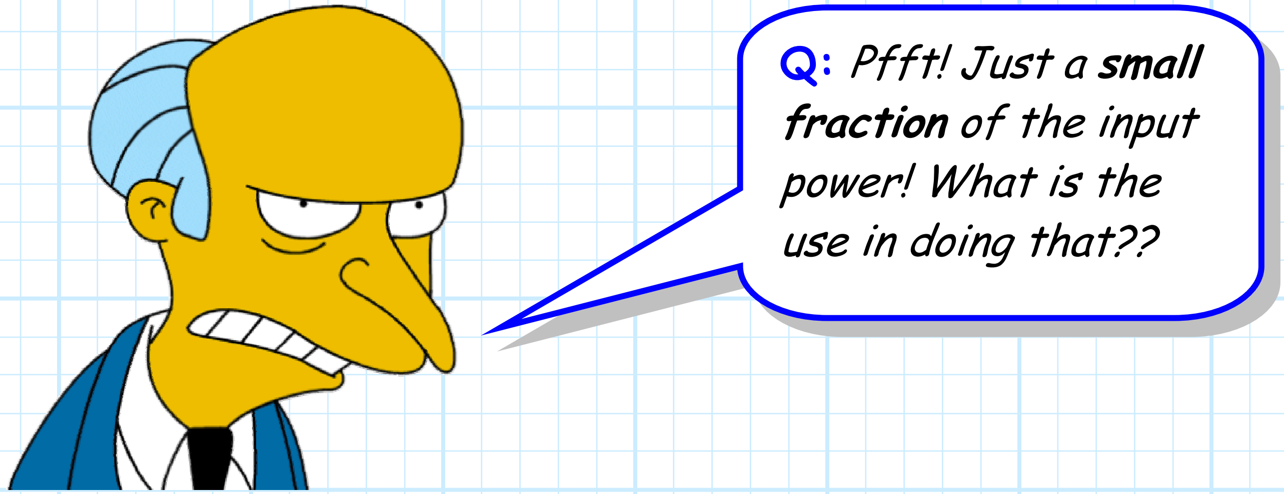
Input	*Through*	*Coupled*	*Isolation*
Port 1	Port 2	Port 3	Port 4
Port 2	Port 1	Port 4	Port 3
Port 3	Port 4	Port 1	Port 2
Port 4	Port 3	Port 2	Port 1

Typically, the coupling coefficients for a directional coupler are in the range of approximately:

$$0.25 > c^2 > 0.0001$$

As a result, we find that $\sqrt{1-c^2} \approx 1$. What this means is that the power out of the **through** port is just **slightly smaller** (typically) than the power incident on the input port.

Likewise, the power out of the **coupling** port is typically a **small fraction** of the power incident on the input port.



A: A directional coupler is often used for **sampling** a small portion of the signal power. For example, we might **measure** the output power of the **coupled** port (e.g., P_3^-) and then we can determine the amount of signal power flowing through the device (e.g., $P_1^+ = P_3^- / c^2$)

Unfortunately, the **ideal** directional coupler **cannot** be built! For example, the input match is never **perfect**, so that the diagonal elements of the scattering matrix, although **very small**, are not zero.

Likewise, the isolation port is never **perfectly** isolated, so that the values S_{41}, S_{32}, S_{23} and S_{14} are also non-zero—some **small** amount of power leaks out!

As a result, the through port will be **slightly less** than the value $\sqrt{1-c^2}$. The scattering matrix for a **non-ideal coupler** would therefore be:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{21} & jc & S_{41} \\ S_{21} & S_{11} & S_{41} & jc \\ jc & S_{41} & S_{11} & S_{21} \\ S_{41} & jc & S_{21} & S_{11} \end{bmatrix}$$

From **this** scattering matrix, we can extract **some important parameters** about directional couplers:

Coupling C

The coupling value is the ratio of the coupled output power to the input power, in dB:

$$C\,(dB) = 10\log_{10}\left[\frac{P_1^+}{P_3^-}\right] = -10\log_{10}|\,jc\,|^2$$

This is the **primary** specification of a directional coupler!

Directivity D

The directivity is the ratio of the power out of the coupling port to the power out of the isolation port, in dB. This value indicates how effective the device is in "**directing**" the coupled energy into the correct port. The **higher** the directivity, the better.

$$D(dB) = 10\log_{10}\left[\frac{P_3^-}{P_4^-}\right] = 10\log_{10}\left[\frac{|jc|^2}{|S_{41}|^2}\right]$$

Isolation I

Isolation is the ratio of the input power to the power out of the isolation port, in dB. This value indicates how "isolated" the isolation port actually is. The **higher** the isolation, the better.

$$I(dB) = 10\log_{10}\left[\frac{P_1^+}{P_4^-}\right] = -10\log_{10}\left[|S_{41}|^2\right]$$

Note that isolation, directivity, and coupling are **not** independent values! **You** should be able to quickly show that:

$$I(dB) = C(dB) + D(dB)$$

Mainline Loss ML

The mainline loss is the ratio of the input power to the power out of the through port, in dB. It indicates how much power the signal **loses** as it travels from the input to the through port.

$$ML(dB) = 10\log_{10}\left[\frac{P_1^+}{P_2^-}\right] = -10\log_{10}\left[|S_{21}|^2\right]$$

Coupling Loss *ML*

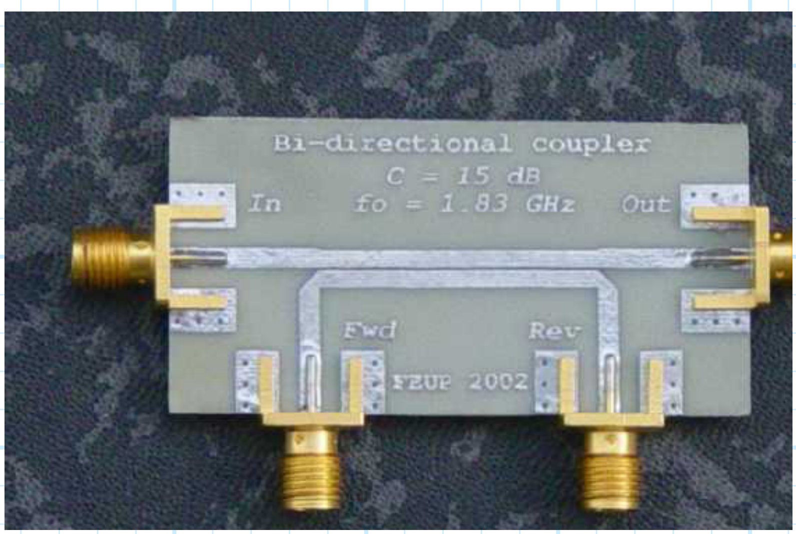
The coupling loss indicates the portion of the mainline loss that is due to coupling some of the input power into the coupling port. Conservation of energy makes this loss is **unavoidable.**

$$CL(dB) = -10\log_{10}\left[1 - |jc|^2\right]$$

Insertion Loss *IL*

The coupling loss indicates the portion of the mainline loss that is **not** due to coupling some of the input power into the coupling port. This loss **is** avoidable, and thus the **smaller** the insertion loss, the better.

$$IL(dB) = ML(dB) - CL(dB)$$



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