7.2 – The T-Junction Power Divider

Reading Assignment: *pp. 315-318*

Let's look specifically at 3-port couplers that can be used as power **dividers**.

HO: THE T-JUNCTION POWER DIVIDER

We will study **three** standard T-Junction power dividers:

HO: THE RESISTIVE DIVIDER

HO: THE LOSSLESS DIVIDER

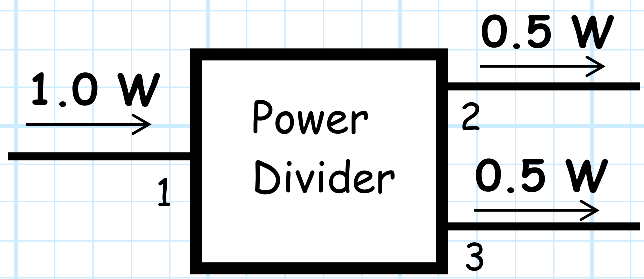
The third type of power divider that we will study is the **Wilkinson Power Divider**—the subject of the next section.

The T-Junction Power Divider

Recall that we **cannot** build a matched, lossless reciprocal **three**-port device.

So let's consider one of the most useful three-port devices that we **can** build!

Recall that a fundamental three-port is the power **divider**.



The Power Divider

To **efficiently** divide the power incident on the input port, the port (port 1) must be **matched**:

$$S_{11} = 0$$

Likewise, the divided power must be efficiently (i.e., **without loss**) delivered to the output ports of the divider:

$$|S_{21}|^2 + |S_{31}|^2 = 1$$

In addition, it is **desirable** that ports 2 and 3 be **matched**:

$$S_{22} = S_{33} = 0$$

And also **desirable** that ports 2 and 3 be **isolated**:

$$S_{23} = S_{32} = 0$$

This last requirement ensures that no signal incident on port 2 (e.g., reflected from a load) will **"leak"** into port 3—and vice versa.

These conditions completely describe an ideal power divider—conditions that **can** be met (we can **build** an ideal power divider!). For example an ideal power divider **could** have the form:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

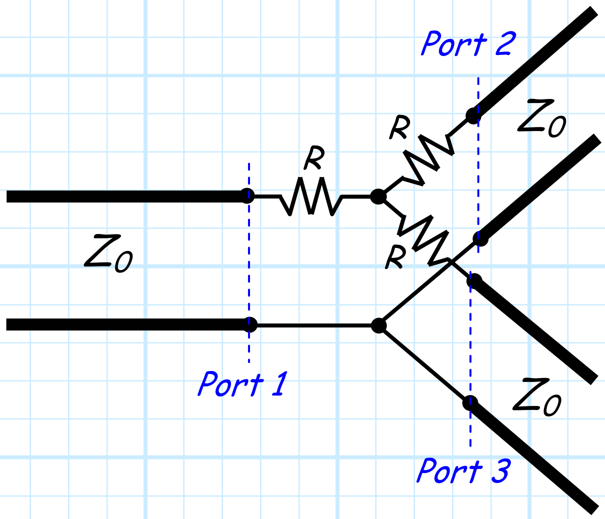
Note that **this** device would take the power into port 1 and divide into **two equal parts**—half exiting **port 2**, and half exiting **port3** (provided ports 2 and 3 are terminated in matched loads!).

$$P_2^- = |S_{21}|^2 P_1^+ = 0.5 P_1^+ \qquad P_3^- = |S_{31}|^2 P_1^+ = 0.5 P_1^+$$

Nonetheless, this divider is clearly a **lossy** device (see columns 2 and 3!)—it must have some lossy element (e.g., **resistors**)!

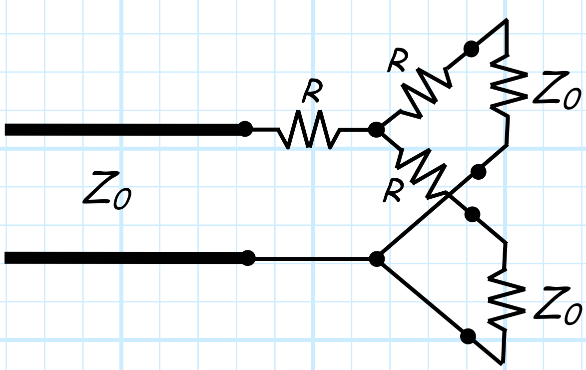
The Resistive Divider

Consider the **resistive** power divider:



This power divider will be matched at port 1 **if** R is selected as:

$$
\begin{aligned}
Z_0 &= R + (R+Z_0) \| (R+Z_0) \\
&= R + \frac{R+Z_0}{2} \\
&= 1.5R + \frac{Z_0}{2}
\end{aligned}
$$



Solving this equation, we find that port 1 is matched **if**:

$$\boxed{R = \frac{Z_0}{3}}$$

From the **symmetry** of the circuit, we find that all the **other** ports will be matched as well (i.e., $S_{11} = S_{22} = S_{33} = 0$). Moreover, it can be shown that:

$$S_{12} = S_{21} = S_{31} = S_{31} = S_{23} = S_{32} = 1/2$$

So:

$$\mathcal{S} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

Note the magnitude of each column is less than one. E.G.,:

$$|S_{21}|^2 + |S_{31}|^2 = 1/2 < 1$$

Therefore this power divider is **lossy**!

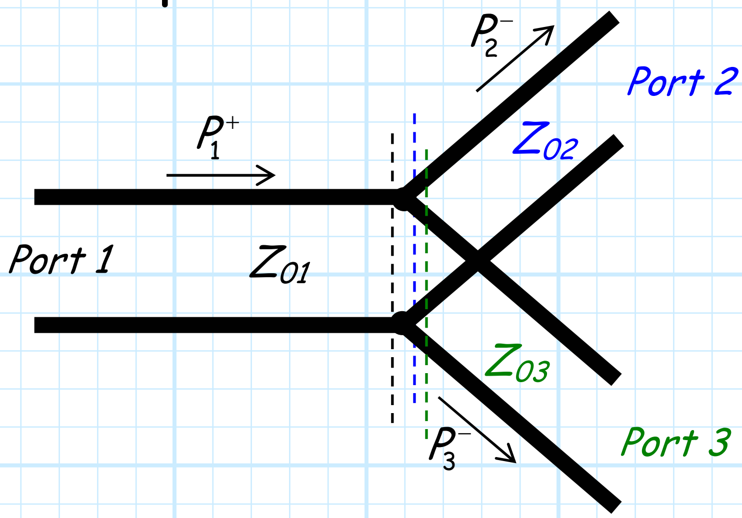
In fact, we find that the power out of each port is just **one-quarter** of the input power:

$$P_2^- = P_3^+ = \frac{P_1^+}{4}$$

In other words, **half** the input power is **absorbed** by the divider!

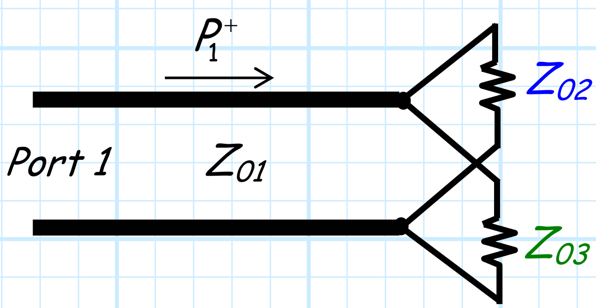
The Lossless Divider

Consider the **lossless power divider**:



To be ideal, we want $S_{11} = 0$. Thus, when ports 2 and port 3 are **terminated** in matched loads, the input impedance at port 1 must be equal to Z_{01}. This will only be true if the values Z_{02} and Z_{03} are selected such that:

$$Z_{01} = \left(\frac{1}{Z_{02}} + \frac{1}{Z_{03}} \right)^{-1} = \frac{Z_{02}\, Z_{03}}{Z_{02} + Z_{03}}$$



Note however that this circuit is **not** symmetric, thus we find that $S_{22} \neq 0$ and $S_{33} \neq 0$!

It is evident that this divider is **lossless** (not resistive components), so that:

$$P_1^+ = P_2^- + P_3^-$$

where P_1^+ is the power incident (and absorbed if $S_{11} = 0$) on port 1, and P_2^- and P_3^- is the power incident on the matched loads of ports 2 and 3. Note this is likewise the power **absorbed** by these loads, since they are **matched**.

Now, we **can** divide the power incident on port 1 equally. I.E.:

$$P_2^- = P_3^- = P_1^+ / 2$$

Clearly, this 3dB division is true only if $Z_{02} = Z_{03}$. However, we do **not** have to divide the power evenly between ports 2 and 3!

It can be shown that a division ration α is :

$$\alpha = \frac{P_2^-}{P_3^-} = \frac{Z_{03}}{Z_{02}}$$

Thus, if we desire an **ideal** ($S_{11} = 0$) divider with a specific division ratio α, we find that:

$$Z_{02} = Z_{01}\left(1 + \frac{1}{\alpha}\right)$$

and:

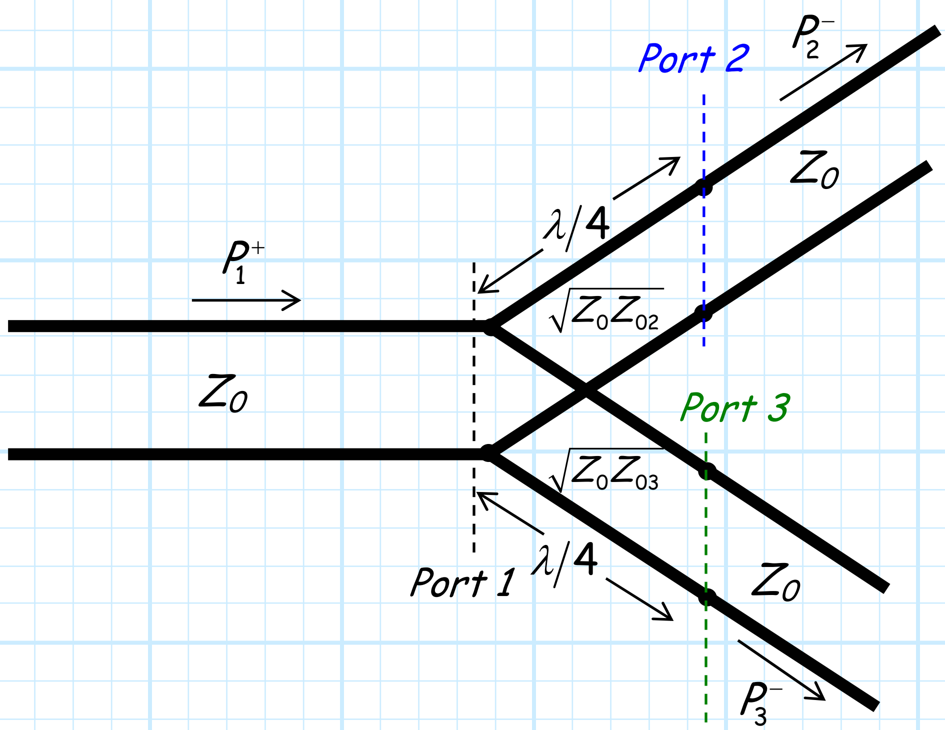
$$Z_{03} = Z_{01}\left(1 + \alpha\right)$$

Note that if we desire a **3 dB** divider (i.e., $\alpha = 1$), then:

$$Z_{02} = Z_{03} = 2\,Z_{01}$$

Q: *I don't understand how this is helpful. Don't we typically want the characteristic impedance of all three ports to be equal to the **same** value (e.g., $Z_{01} = Z_{02} = Z_{03} = Z_0$)?*

A: True ! A more practical way to implement this divider is to use a **matching network**, such as a quarter wave transformer:



But beware! Recall that this matching network will work perfectly at only **one** frequency.

Finally, note this three-port device is lossless and reciprocal, but is matched **only** at port 1, and ports 2 and 3 are **not** isolated!

Therefore a **3 dB** lossless divider has a scattering matrix (at the design frequency) of this form:

$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}\!/\!\sqrt{2} & {}^{-j}\!/\!\sqrt{2} \\ {}^{-j}\!/\!\sqrt{2} & S_{22} & S_{23} \\ {}^{-j}\!/\!\sqrt{2} & S_{32} & S_{33} \end{bmatrix}$$

If we desire this **3dB** lossless divider (where $Z_{02} = Z_{03} = 2\,Z_{01}$), we arrive at this design: