7.3 – The Wilkinson Power Divider

Reading Assignment: *pp. 318-323*

The Wilkinson power divider is the most **popular** power divider designs.

It is very similar to a lossless 3dB divider, but has one **additional** component!

This additional component gives this power **divider** many of the important attributes of a power **combiner**.

HO: THE WILKINSON POWER DIVIDER

Q: *I don't see how the Wilkinson power divider design provides the scattering matrix you claim. Is there any way to* ***analyze*** *this structure to verify its performance?*

A: Yes! But first we must learn about two very important and related concepts in microwave engineering—circuit symmetry and **odd/even mode analysis**.

HO: SYMMETRCI CIRCUIT ANALYSIS
HO: ODD/EVEN MODE ANALYSIS

Now we can analyze a Wilkinson power divider!

HO: WILKINSON DIVIDER EVEN/ODD MODE ANALYSIS

The Wilkinson Power Divider

The **Wilkinson power divider** is a 3-port device with a scattering matrix of:

$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}\!/\!_{\sqrt{2}} & {}^{-j}\!/\!_{\sqrt{2}} \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

Note this device is **matched** at port 1 ($S_{11} = 0$), and we find that magnitude of column 1 is:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$$

Thus, just like the lossless divider the incident power on port 1 is **evenly** and **efficiently** divided between the outputs of port 2 and port 3:

$$P_2^- = |S_{21}|^2 P_1^+ = \frac{P_1^+}{2} \qquad\qquad P_3^- = |S_{31}|^2 P_1^+ = \frac{P_1^+}{2}$$

But now look closer at the scattering matrix. We also note that the ports 2 and 3 of this device are **matched** !

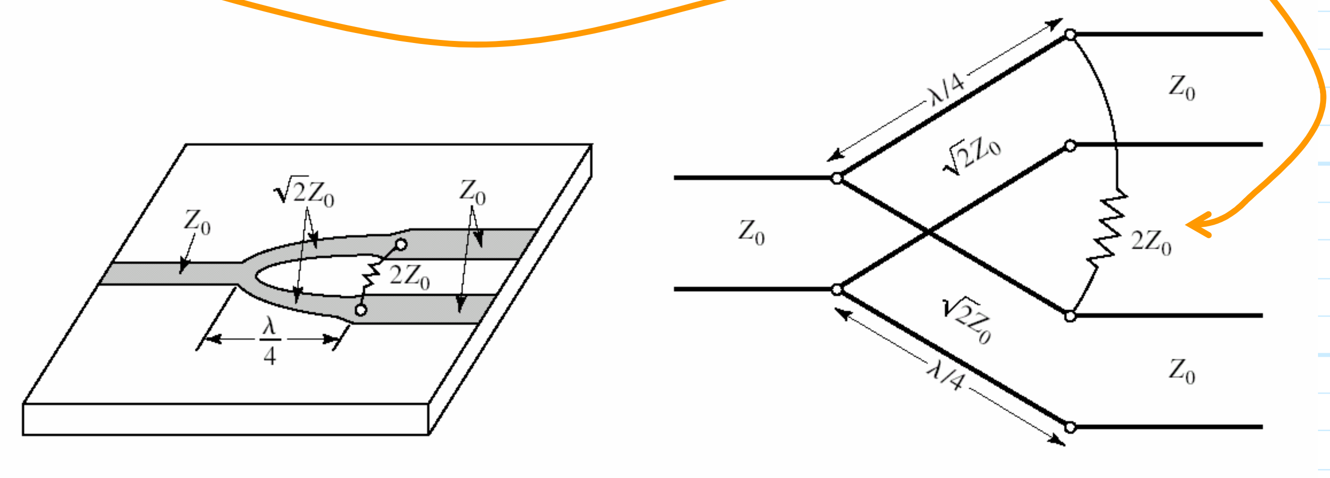
$$S_{22} = S_{33} = 0$$

Likewise, we note that ports 2 and ports 3 are **isolated**:

$$S_{23} = S_{32} = 0$$

Q: *So just how do we **make** this Wilkinson power divider?*

It looks a lot like a **lossless 3dB divider**, only with an additional **resistor** of value $2Z_0$ between ports 2 and 3:



(a) (b)

This resistor is the secret to the Wilkinson power divider, and is the reason that it is **matched** at ports 2 and 3, and the reason that ports 2 and 3 are **isolated**.

Note however, that the **quarter-wave** transmission line sections make the Wilkinson power divider a **narrow-band** device.

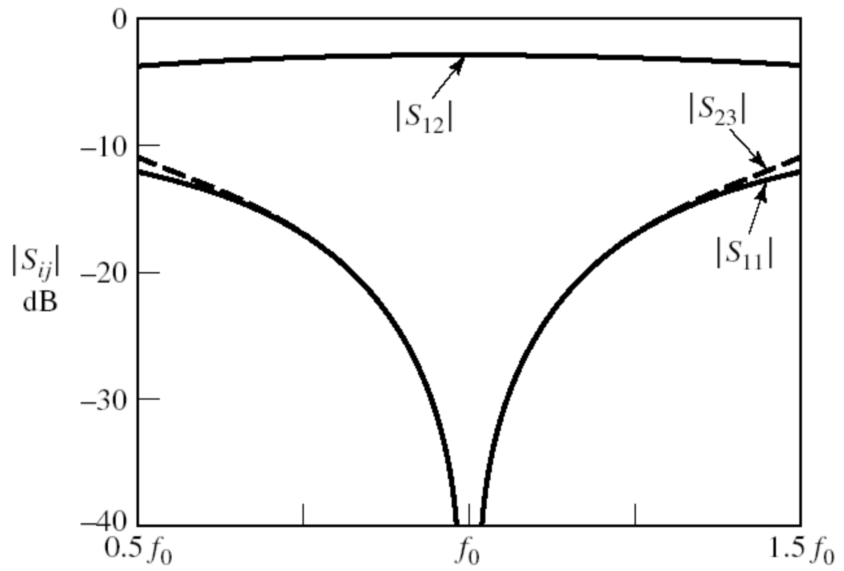
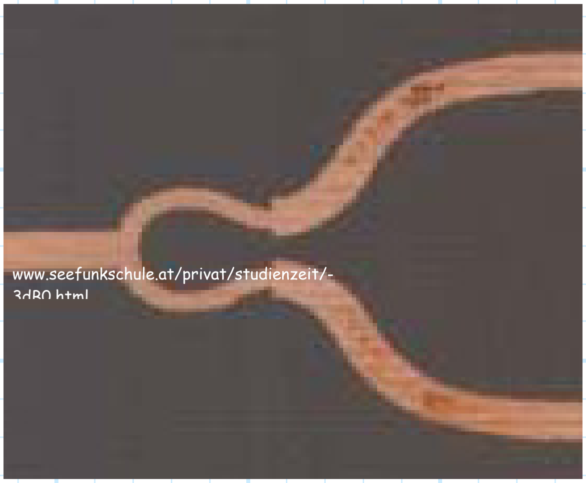
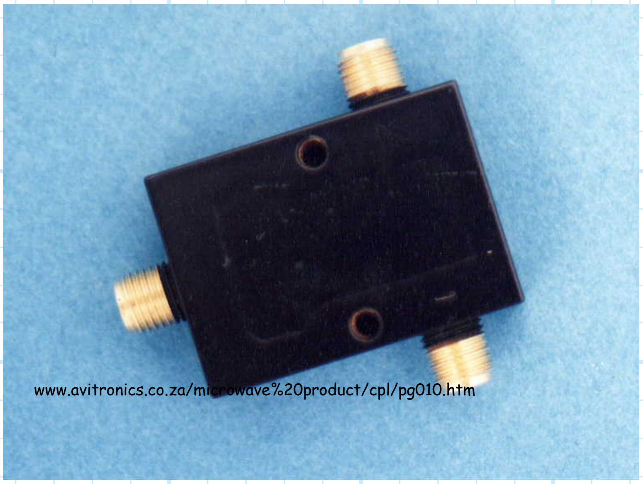


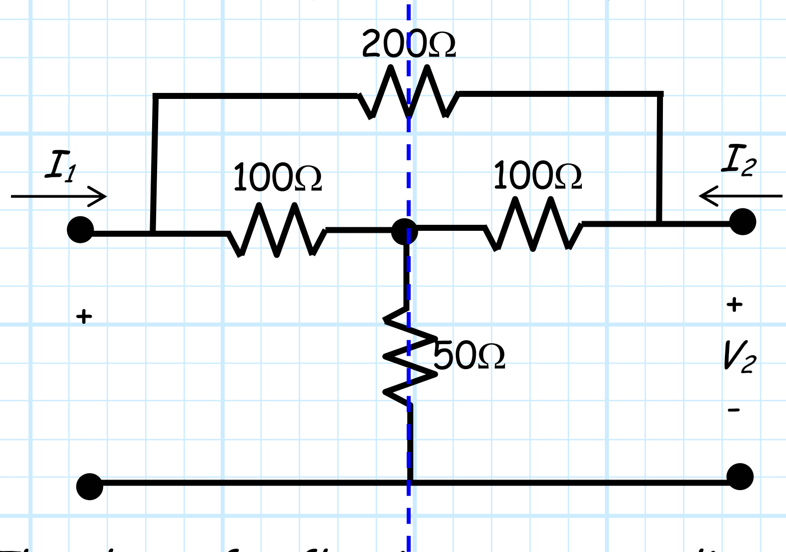
Figure 7.12 (p. 322)
Frequency response of an equal-split Wilkinson power divider. Port 1 is the input port; ports 2 and 3 are the output ports.





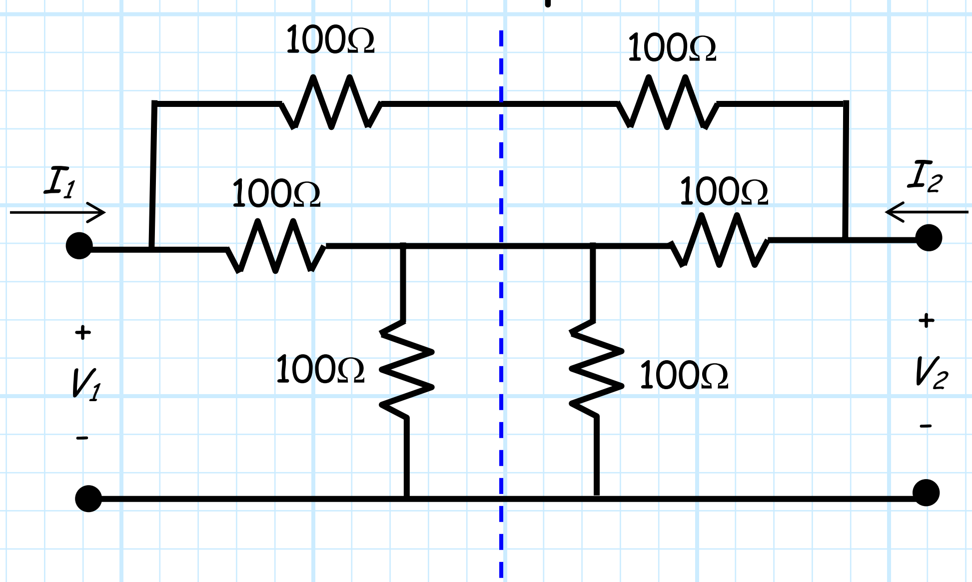
Symmetric Circuit Analysis

Consider the following D_1 symmetric **two-port** device:



Q: *Yikes! The plane of reflection symmetry slices through two resistors. What can we do about that?*

A: Resistors are easily split into two equal pieces: the 200Ω resistor into two 100Ω resistors in **series**, and the 50Ω resistor as two 100 Ω resistors in **parallel**.



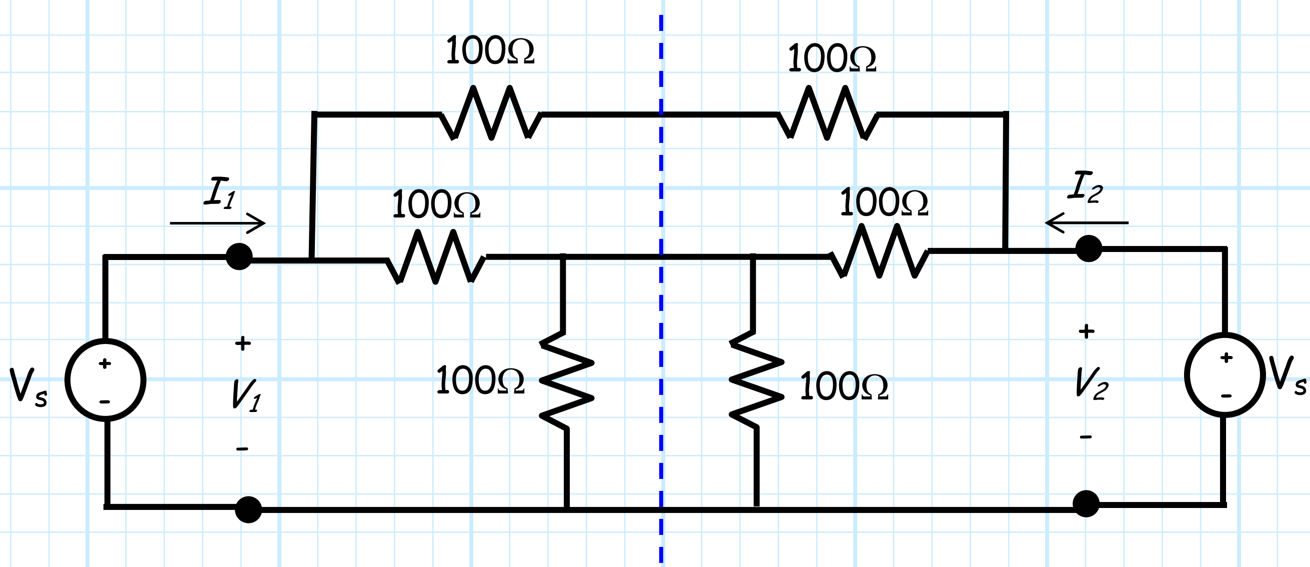
Recall that the **symmetry** of this 2-port device leads to **simplified** network matrices:

$$\mathcal{S}=\begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix} \qquad \mathcal{Z}=\begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix} \qquad \mathcal{Y}=\begin{bmatrix} Y_{11} & Y_{21} \\ Y_{21} & Y_{11} \end{bmatrix}$$

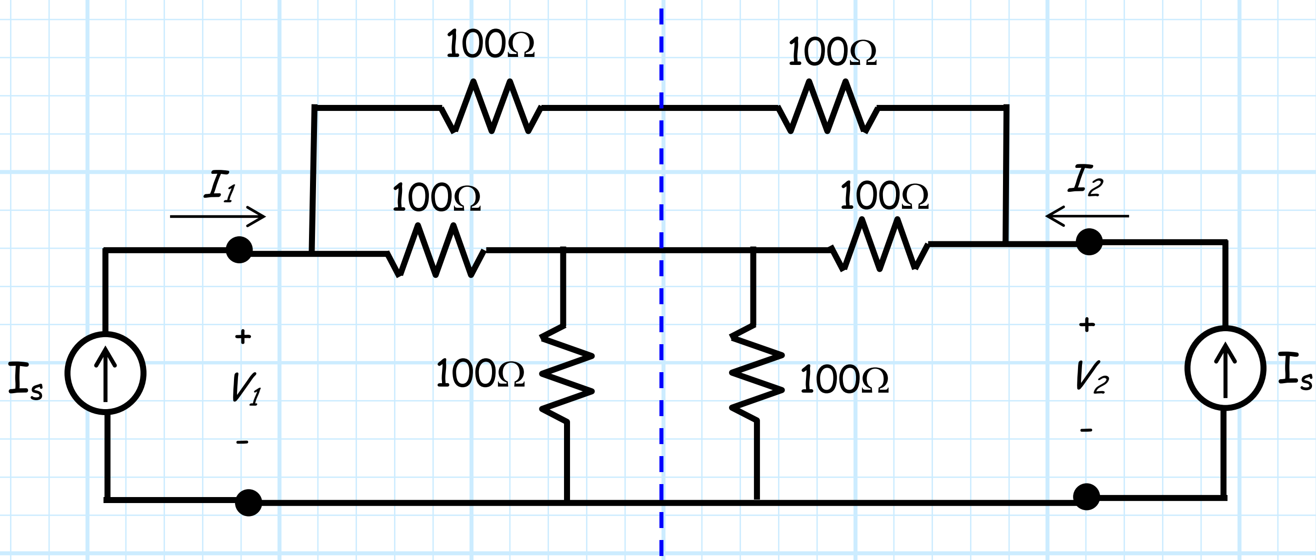
Q: *Yes, but can circuit symmetry likewise simplify the procedure of **determining** these elements? In other words, can symmetry be used to **simplify circuit analysis**?*

A: You bet!

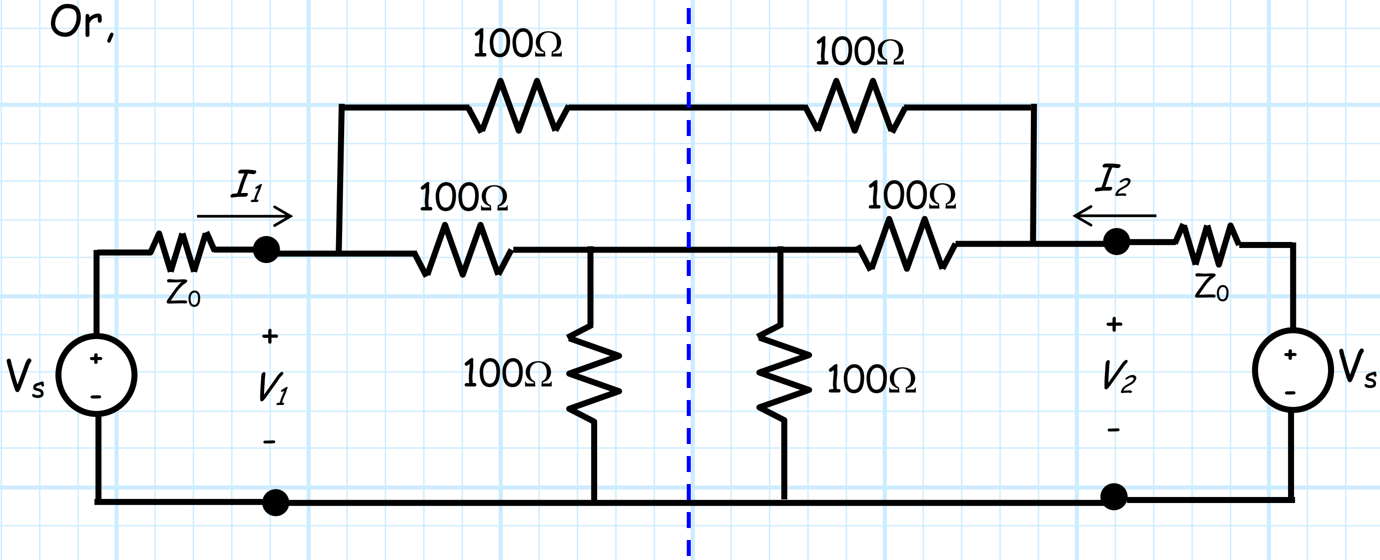
First, consider the case where we **attach sources** to circuit in a way that **preserves** the circuit **symmetry**:



Or,

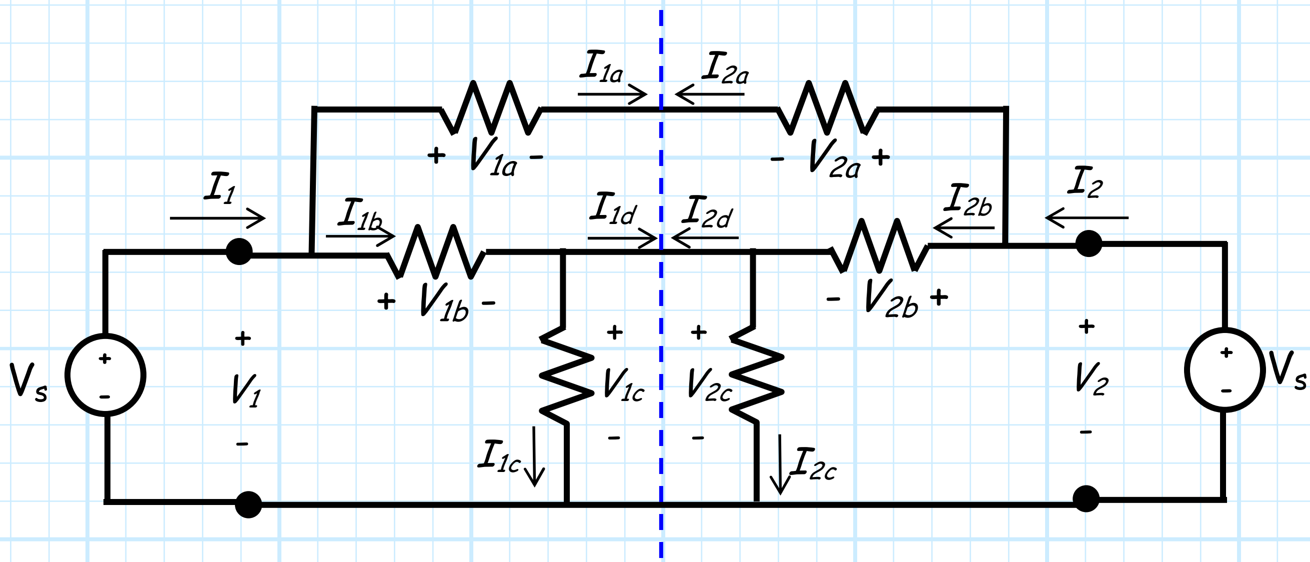


Or,



But remember! In order for **symmetry to be preserved**, the source values on both sides (i.e, I_s, V_s, Z_0) must be **identical**!

Now, consider the **voltages** and **currents** within this circuit under this symmetric configuration:



Since this circuit possesses **bilateral** (reflection) symmetry ($1 \rightarrow 2, 2 \rightarrow 1$), symmetric currents and voltages must be equal:

$$V_1 = V_2 \qquad I_1 = I_2$$
$$V_{1a} = V_{2a} \qquad I_{1a} = I_{2a}$$
$$V_{1b} = V_{2b} \qquad I_{1b} = I_{2b}$$
$$V_{1c} = V_{2c} \qquad I_{1c} = I_{2c}$$
$$I_{1d} = I_{2d}$$

Q: ***Wait!*** *This **can't** possibly be correct! Look at currents I_{1a} and I_{2a}, as well as currents I_{1d} and I_{2d}. From KCL, **this** must be true:*

$$I_{1a} = -I_{2a} \qquad I_{1d} = -I_{2d}$$

*Yet **you** say that **this** must be true:*

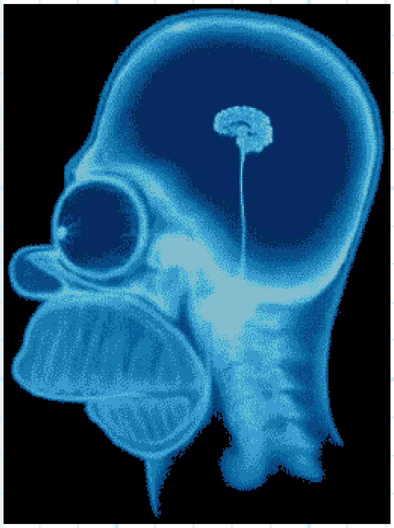
$$I_{1a} = I_{2a} \qquad I_{1d} = I_{2d}$$

*There is an **obvious contradiction** here! There is **no way** that both sets of equations can simultaneously be correct, **is there?***

A: Actually there **is**! There is **one** solution that will satisfy **both** sets of equations:

$$I_{1a} = I_{2a} = 0 \qquad\qquad I_{1d} = I_{2d} = 0$$

The currents are **zero**!

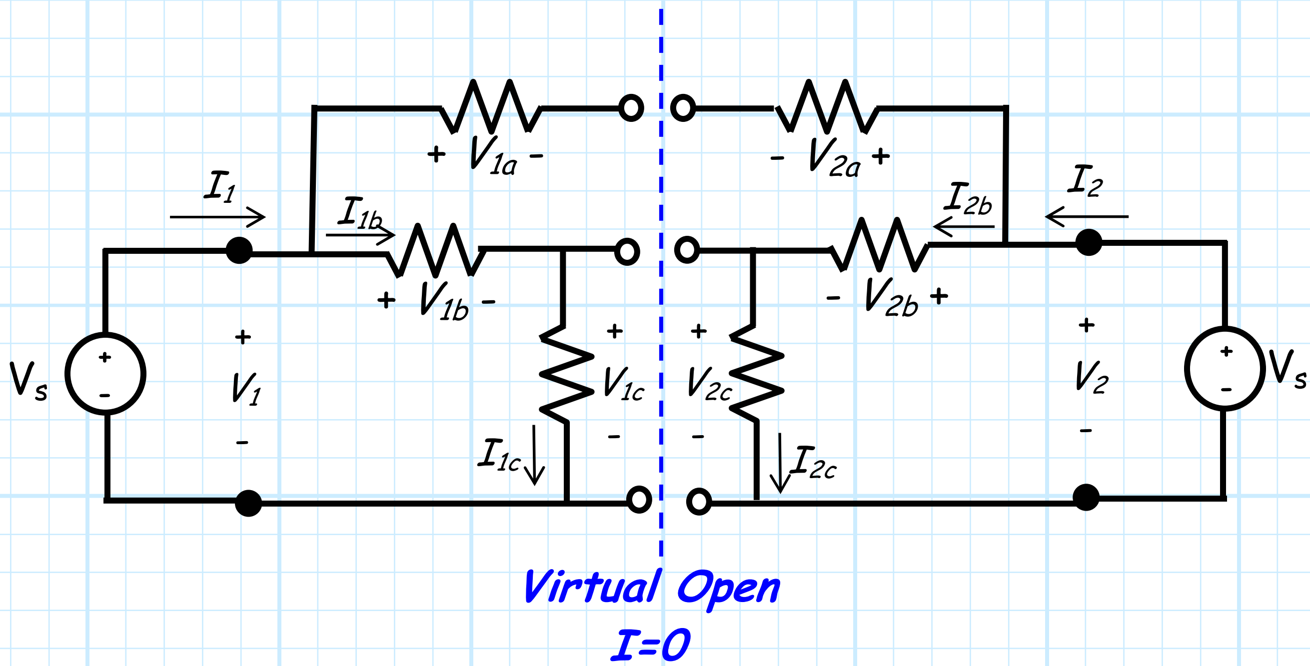


If you **think** about it, this makes **perfect sense**! The result says that **no current** will flow from one side of the symmetric circuit into the other.

If current **did** flow across the symmetry plane, then the circuit symmetry would be **destroyed**—one side would effectively become the "**source side**", and the other the "**load side**" (i.e., the source side delivers current to the load side).

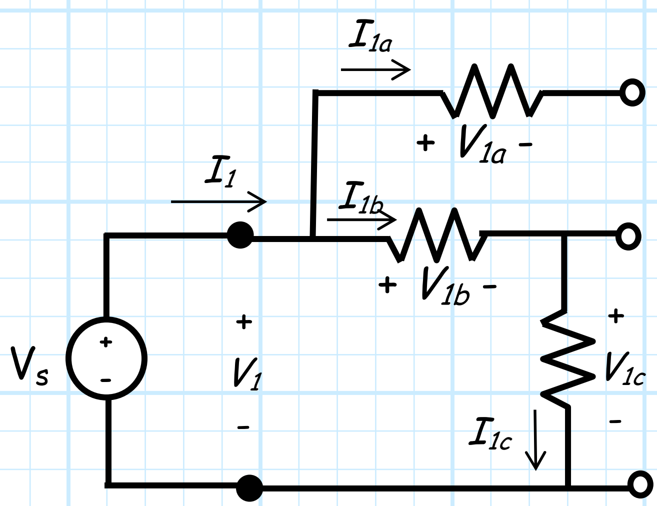
Thus, **no current** will flow **across** the reflection symmetry plane of a **symmetric circuit**—the symmetry plane thus acts as a **open circuit**!

The plane of symmetry thus becomes a **virtual open**!



Q: *So what?*

A: So what! This means that our circuit can be **split apart** into **two separate** but **identical** circuits. Solve **one** half-circuit, and you have **solved** the other!



$$V_1 = V_2 = V_s$$

$$V_{1a} = V_{2a} = 0$$

$$V_{1b} = V_{2b} = V_s/2$$

$$V_{1c} = V_{2c} = V_s/2$$

$$I_1 = I_2 = V_s/200$$

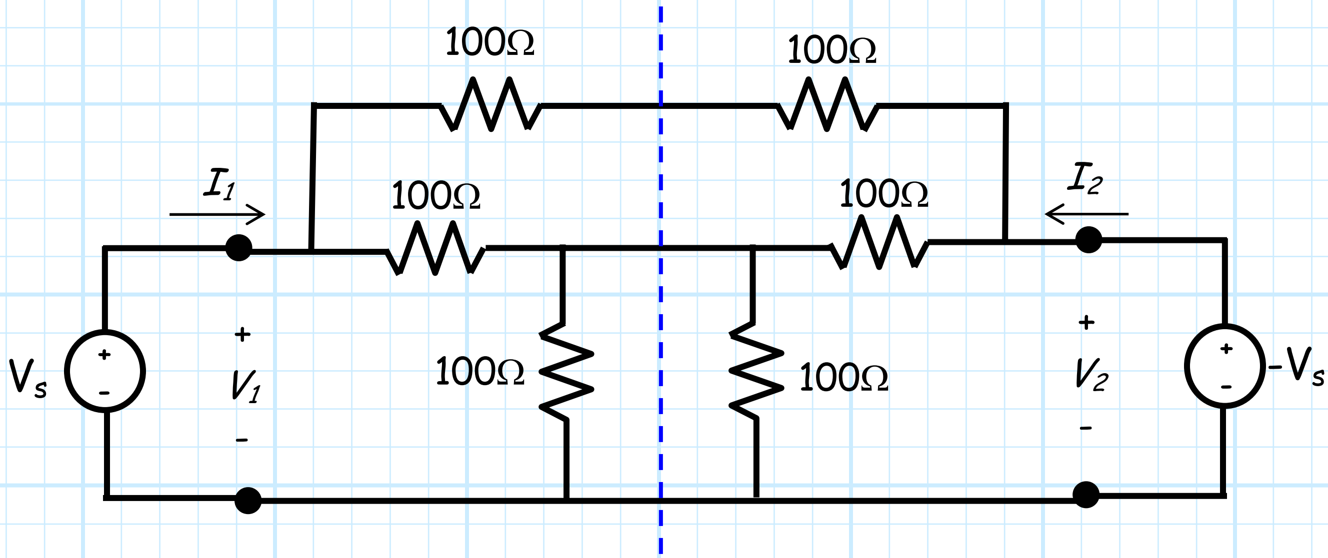
$$I_{1a} = I_{2a} = 0$$

$$I_{1b} = I_{2b} = V_s/200$$

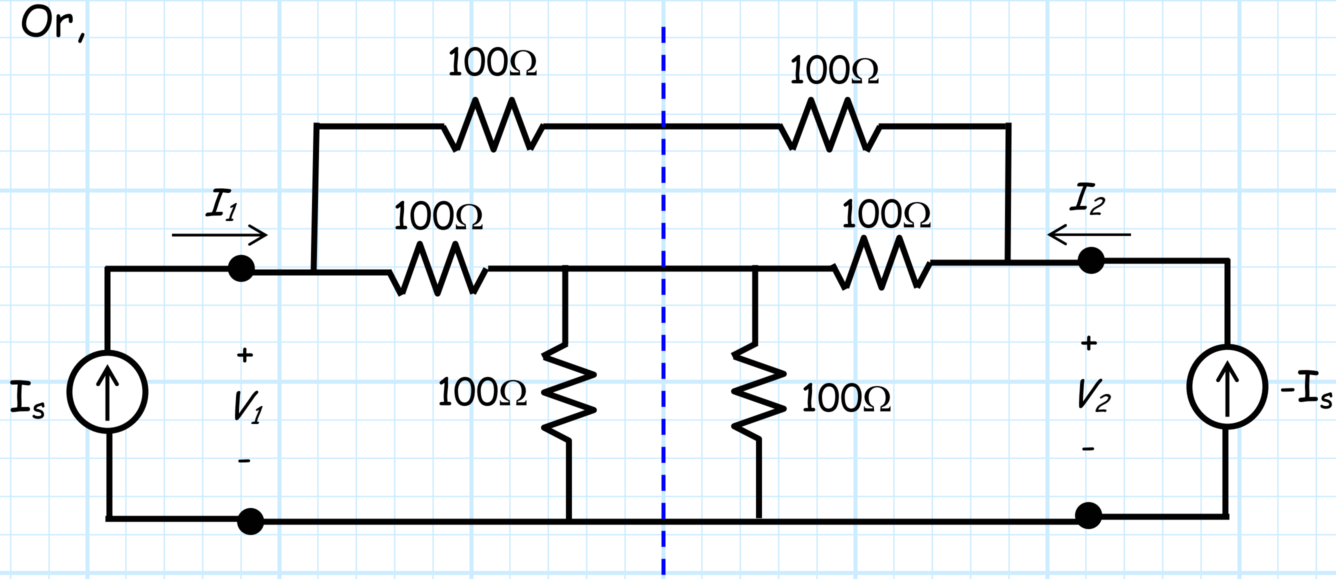
$$I_{1c} = I_{2c} = V_s/200$$

$$I_{1d} = I_{2d} = 0$$

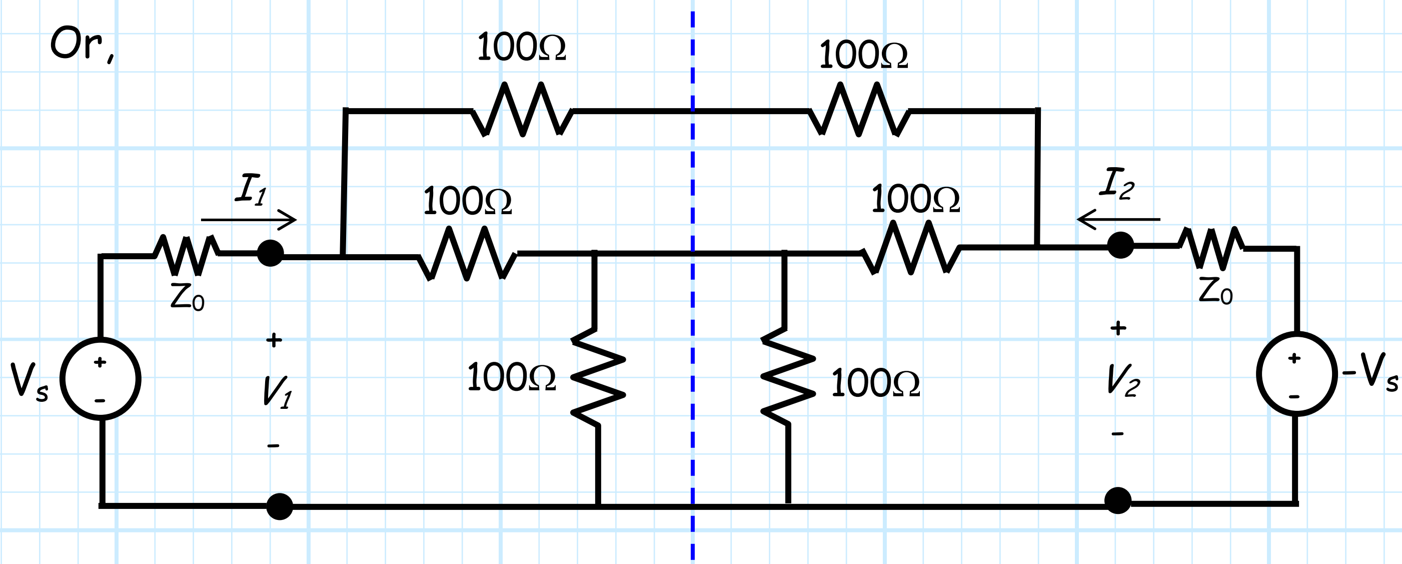
Now, consider **another** type of symmetry, where the sources are **equal** but **opposite** (i.e., **180 degrees** out of phase).



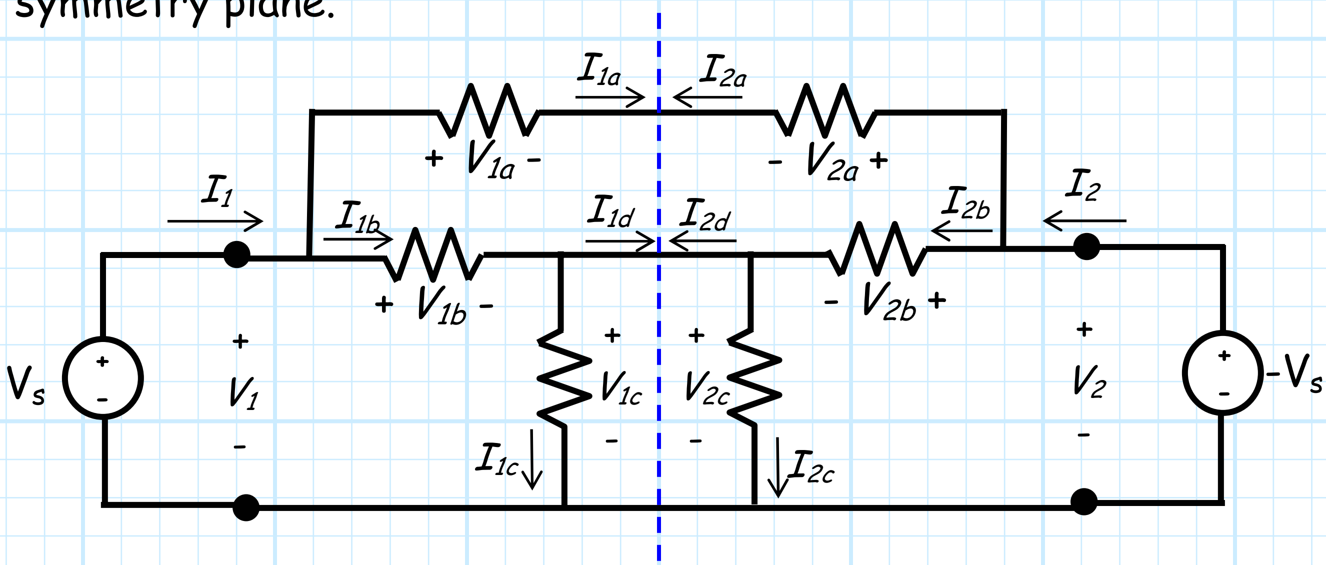
Or,



Or,



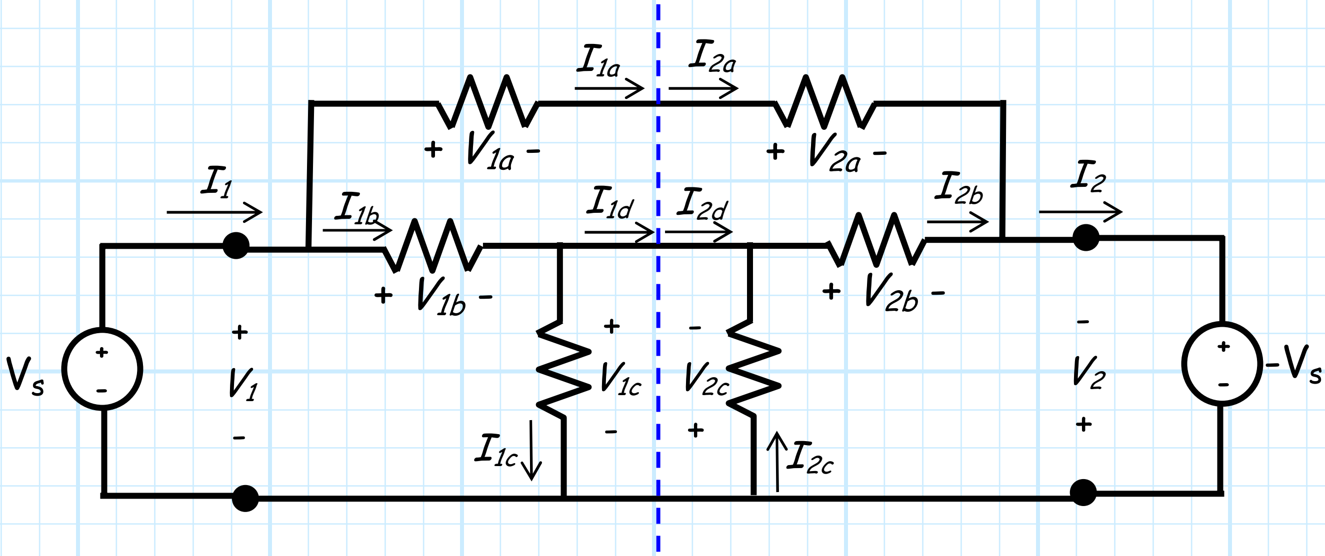
This situation still preserves the **symmetry** of the circuit—**somewhat.** The **voltages** and **currents** in the circuit will now posses **odd symmetry**—they will be **equal but opposite** (180 degrees out of phase) at symmetric points across the symmetry plane.



$$V_1 = -V_2$$
$$V_{1a} = -V_{2a}$$
$$V_{1b} = -V_{2b}$$
$$V_{1c} = -V_{2c}$$

$$I_1 = -I_2$$
$$I_{1a} = -I_{2a}$$
$$I_{1b} = -I_{2b}$$
$$I_{1c} = -I_{2c}$$
$$I_{1d} = -I_{2d}$$

Perhaps it would be easier to **redefine** the circuit variables as:



$$V_1 = V_2$$
$$V_{1a} = V_{2a}$$
$$V_{1b} = V_{2b}$$
$$V_{1c} = V_{2c}$$

$$I_1 = I_2$$
$$I_{1a} = I_{2a}$$
$$I_{1b} = I_{2b}$$
$$I_{1c} = I_{2c}$$
$$I_{1d} = I_{2d}$$

Q: *But wait! **Again** I see a problem. By **KVL** it is evident that:*

$$V_{1c} = -V_{2c}$$

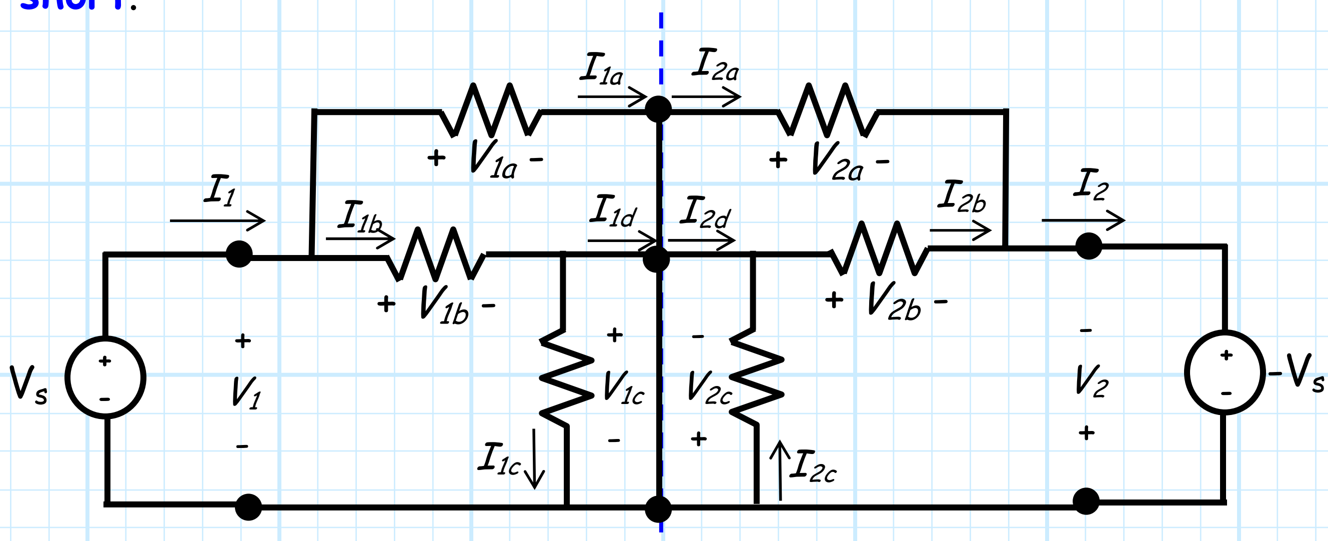
*Yet **you** say that $V_{1c} = V_{2c}$ must be true!*

A: Again, the solution to **both** equations is **zero**!

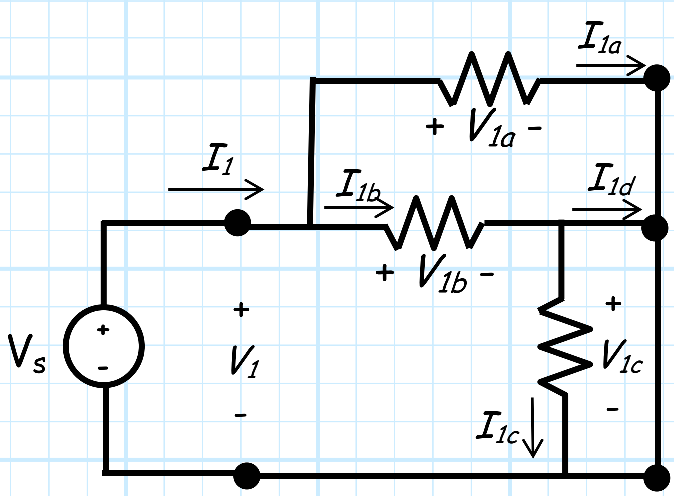
$$V_{1c} = V_{2c} = 0$$

For the case of **odd symmetry**, the symmetric plane must be a plane of **constant potential** (i.e., constant voltage)—just like a **short circuit**!

Thus, for odd symmetry, the symmetric plane forms a **virtual short**.



This **greatly** simplifies things, as we can again **break** the circuit into **two** independent and (effectively) identical circuits!



$$V_1 = V_s$$
$$V_{1a} = V_s$$
$$V_{1b} = V_s$$
$$V_{1c} = 0$$

$$I_1 = V_s/50$$
$$I_{1a} = V_s/100$$
$$I_{1b} = V_s/100$$
$$I_{1c} = 0$$
$$I_{1d} = V_s/100$$

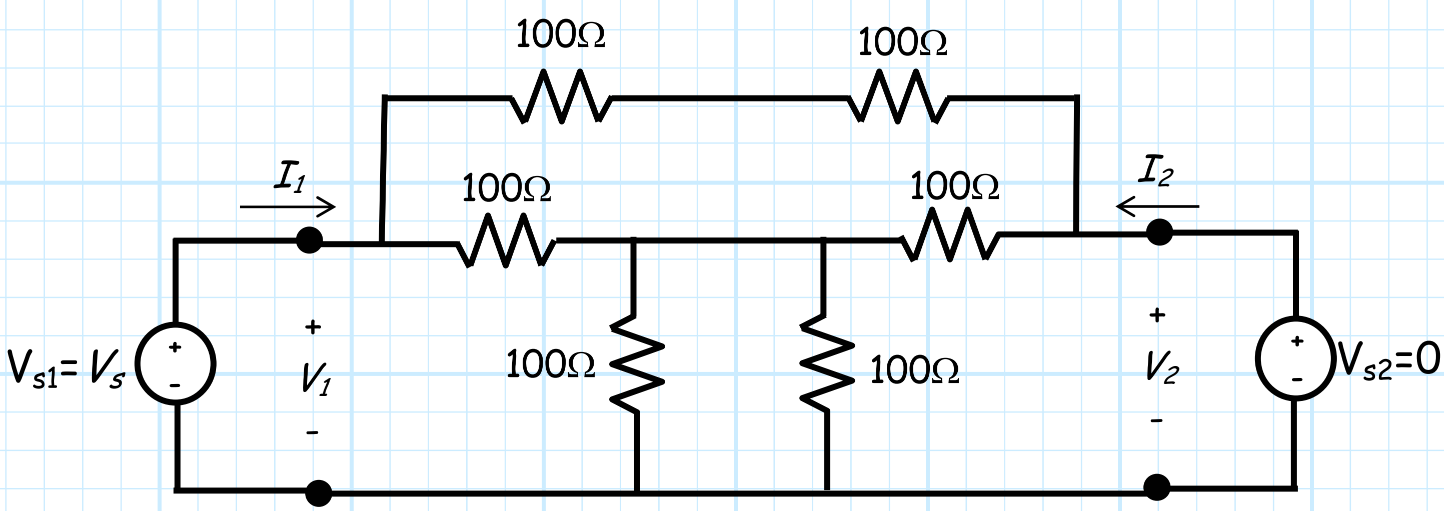
Odd/Even Mode Analysis

Q: *Although symmetric **circuits** appear to be plentiful in microwave engineering, it seems **unlikely** that we would often encounter symmetric **sources**. Do virtual shorts and opens typically ever occur?*

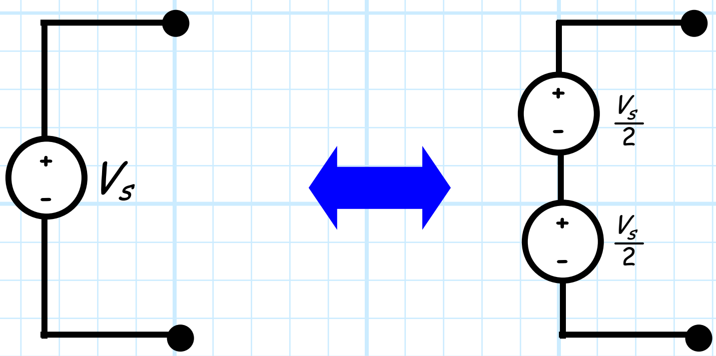
A: One word—**superposition**!

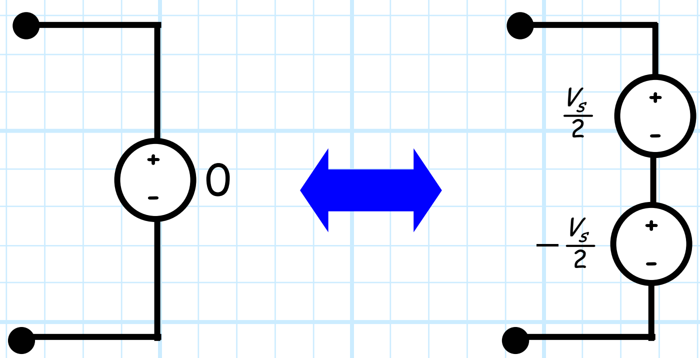
If the elements of our circuit are **independent** and **linear**, we can apply superposition to analyze **symmetric circuits** when **non-symmetric** sources are attached.

For example, say we wish to determine the **admittance matrix** of this circuit. We would place a **voltage source at port 1**, and a **short circuit at port 2**—a set of **asymmetric** sources if there ever was one!

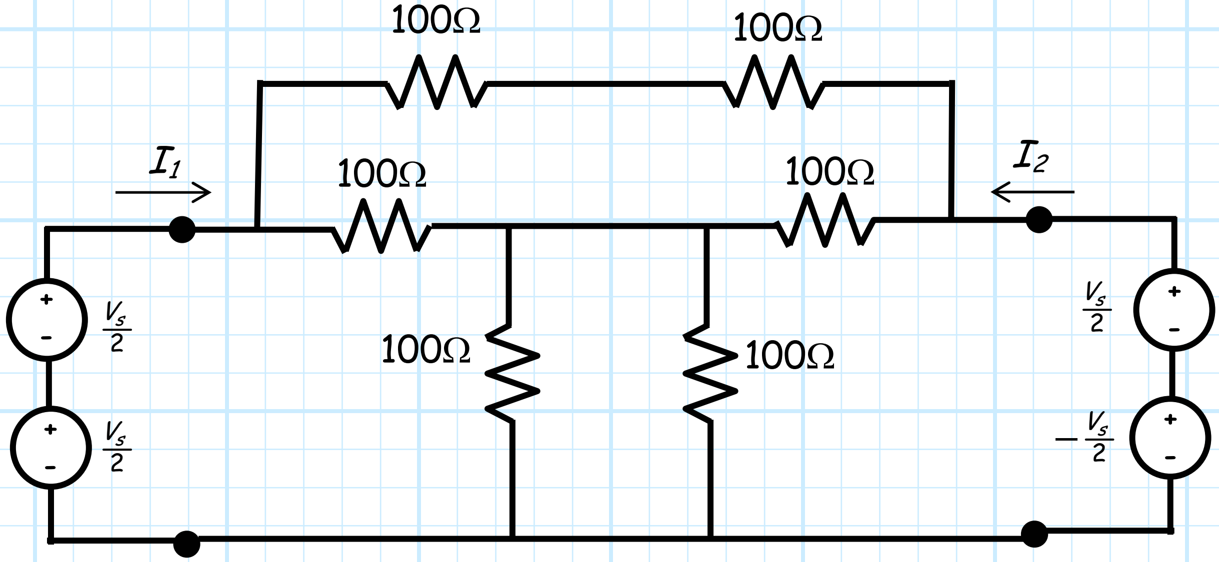


Here's the really **neat** part. We find that the source on port 1 can be model as **two equal** voltage sources in series, whereas the source at port 2 can be modeled as **two equal but opposite** sources in series.

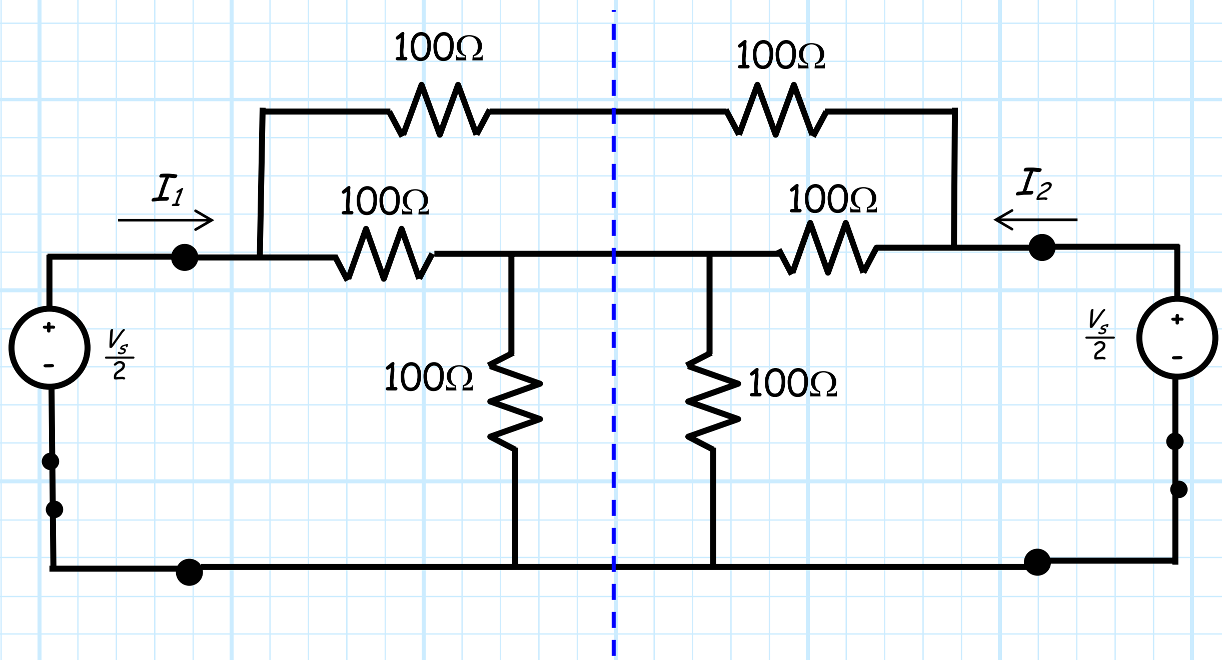




Therefore an **equivalent** circuit is:



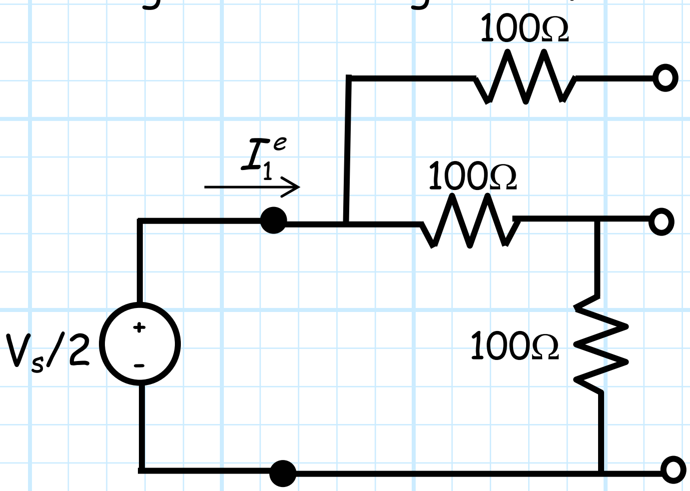
Now, the above circuit (due to the sources) is obviously **asymmetric**—**no** virtual ground, **nor** virtual short is present. But, let's say we **turn off** (i.e., set to V=0) the **bottom** source on **each side** of the circuit:



Our **symmetry** has been **restored**! The symmetry plane is a **virtual open**.

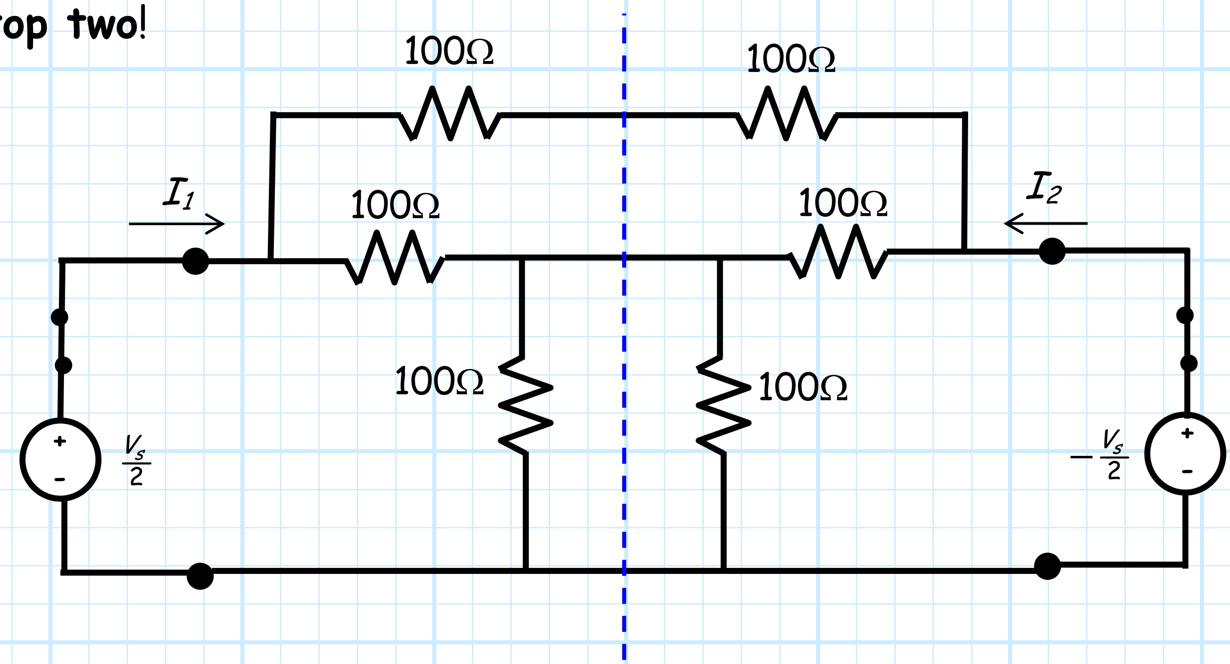
This circuit is referred to as its **even mode**, and analysis of it is known as the **even mode analysis**. The solutions are known as the even mode **currents** and **voltages**!

Evaluating the resulting **even mode** half circuit we find:



$$I_1^e = \frac{V_s}{2}\frac{1}{200} = \frac{V_s}{400} = I_2^e$$

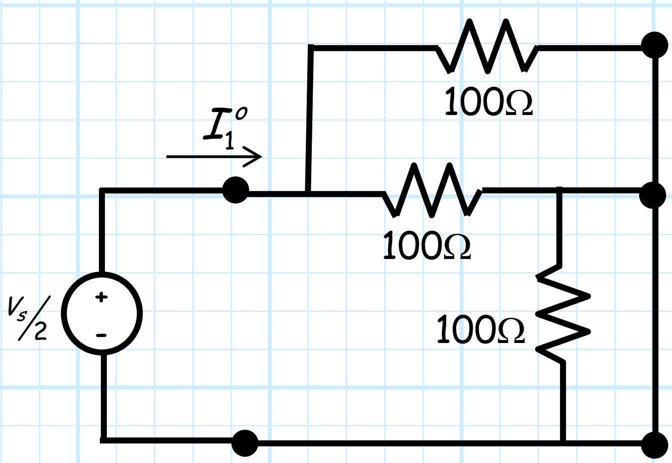
Now, let's turn the bottom sources **back on**—but turn **off** the **top two**!



We now have a circuit with **odd symmetry**—the symmetry plane is a **virtual short**!

This circuit is referred to as its **odd mode**, and analysis of it is known as the **odd mode analysis**. The solutions are known as the odd mode **currents** and **voltages**!

Evaluating the resulting **odd mode** half circuit we find:



$$I_1^o = \frac{V_s}{2}\frac{1}{50} = \frac{V_s}{100} = -I_2^o$$

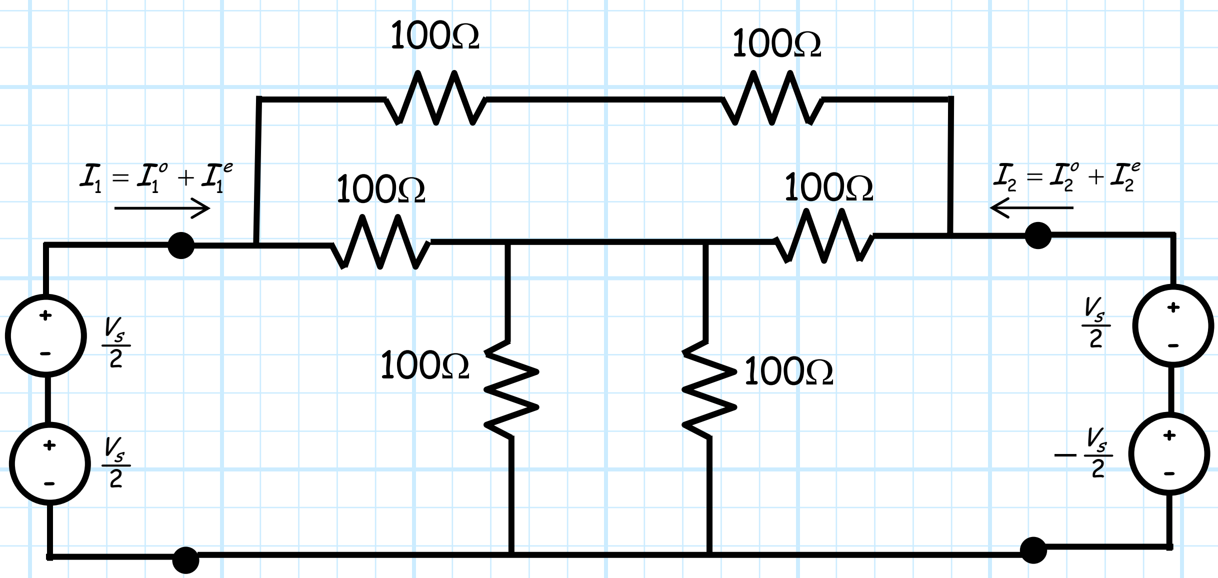
Q: *But what good is this "even mode" and "odd mode" analysis? After all, the source on port 1 is $V_{s1} = V_s$, and the source on port 2 is $V_{s2} = 0$. What are the currents I_1 and I_2 for **these** sources?*

A: Recall that these sources are the **sum** of the even and odd mode sources:

$$V_{s1} = V_s = \frac{V_s}{2} + \frac{V_s}{2} \qquad\qquad V_{s2} = 0 = \frac{V_s}{2} - \frac{V_s}{2}$$

and thus—since all the devices in the circuit are **linear**—we know from superposition that the currents I_1 and I_2 are simply the **sum** of the **odd** and **even** mode currents !

$$I_1 = I_1^e + I_1^o \qquad\qquad I_2 = I_2^e + I_2^o$$



Thus, **adding** the odd and even mode analysis results together:

$$\begin{aligned} I_1 &= I_1^e + I_1^o \\ &= \frac{V_s}{400} + \frac{V_s}{100} \\ &= \frac{V_s}{80} \end{aligned} \qquad \begin{aligned} I_2 &= I_2^e + I_2^o \\ &= \frac{V_s}{400} - \frac{V_s}{100} \\ &= -\frac{3\,V_s}{400} \end{aligned}$$

And then the **admittance parameters** for this two port network is:

$$Y_{11} = \frac{I_1}{V_{s1}}\bigg|_{V_{s2}=0} = \frac{V_s}{80}\frac{1}{V_s} = \frac{1}{80}$$

$$Y_{21} = \frac{I_2}{V_{s1}}\bigg|_{V_{s2}=0} = -\frac{3\,V_s}{400}\frac{1}{V_s} = \frac{-3}{400}$$

And from the **symmetry** of the device we know:

$$Y_{22} = Y_{11} = \frac{1}{80}$$

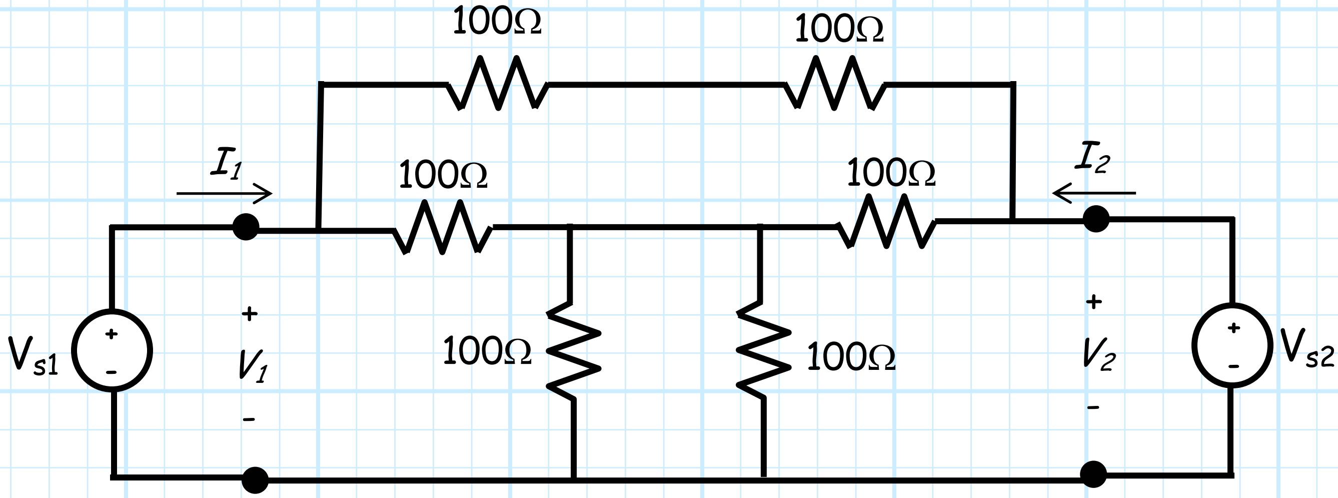
$$Y_{12} = Y_{21} = \frac{-3}{400}$$

Thus, the full **admittance matrix** is:

$$\mathcal{Y} = \begin{bmatrix} 1/80 & -3/400 \\ -3/400 & 1/80 \end{bmatrix}$$

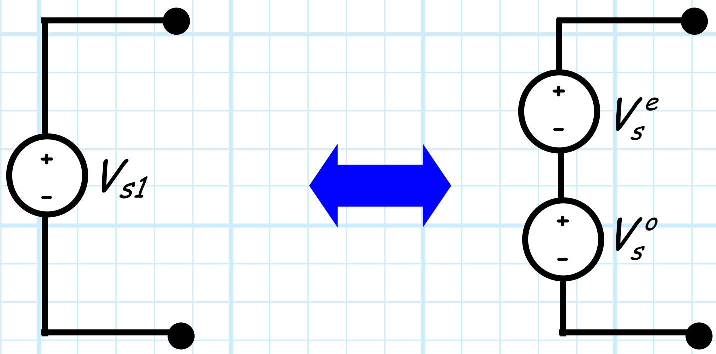
Q: *What happens if both sources are non-zero? Can we use symmetry then?*

A: Absolutely! Consider the problem below, where neither source is equal to zero:

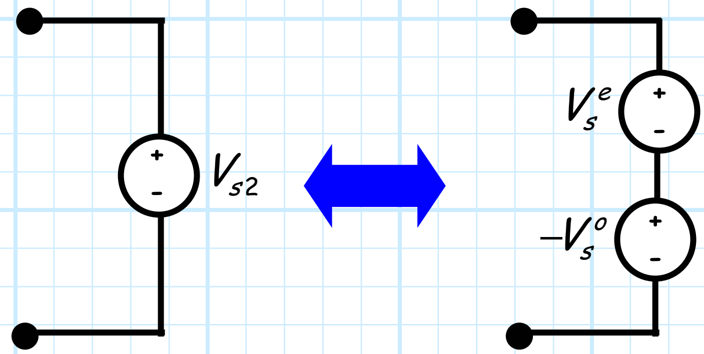


In this case we can define an even mode and an odd mode source as:

$$V_s^e = \frac{V_{s1} + V_{s2}}{2} \qquad\qquad V_s^o = \frac{V_{s1} - V_{s2}}{2}$$

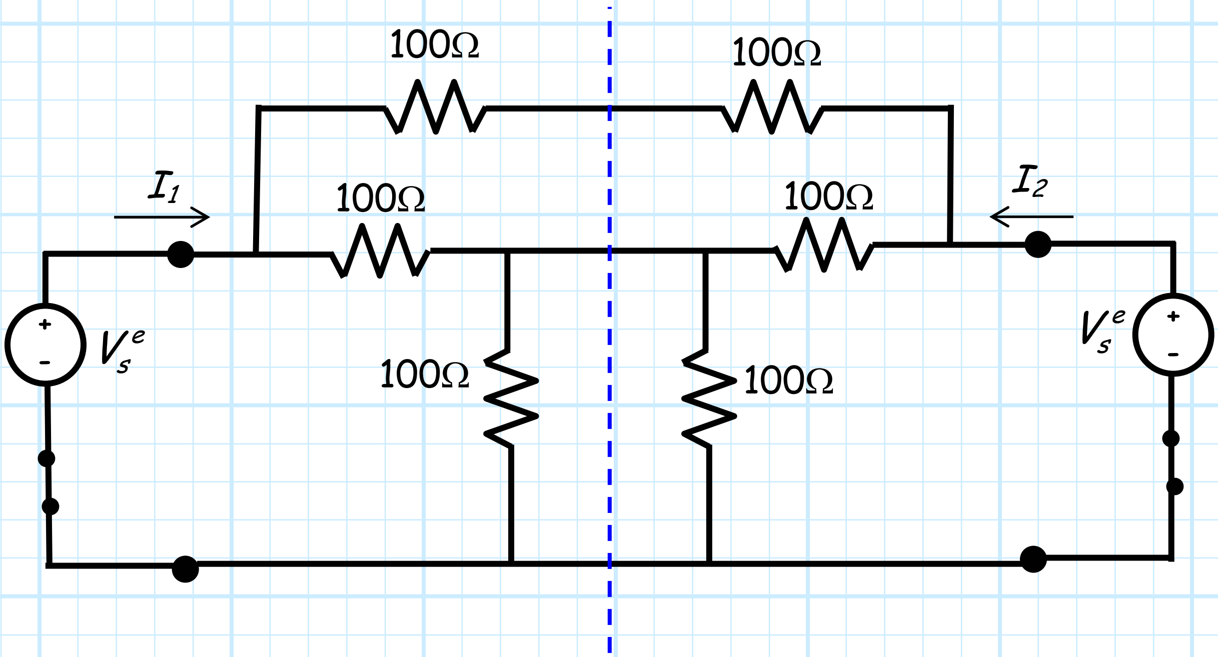


$$V_{s1} = V_s^e + V_s^o$$

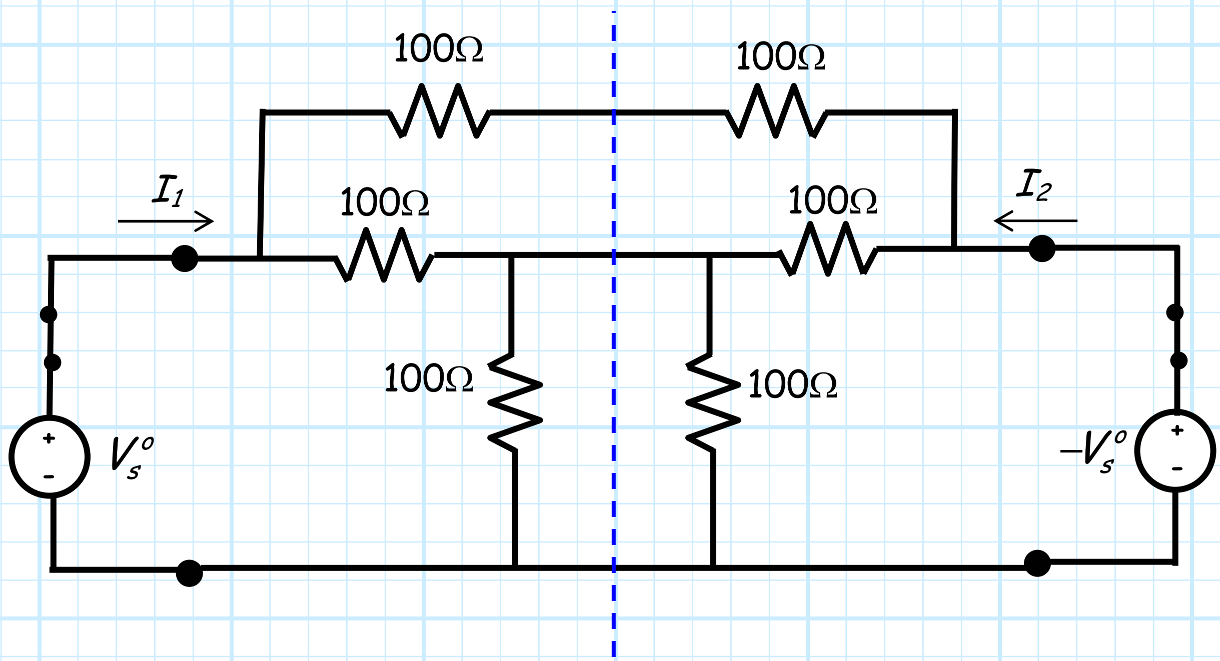


$$V_{s2} = V_s^e - V_s^o$$

We then can analyze the **even mode** circuit:



And then the **odd mode** circuit:

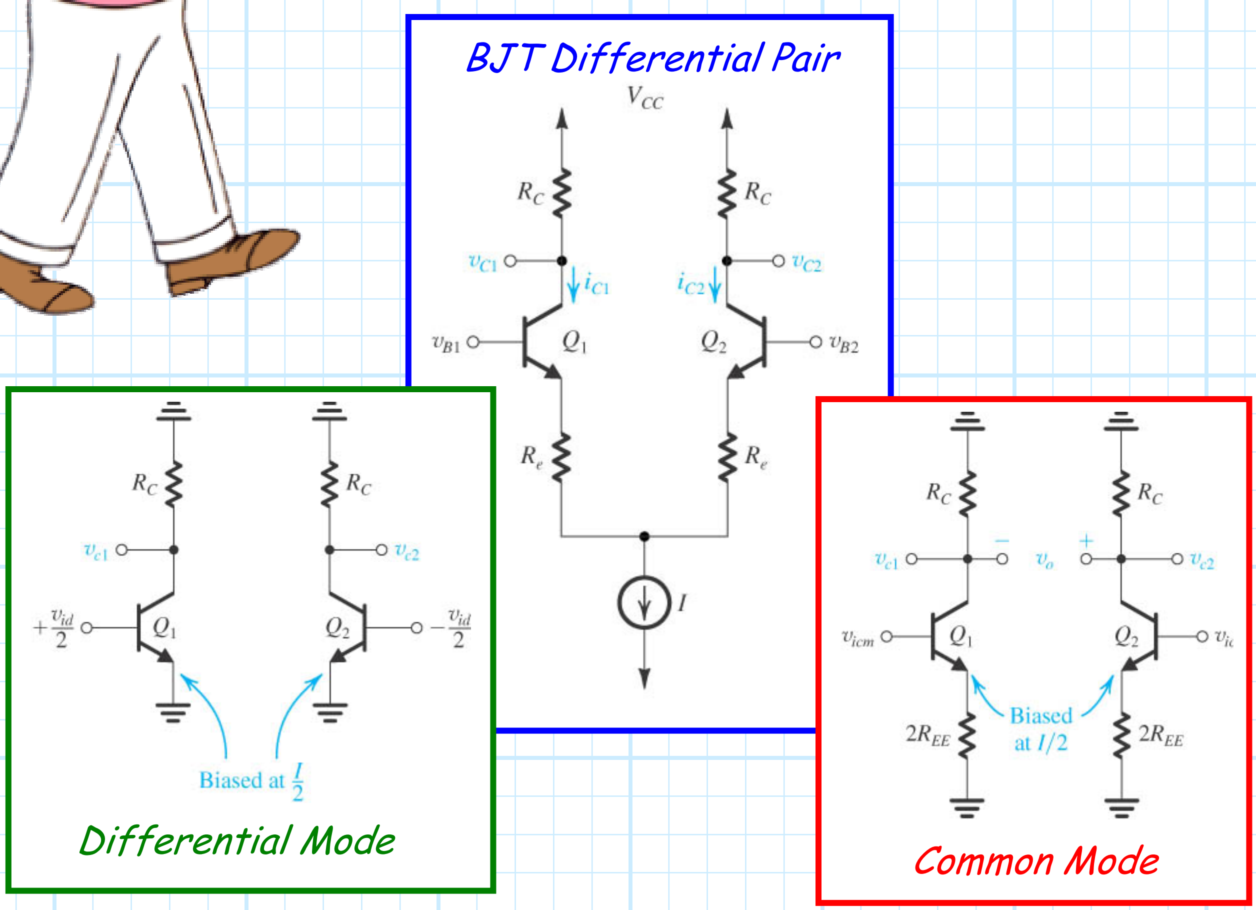


And then combine these results in a **linear superposition**!

One **final** word (I promise!) about circuit symmetry and even/odd mode analysis: **precisely** the **same** concept exits in **electronic circuit** design!

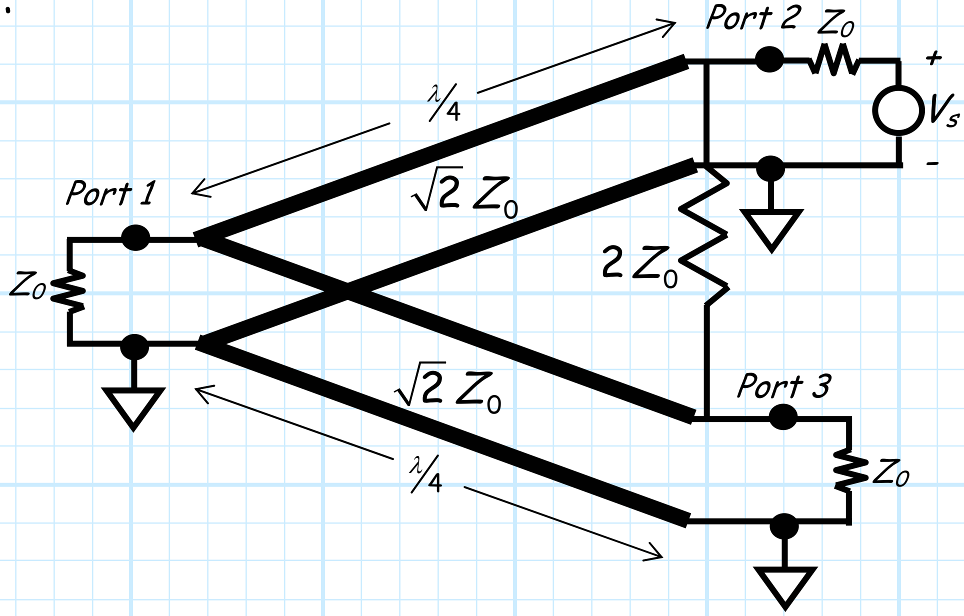
Specifically, the **differential** (odd) and **common** (even) **mode** analysis of bilaterally symmetric electronic circuits, such as **differential amplifiers**!

*Hi! You might remember differential and common mode analysis from such classes as "**EECS 412**- Electronics II", or handouts such as "**Differential Mode** Small-Signal Analysis of **BJT Differential Pairs**"*

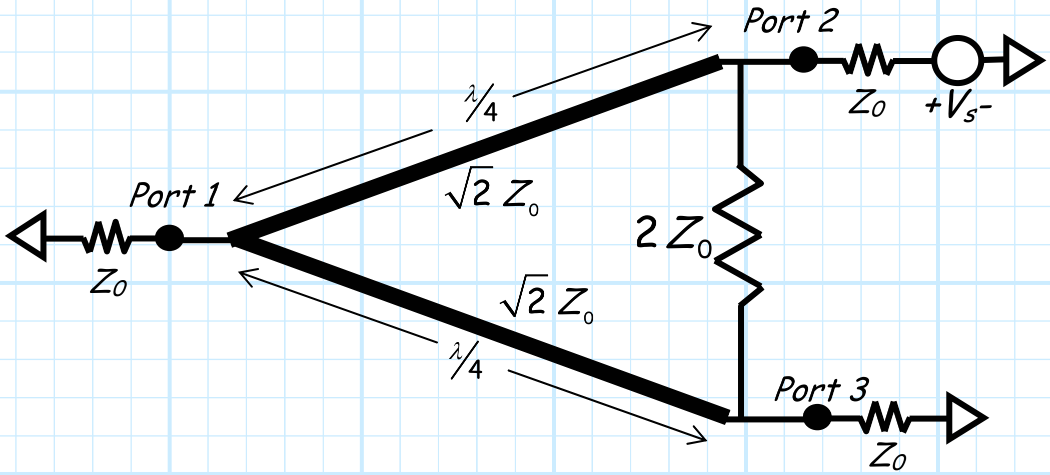


Even/Odd Mode Analysis of the Wilkinson Divider

Consider a matched **Wilkinson power divider**, with a **source at port 2**:



Too **simplify** this schematic, we **remove** the ground plane, which includes the **bottom conductor** of the transmission lines:



Q: *How do we* ***analyze*** *this circuit ?*

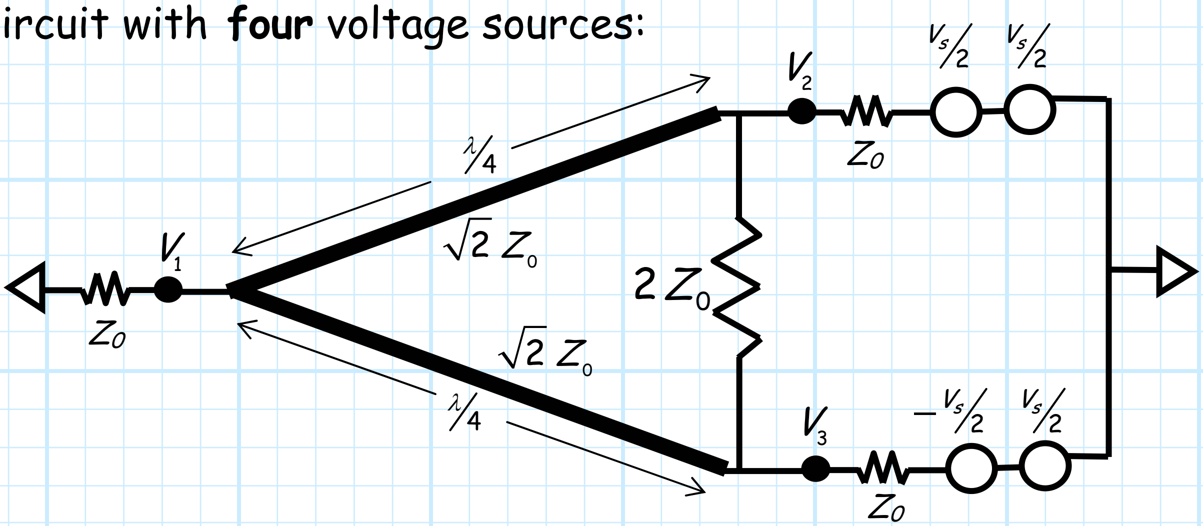
A: Use **Even-Odd mode** analysis!

Remember, even-odd mode analysis uses **two** important principles:

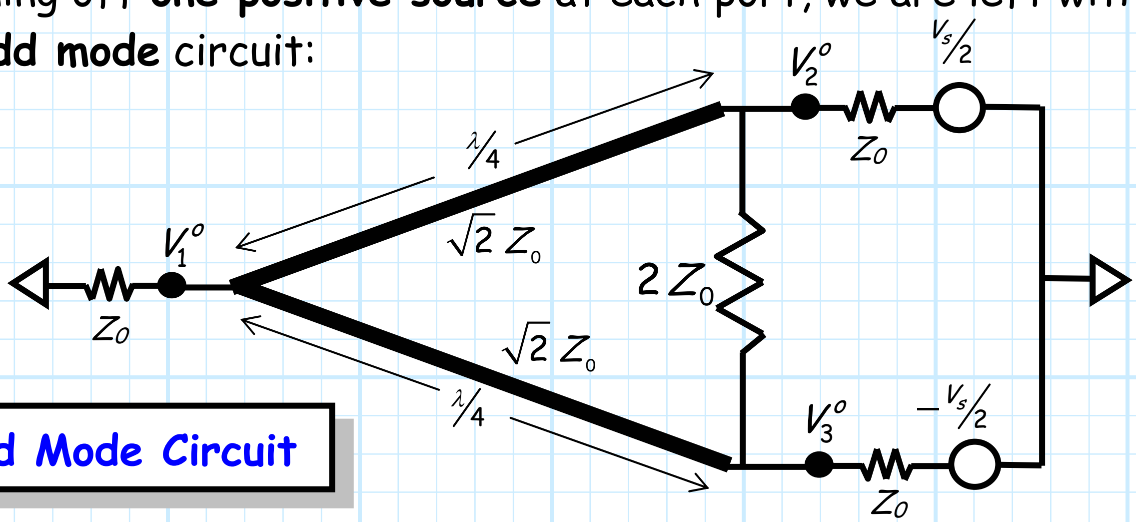
a) **superposition**

b) circuit **symmetry**

To see how we apply these principles, let's first rewrite the circuit with **four** voltage sources:

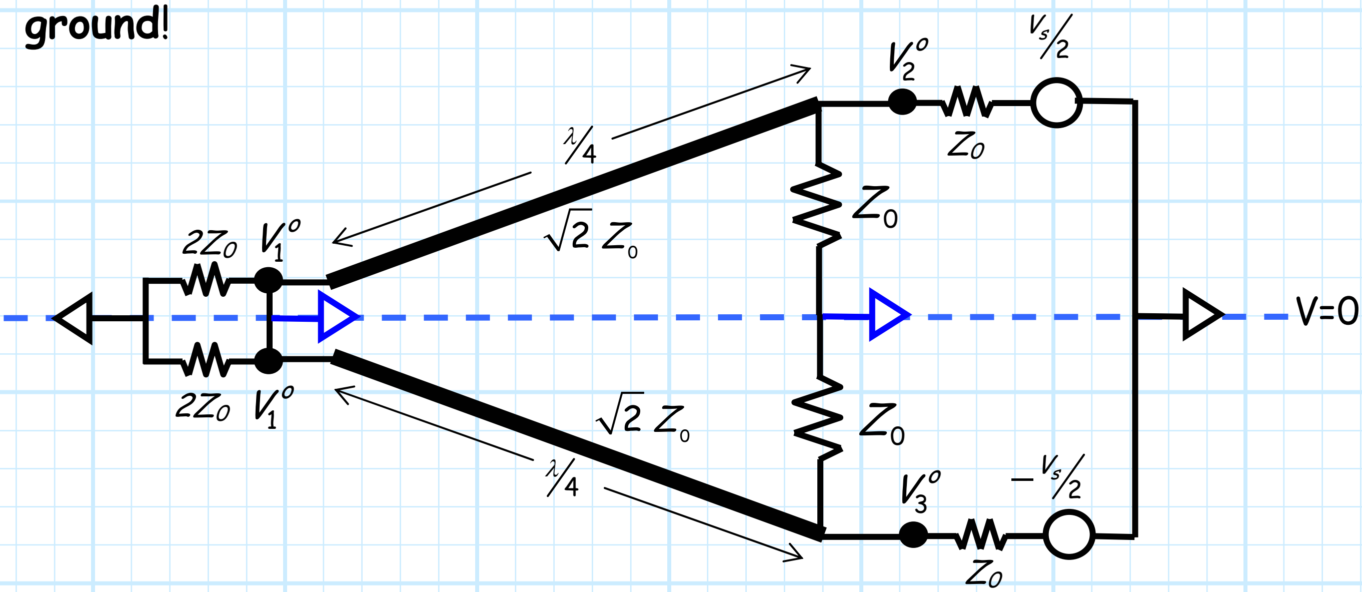


Turning off **one positive source** at each port, we are left with an **odd mode** circuit:

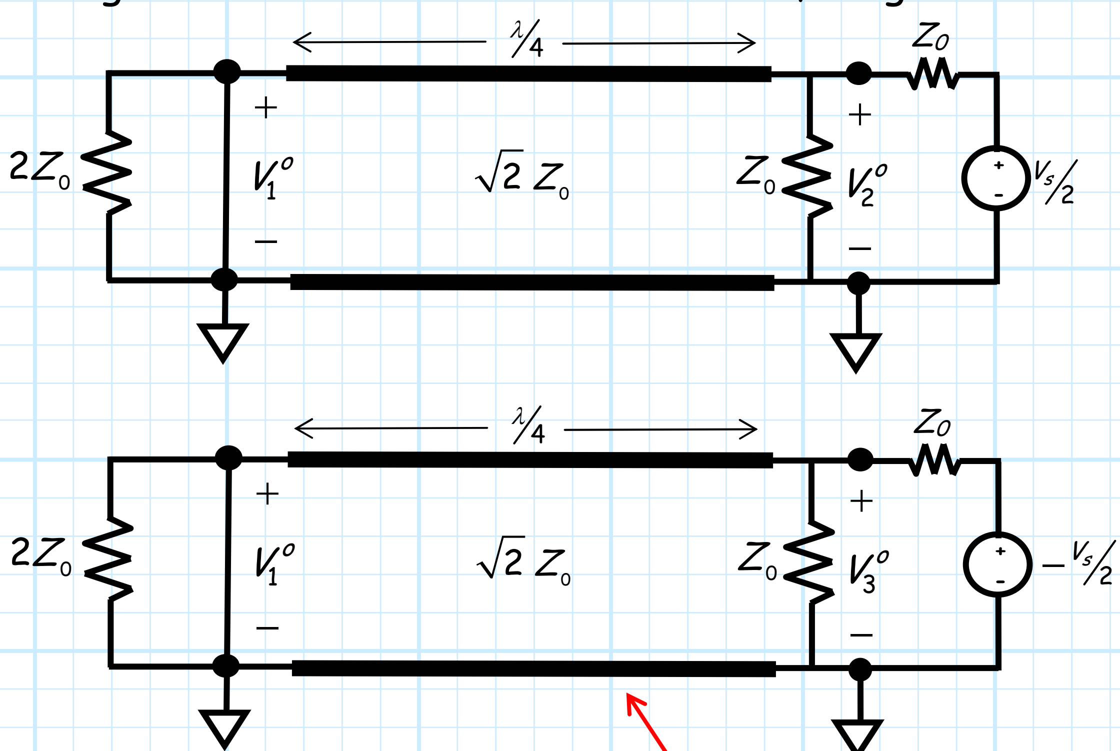


Odd Mode Circuit

Note the circuit has **odd symmetry**, and thus the plane of symmetry becomes a **virtual short**, and in this case, a virtual **ground**!

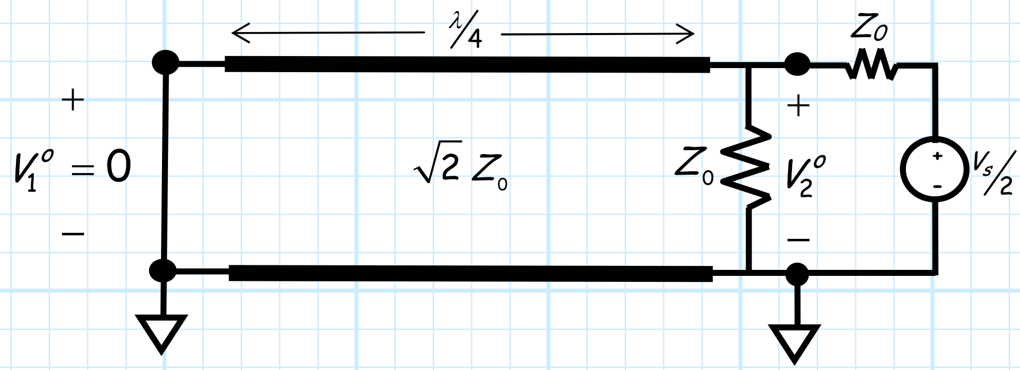


Dividing the circuit into two **half-circuits**, we get:



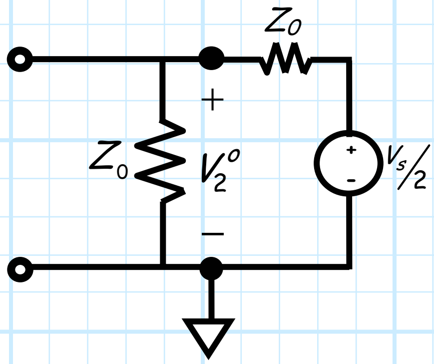
Note we have again drawn the **bottom conductor** of the transmission line (a **ground plane**) to enhance **clarity** (I **hope**!).

Analyzing the top circuit, we find that the transmission line is terminated in a **short** circuit in **parallel** with a **resistor** of value $2Z_0$. Thus, the transmission line is terminated in a **short circuit**!



This of course makes determining V_1^o **trivial** (**hint:** $V_1^o = 0$).

Now, since the transmission line is a **quarter wavelength**, this **short** circuit at the **end** of the transmission line transforms to an **open** circuit at the **beginning**!



As a result, determining voltage V_2^o is nearly as **trivial** as determining voltage V_1^o. **Hint:**

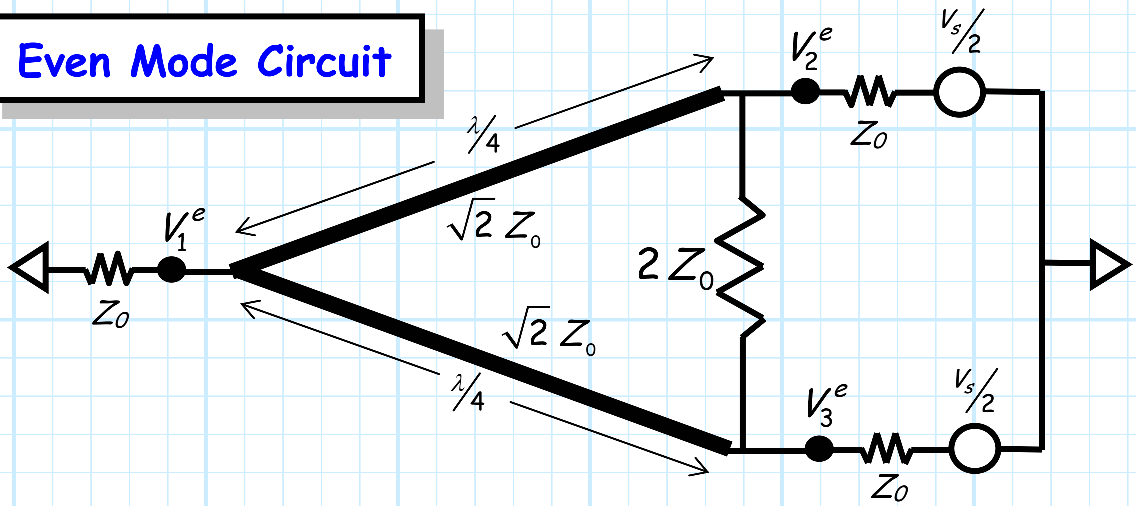
$$V_2^o = \frac{V_s}{2}\,\frac{Z_0}{Z_0 + Z_0} = \frac{V_s}{4}$$

And from the **odd symmetry** of the circuit, we likewise know:

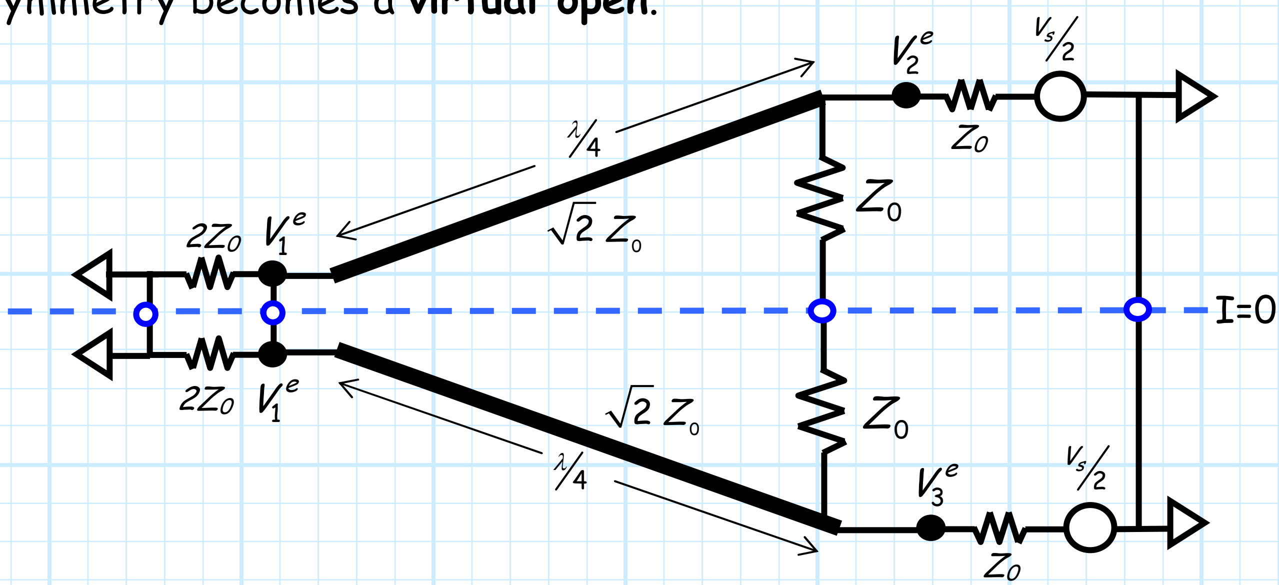
$$V_3^o = -V_2^o = -\frac{V_s}{4}$$

Now, let's turn **off** the **odd mode sources**, and turn back **on** the **even mode sources**.

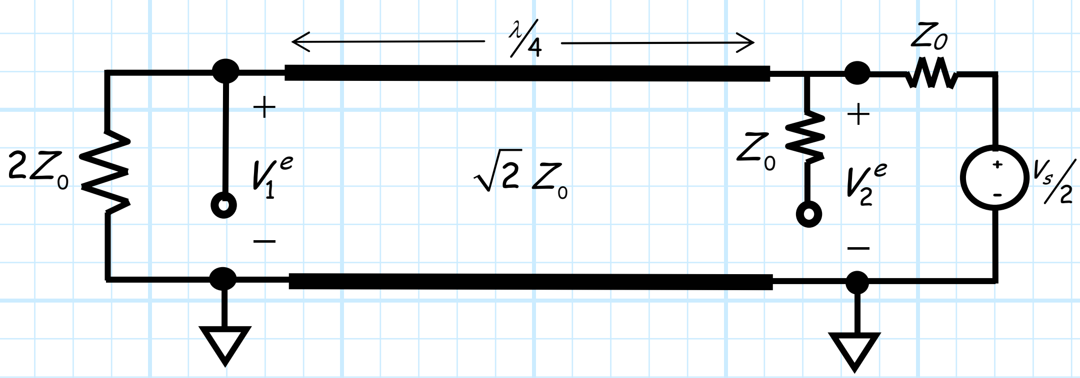
Even Mode Circuit

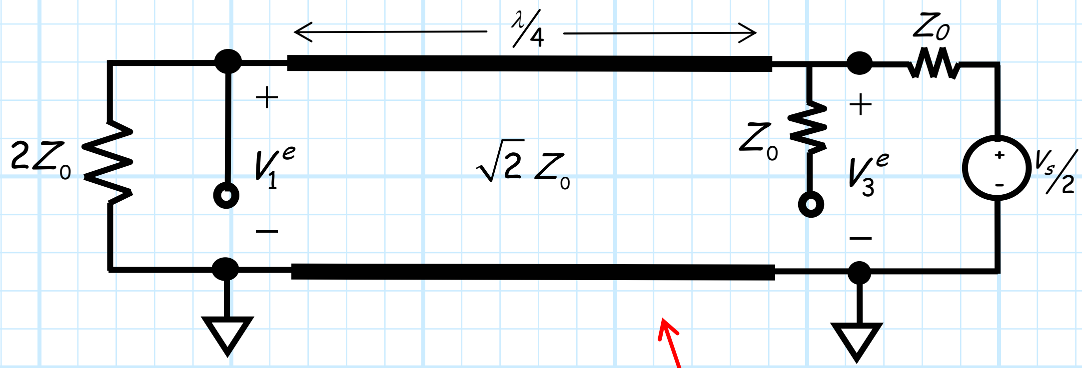


Note the circuit has **even symmetry**, and thus the plane of symmetry becomes a **virtual open**.



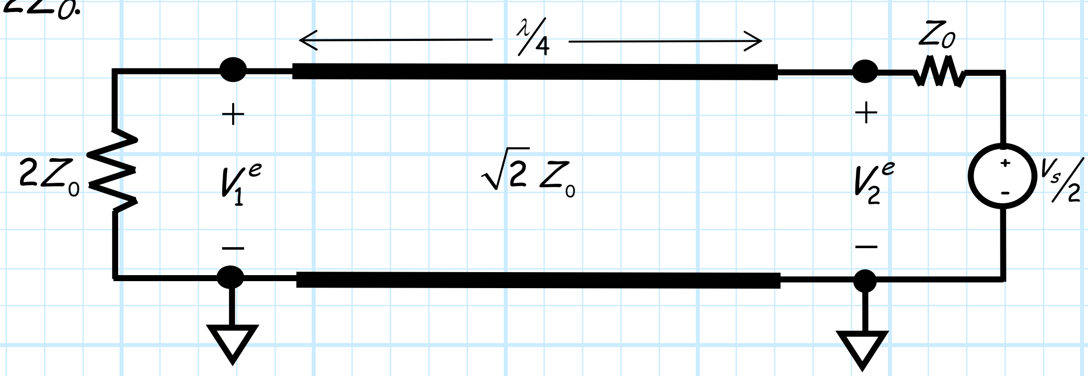
Dividing the circuit into two **half-circuits**, we get:





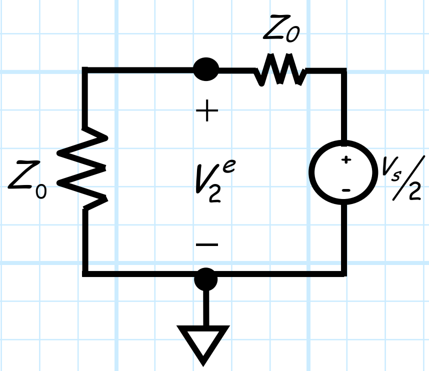
Note we have **again** drawn the **bottom conductor** of the transmission line (a **ground plane**).

Analyzing the top circuit, we find that the transmission line is terminated in a **open** circuit in **parallel** with a **resistor** of value $2Z_0$. Thus, the transmission line is terminated in a **resistor** valued $2Z_0$.



Now, since the transmission line is a **quarter wavelength**, the $2Z_0$ resistor at the **end** of the transmission line transforms to **this** value at the **beginning**:

$$Z_{in} = \frac{\left(\sqrt{2}Z_0\right)^2}{2Z_0} = Z_0$$



Voltage V_2^e can again be determined by **voltage division**:

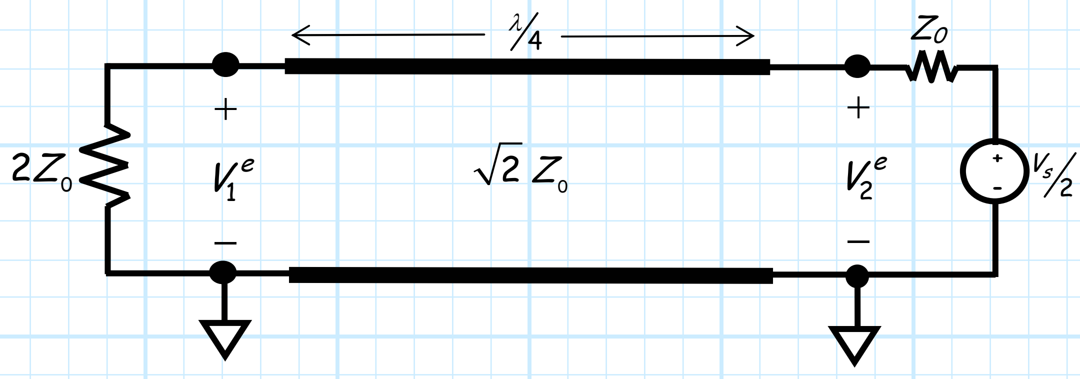
$$V_2^e = \frac{V_s}{2}\frac{Z_0}{Z_0 + Z_0} = \frac{V_s}{4}$$

And then due to the **even symmetry** of the circuit, we know:

$$V_3^e = V_2^e = \frac{V_s}{4}$$

Q: *What about voltage V_1^e? What is its value?*

A: Well, there's **no** direct or easy way to find this value. We must apply our transmission line theory (i.e., the solution to the **telegrapher's equations** + **boundary conditions**) to find this value. This means **applying** the knowledge and skills acquired during our **scholarly** examination of **Chapter 2**!



If we **carefully** and **patiently** analyze the above transmission line circuit, we find that (see if **you** can verify this!):

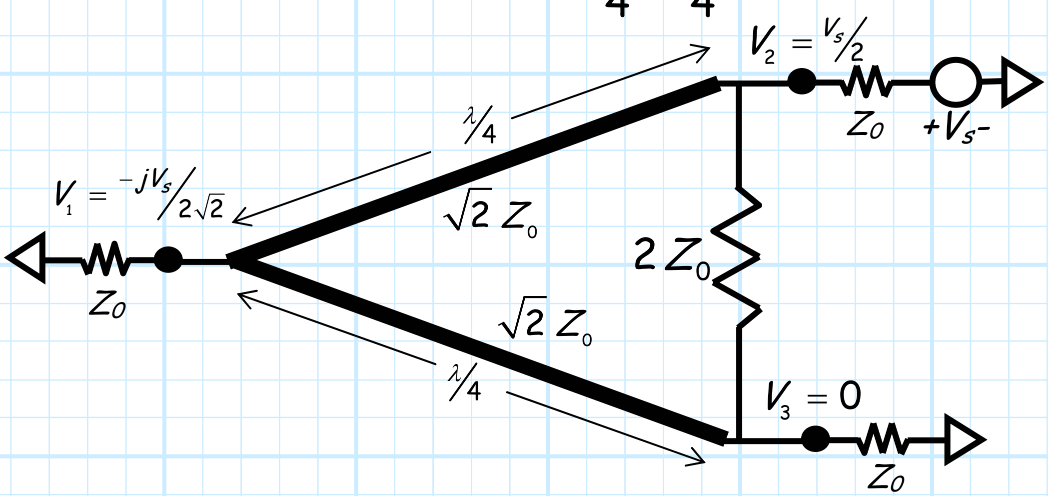
$$V_1^e = \frac{-j\,V_s}{2\sqrt{2}}$$

And thus, completing our **superposition** analysis, the voltages and currents within the circuit is simply found from the **sum** of the solutions of each mode:

$$V_1 = V_1^o + V_1^o = 0 - \frac{jV_s}{2\sqrt{2}} = -\frac{jV_s}{2\sqrt{2}}$$

$$V_2 = V_2^o + V_2^o = \frac{V_s}{4} + \frac{V_s}{4} = \frac{V_s}{2}$$

$$V_3 = V_3^o + V_3^o = -\frac{V_s}{4} + \frac{V_s}{4} = 0$$



Note that the voltages we calculated are **total voltages**—the **sum** of the **incident** and **exiting** waves at each port:

$$V_1 \doteq V_1\left(z_1 = z_{1P}\right) = V_1^+\left(z_1 = z_{1P}\right) + V_1^-\left(z_1 = z_{1P}\right)$$

$$V_2 \doteq V_2\left(z_2 = z_{2P}\right) = V_2^+\left(z_2 = z_{2P}\right) + V_2^-\left(z_2 = z_{2P}\right)$$

$$V_3 \doteq V_3\left(z_3 = z_{3P}\right) = V_3^+\left(z_3 = z_{3P}\right) + V_3^-\left(z_3 = z_{3P}\right)$$

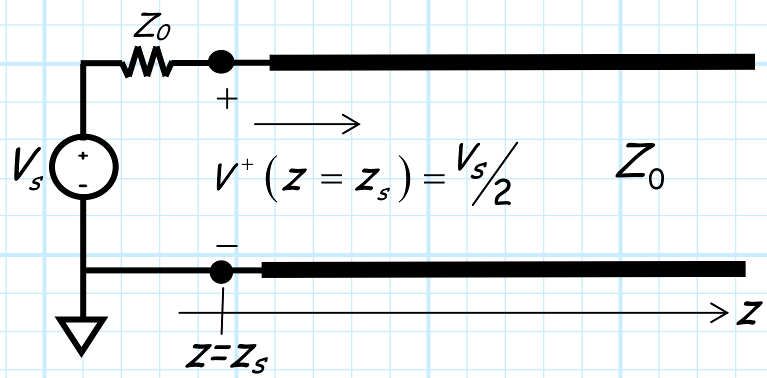
Since ports 1 and 3 are terminated in **matched loads**, we know that the **incident** wave on those ports are **zero**. As a result, the **total** voltage is equal to the value of the exiting waves at those ports:

$$V_1^+(z_1 = z_{1P}) = 0 \qquad V_1^-(z_1 = z_{1P}) = \frac{-jV_s}{2\sqrt{2}}$$

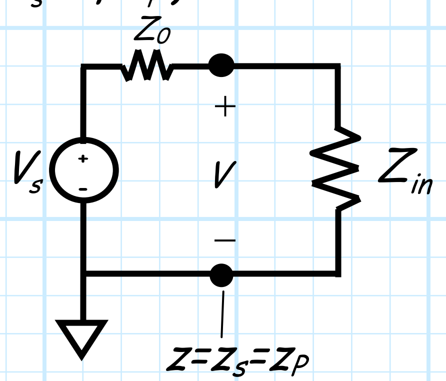
$$V_3^+(z_3 = z_{3P}) = 0 \qquad V_3^-(z_3 = z_{3P}) = 0$$

The problem now is to determine the values of the **incident** and **exiting** waves at port 2 (i.e., $V_2^+(z_2 = z_{2P})$ and $V_2^-(z_2 = z_{2P})$).

Recall however, the specific case where the **source impedance** is **matched** to transmission line characteristic impedance (i.e., $Z_s = Z_0$). We found for this specific case, the incident wave "launched" by the source **always** has the value $V_s/2$ at the source:



Now, if the length of the transmission line connecting a source to a port (or load) is **electrically very small** (i.e., $\beta\ell \ll 1$), then the source is effectively **connected directly** to the source (i.e, $\beta z_s = \beta z_P$):



And thus the **total** voltage is:

$$\begin{aligned} V &= V^+(z = z_P) + V^-(z = z_P) \\ &= V^+(z = z_S) + V^-(z = z_P) \\ &= V_s/2 + V^-(z = z_P) \end{aligned}$$

For the case where a **matched source** (i.e. $Z_s = Z_0$) is connected directly to a port, we can thus conclude:

$$V^+(z = z_P) = V_s/2$$

$$V^-(z = z_P) = V - V_s/2$$

Thus, for port 2 we find:

$$V_2^+(z_2 = z_{2P}) = V_s/2$$

$$V_2^-(z_2 = z_{2P}) = V_2 - V_s/2 = V_s/2 - V_s/2 = 0$$

Now, we can **finally** determine the scattering parameters S_{12}, S_{22}, S_{32}:

$$S_{12} = \frac{V_1^-(z_1 = z_{1P})}{V_2^+(z_2 = z_{2P})} = \left(\frac{-jV_s}{2\sqrt{2}}\right)\frac{2}{V_s} = \frac{-j}{\sqrt{2}}$$

$$S_{22} = \frac{V_2^-(z_2 = z_{2P})}{V_2^+(z_2 = z_{2P})} = (0)\frac{2}{V_s} = 0$$

$$S_{32} = \frac{V_3^-(z_3 = z_{3P})}{V_2^+(z_2 = z_{2P})} = (0)\frac{2}{V_s} = 0$$

Q: *Wow! That seemed like a **lot** of hard work, and we're only ⅓ of the way done. Do we **have** to move the source to port 1 and then port 3 and perform similar analyses?*

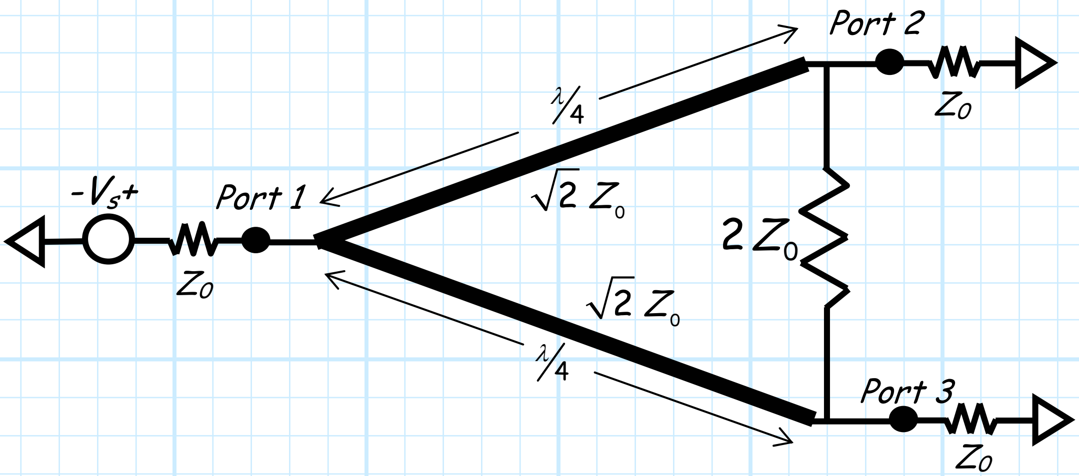
A: Nope! Using the bilateral **symmetry** of the circuit $(1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2)$, we can conclude:

$$S_{13} = S_{12} = \frac{-j}{\sqrt{2}} \qquad S_{33} = S_{22} = 0 \qquad S_{23} = S_{32} = 0$$

and from **reciprocity**:

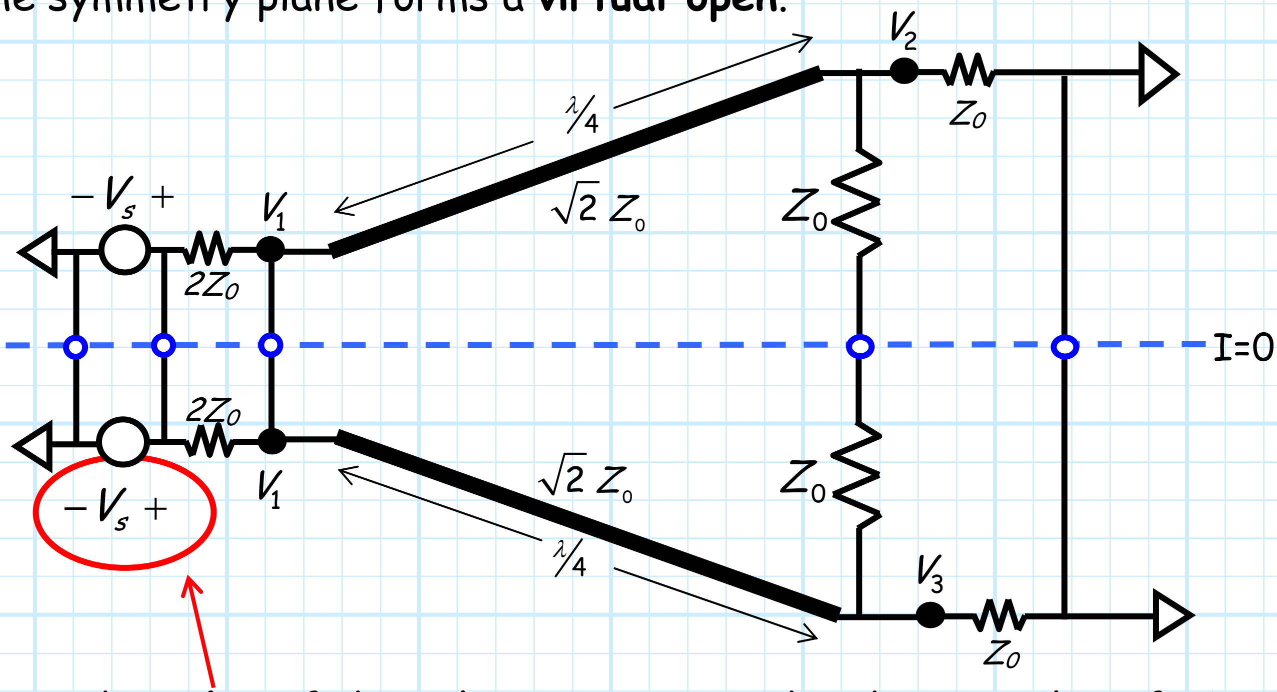
$$S_{21} = S_{12} = \frac{-j}{\sqrt{2}} \qquad S_{31} = S_{13} = \frac{-j}{\sqrt{2}}$$

We thus have determined **8** of the **9** scattering parameters needed to characterize this 3-port device. The **remaining** holdout is the scattering parameter S_{11}. To find this value, we must move the **source to port 1** and analyze.

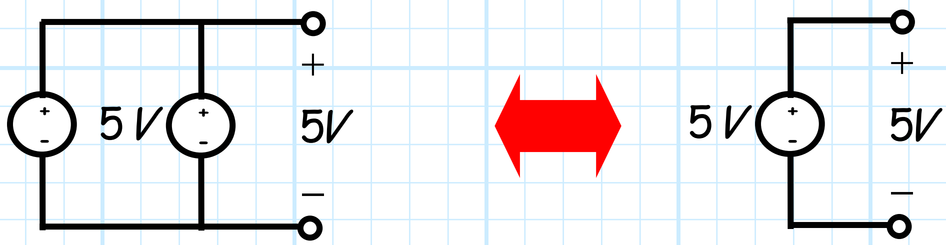


Note this source does **not** alter the bilateral symmetry of the circuit. We can thus use this symmetry to **help analyze** the circuit, **without** having to specifically define odd and even mode sources.

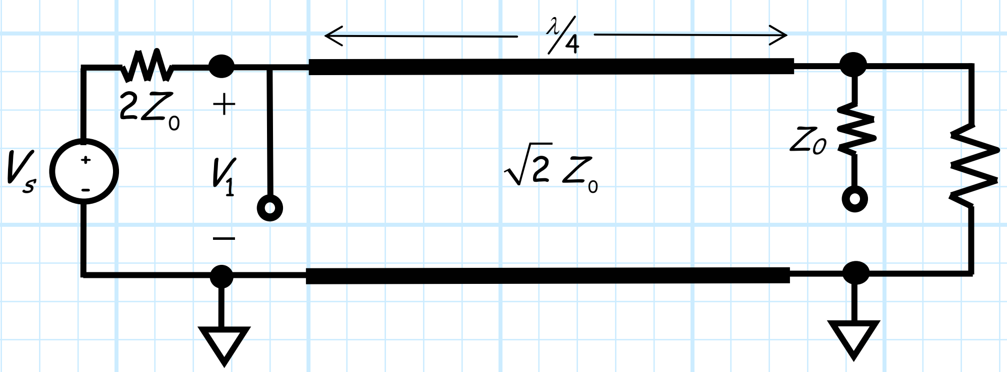
Since the circuit has (even) bilateral symmetry, we know that the symmetry plane forms a **virtual open**.



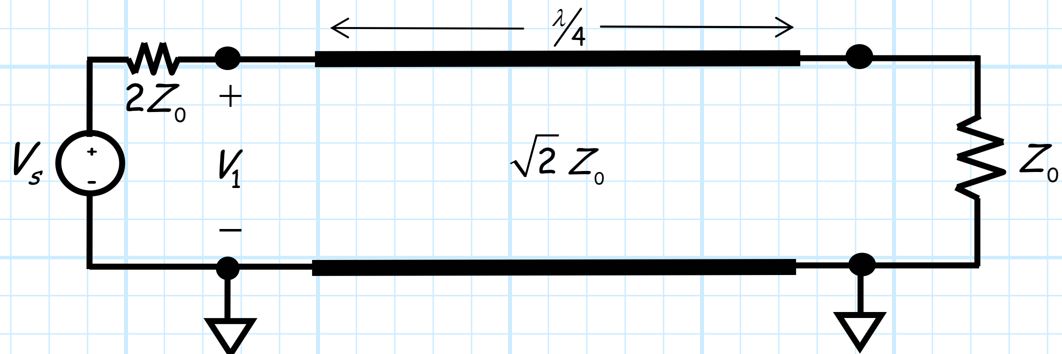
Note the **value** of the voltage sources. They have a value of V_s (as **opposed** to, say, $2V_s$ or $V_s/2$) because two equal voltage sources in **parallel** is equivalent to one voltage source of the **same value**. E.G.:



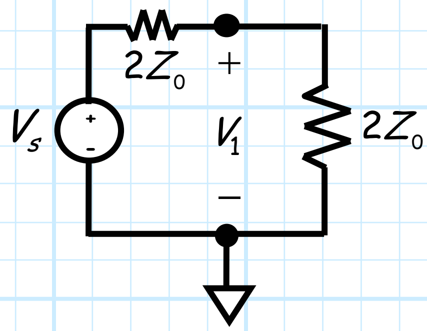
Now **splitting** the circuit into **two** half-circuits, we find the **top** half-circuit to be:



Which **simplifies** to:



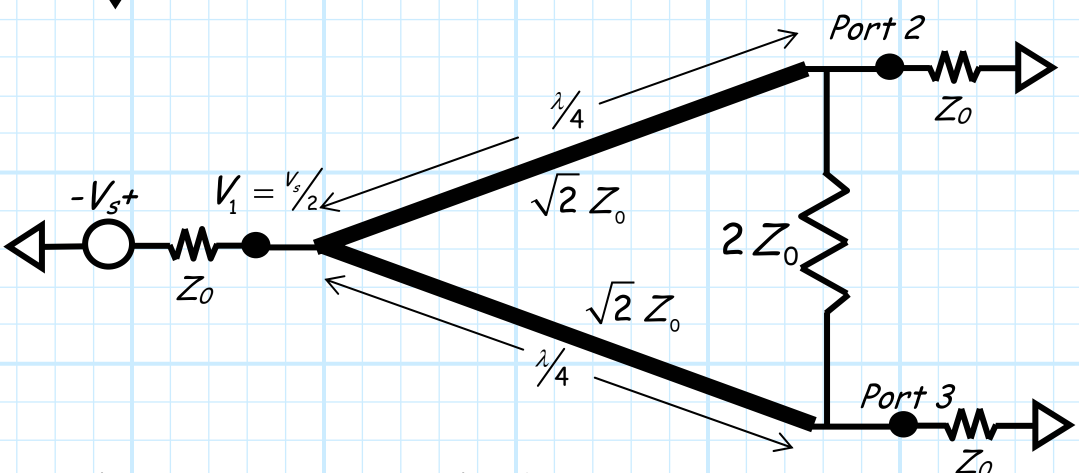
And **transforming** the load resistor at the end of the ¼ wave line back to its **beginning**:



Finally, we use **voltage division** to determine that:

$$V_1 = V_s\left(\frac{2Z_0}{2Z_0 + 2Z_0}\right) = \frac{V_s}{2}$$

Thus,



And since the **source is matched**:

$$V_1^+\left(z_1 = z_{1P}\right) = \frac{V_s}{2}$$

$$V_1^-\left(z_1 = z_{1P}\right) = V_1 - \frac{V_s}{2} = \frac{V_s}{2} - \frac{V_s}{2} = 0$$

So our **final** scattering element is revealed!

$$S_{11} = \frac{V_1^-(z_1 = z_{1P})}{V_1^+(z_1 = z_{1P})} = (0)\frac{2}{V_s} = 0$$

So the scattering matrix of a **Wilkinson power divider** has been **confirmed**:

$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}\!/\!_{\sqrt{2}} & {}^{-j}\!/\!_{\sqrt{2}} \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \\ {}^{-j}\!/\!_{\sqrt{2}} & 0 & 0 \end{bmatrix}$$