

7.6 - Coupled-Line Directional Couplers

Reading Assignment: pp. 337-348

Q: *The Quadrature Hybrid is a 3dB coupler. How do we build couplers with less coupling, say 10dB, 20dB, or 30 dB?*

A: Directional couplers are typically built using **coupled lines**.

HO: COUPLED LINE COUPLERS

Q: *How can we **design** a coupled line couplers so that is an ideal directional coupler with a **specific** coupling value?*

A: **HO: ANALYSIS AND DESIGN OF COUPLED-LINE COUPLERS**

Q: *Like all devices with quarter-wavelength sections, a coupled line coupler would seem to be inherently narrow band. Is there some way to **increase coupler bandwidth**?*

A: **Yes!** We can add more coupled-line sections, just like with **multi-section** matching transformers.

HO: MULTI-SECTION COUPLED LINE COUPLERS

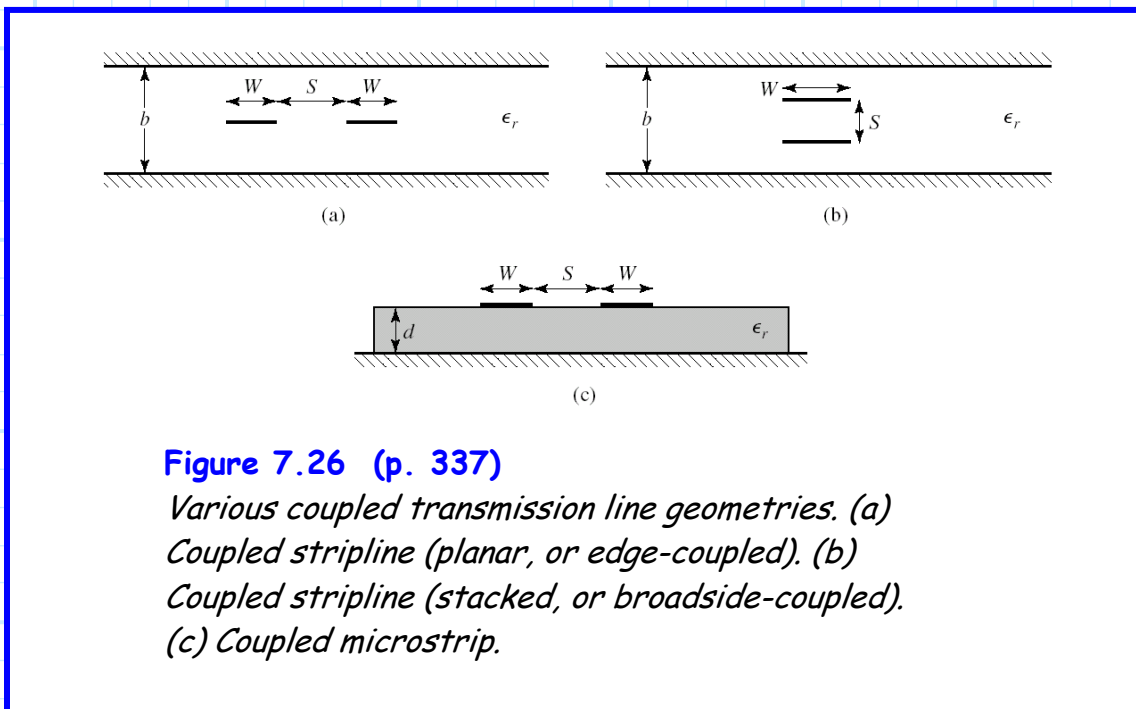
Q: *How do we **design** these multi-section couplers?*

A: All the requisite design examples were provided in the last handout, and there are two good design examples on pages 345 and 348 of your textbook!

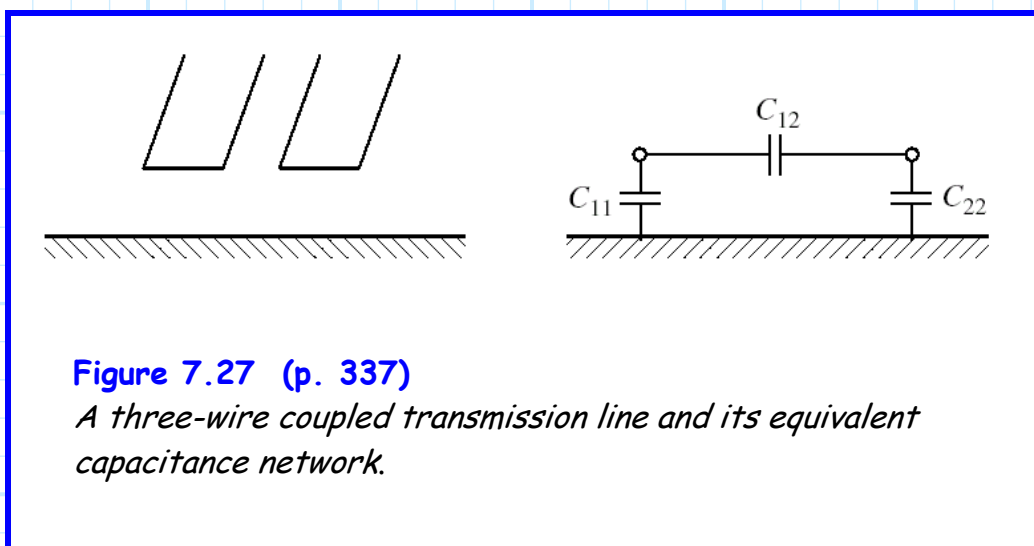
Coupled-Line Couplers

Two transmission lines in **proximity** to each other will **couple** power from one line into another.

This proximity will **modify** the electromagnetic fields (and thus modify voltages and currents) of the propagating wave, and therefore **alter** the characteristic impedance of the transmission line!

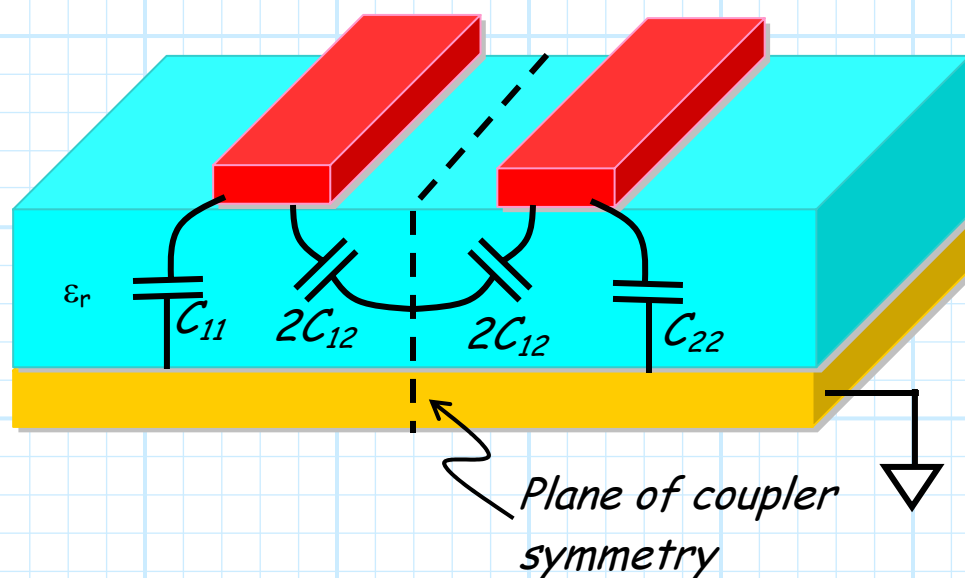


Generally, speaking, we find that this transmission lines are capacitively coupled (i.e., it appears that they are connected by a capacitor):



If the two transmission lines are **identical** (and they typically are), then $C_{11} = C_{22}$.

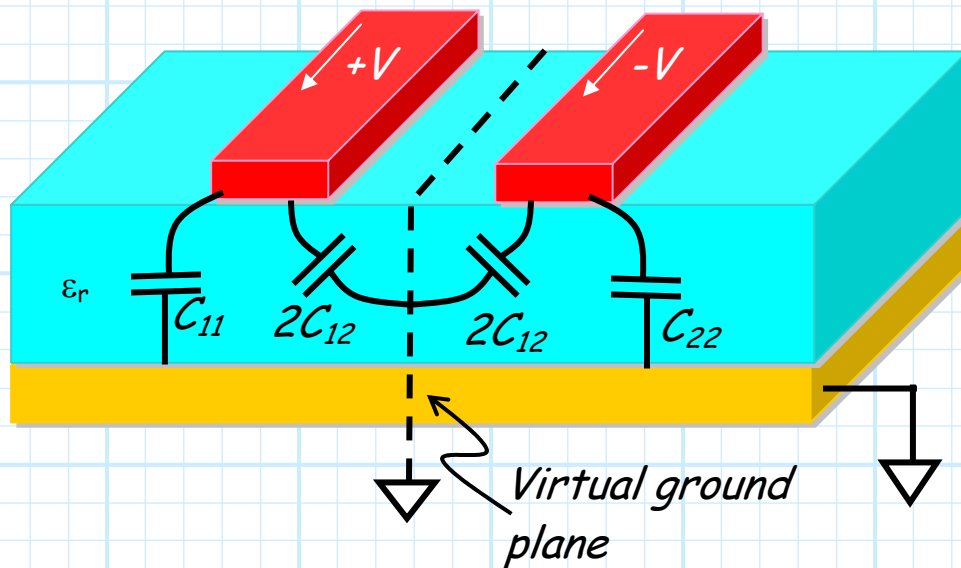
Likewise, if the two transmission lines are identical, then a plane of circuit **symmetry** exists. As a result, we can analyze this circuit using **odd/even mode analysis!**



Note we have divided the C_{12} capacitor into **two series** capacitors, each with a value of $2 C_{12}$.

Odd Mode

If the incident wave along the two transmission lines are **opposite** (i.e., equal magnitude but 180° out of phase), then a **virtual ground plane** is created at the plane of circuit symmetry.



Thus, the capacitance per unit length of each transmission line, in the **odd mode**, is thus:

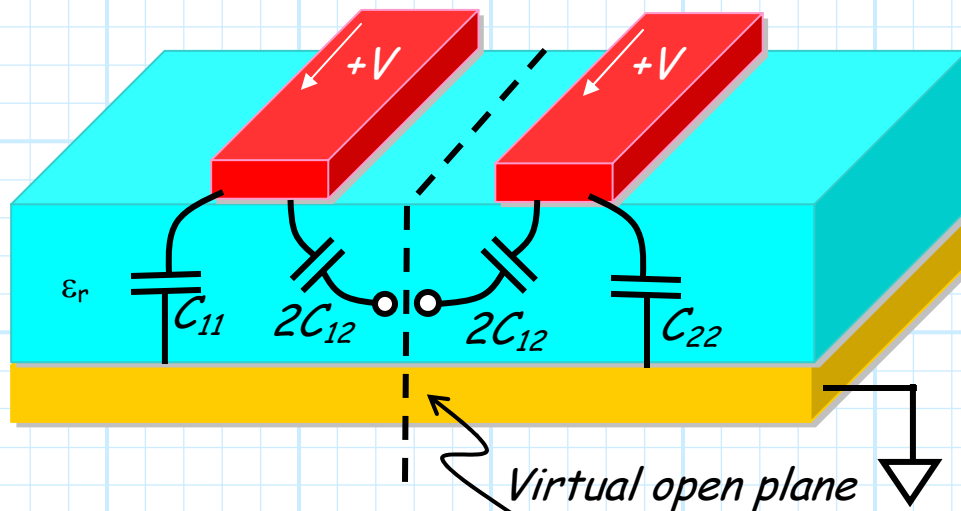
$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

and thus its characteristic impedance is:

$$Z_o = \sqrt{\frac{L}{C_o}}$$

Even Mode

If the incident wave along the two transmission lines are **equal** (i.e., equal magnitude and phase), then a **virtual open plane** is created at the plane of circuit symmetry.



Note the $2C_{12}$ capacitors have been "disconnected", and thus the capacitance per unit length of each transmission line, in the **even mode**, is thus:

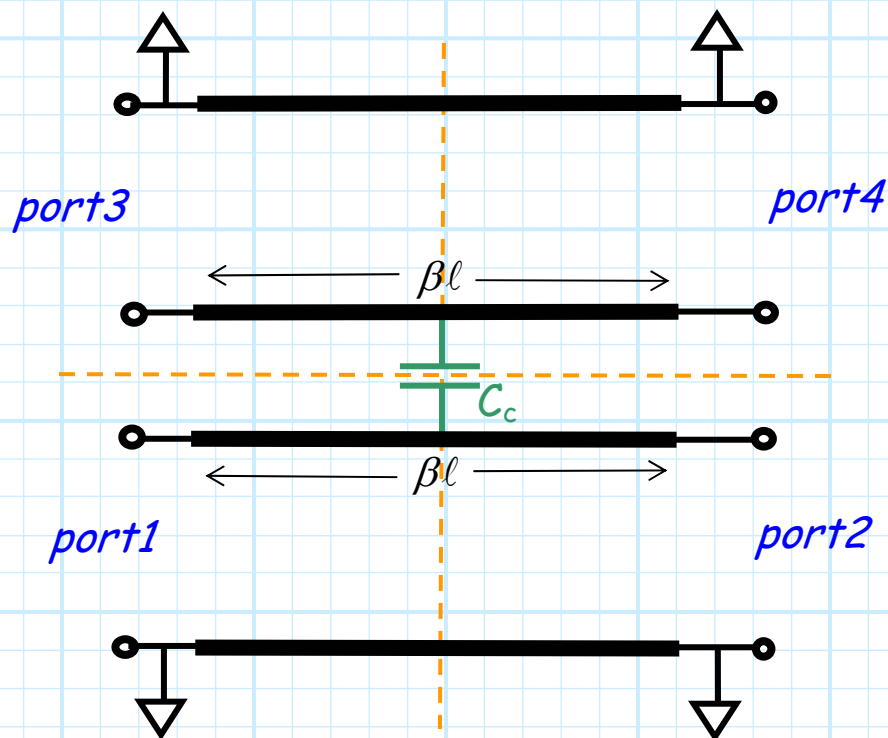
$$C_e = C_{11} = C_{22}$$

and thus its characteristic impedance is:

$$Z_0^e = \sqrt{\frac{L}{C_e}}$$

Analysis and Design of Coupled-Line Couplers

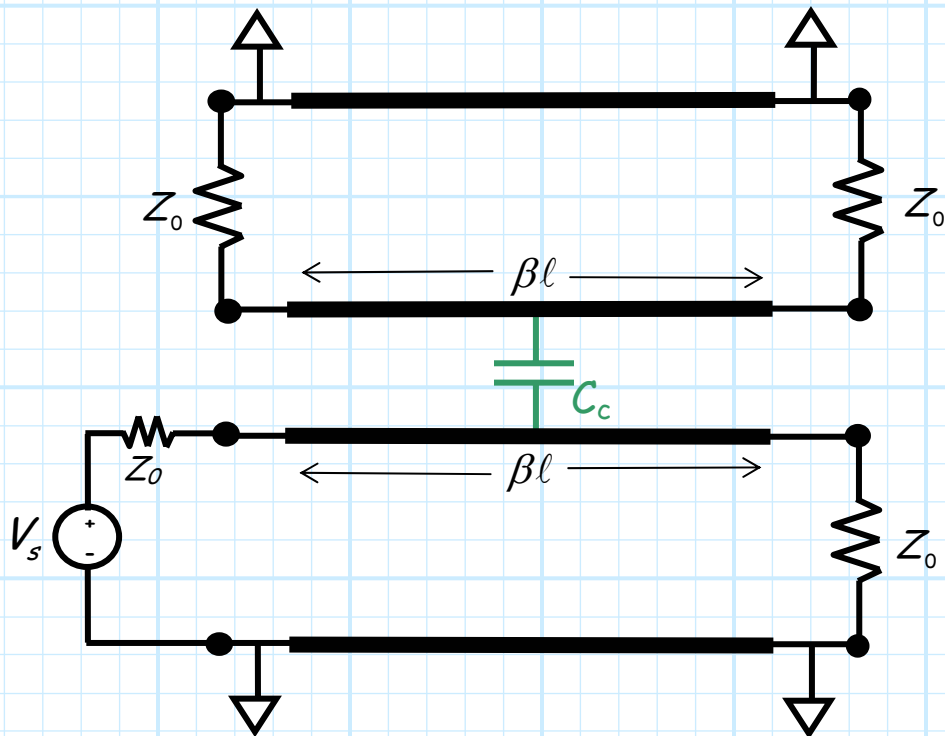
A pair of coupled lines forms a **4-port** device with **two** planes of reflection symmetry—it exhibits D_2 symmetry.



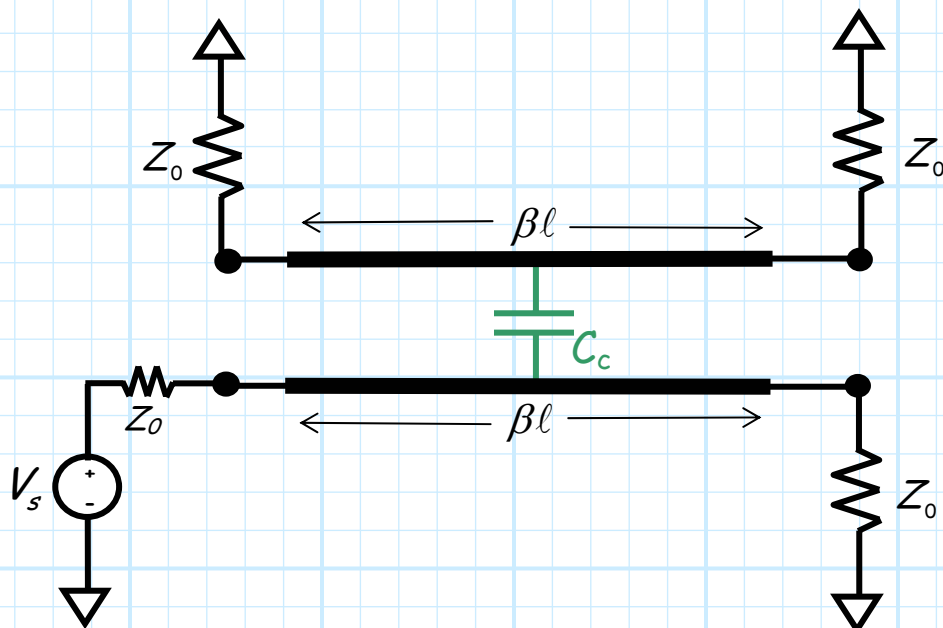
As a result, we know that the **scattering matrix** of this four-port device has just **4 independent** elements:

$$S = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix}$$

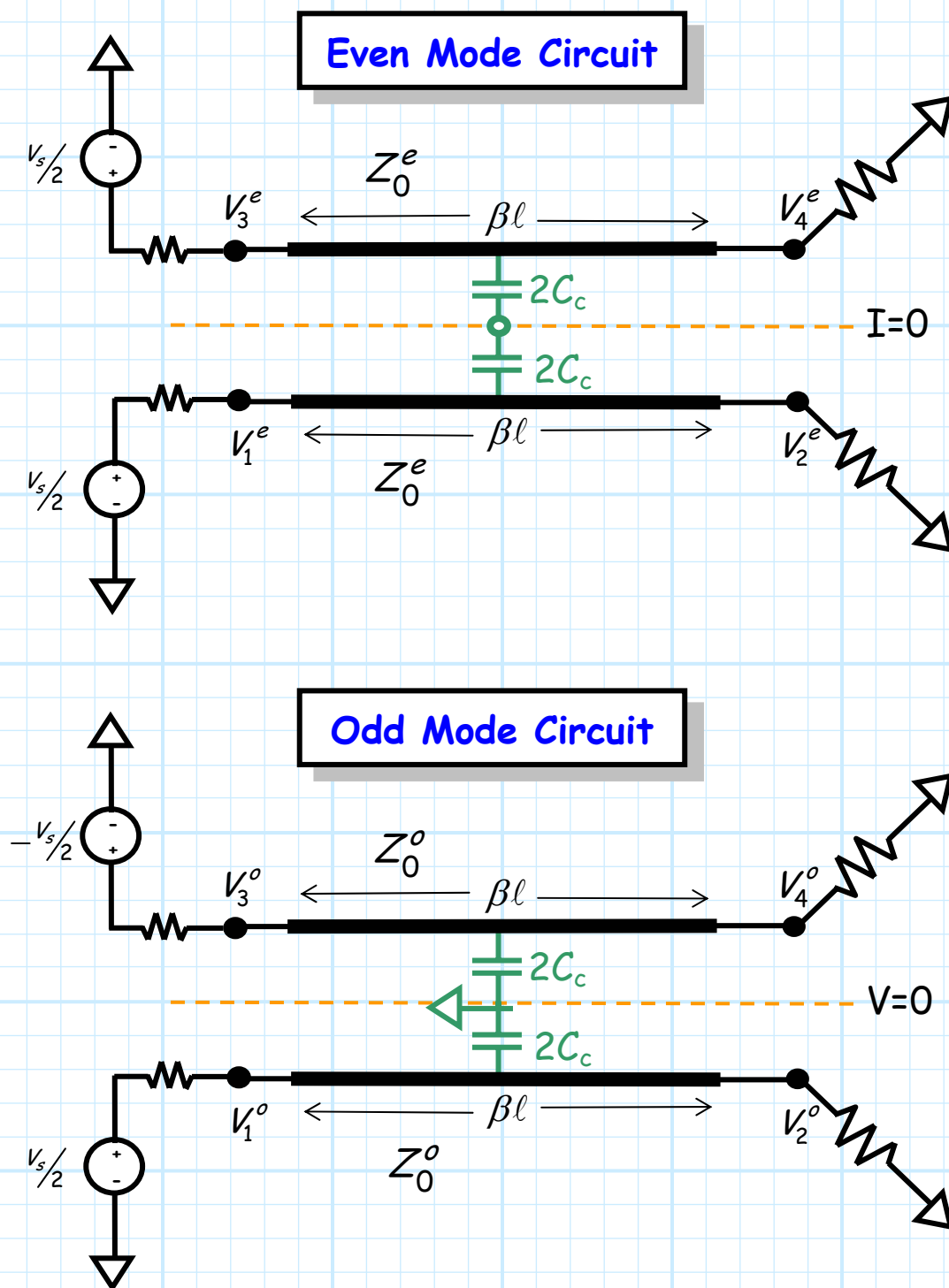
To determine these four elements, we can apply a **source to port 1** and then **terminate all other ports**:



Typically, a coupled-line coupler schematic is drawn **without** explicitly showing the **ground conductors** (i.e., without the ground plane):



To analyze this circuit, we must apply **odd/even mode analysis**. The two circuit analysis modes are:



Note that the **capacitive coupling** associated with these modes are different, resulting in a **different** characteristic impedance of the lines for the two cases (i.e., Z_0^e, Z_0^o).

Q: *So what?*

A: Consider what would happen if the characteristic impedance of each line were **identical** for **each mode**:

$$Z_0^e = Z_0^o = Z_0$$

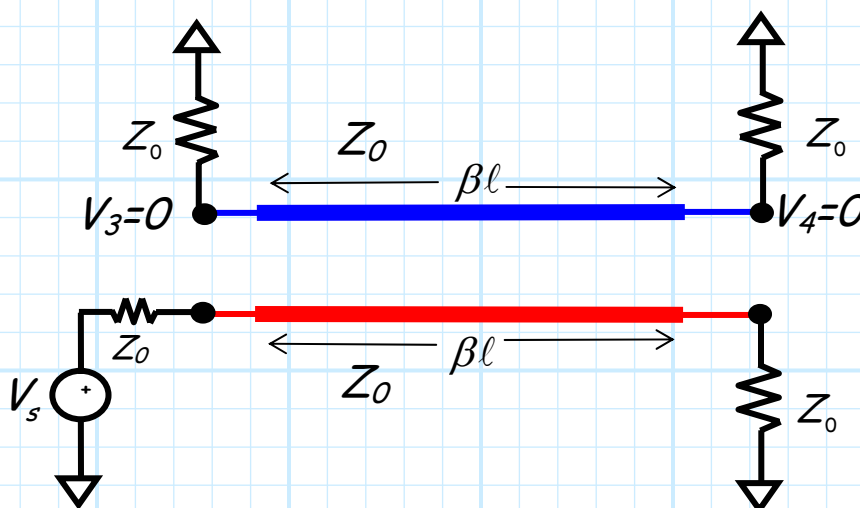
For this situation we would find that:

$$V_3^e = -V_3^o \quad \text{and} \quad V_4^e = -V_4^o$$

and thus when applying **superposition**:

$$V_3 = V_3^e + V_3^o = 0 \quad \text{and} \quad V_4 = V_4^e + V_4^o = 0$$

indicating that **no power is coupled** from the **"energized"** transmission line onto the **"passive"** transmission line.



This makes sense! After all, if no coupling occurs, then the characteristic impedance of each line is **unaltered** by the presence of the other—their characteristic impedance is Z_0 **regardless** of “mode”.

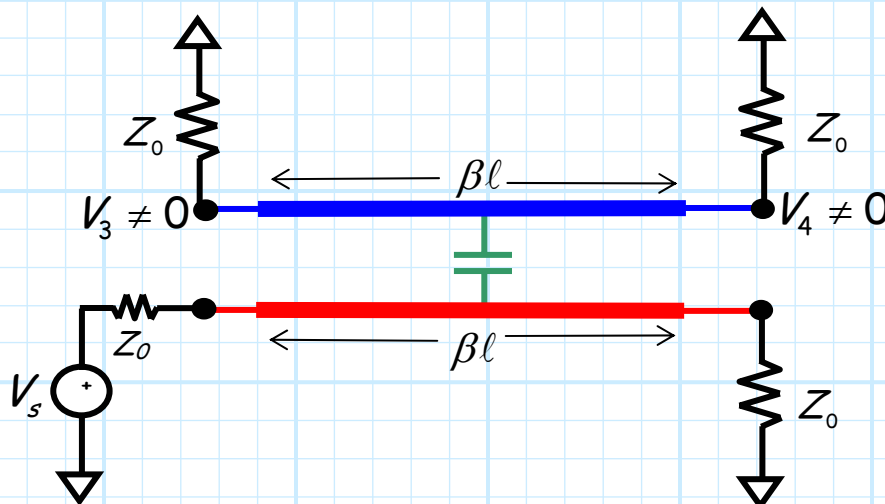
However, if coupling **does** occur, then $Z_0^e \neq Z_0^o$, meaning in general:

$$V_3^e \neq -V_3^o \quad \text{and} \quad V_4^e \neq -V_4^o$$

and thus in general:

$$V_3 = V_3^e + V_3^o \neq 0 \quad \text{and} \quad V_4 = V_4^e + V_4^o \neq 0$$

The odd/even mode analysis thus reveals the amount of **coupling from the energized section onto the passive section!**



Now, our **first step** in performing the odd/even mode analysis will be to determine scattering parameter S_{11} . To accomplish this, we will need to determine voltage V_1 :

$$V_1 = V_1^e + V_1^o$$

The result is a bit complicated, so it won't be presented here. However, a question we might ask is, what value **should** S_{11} be?

Q: *What value should S_{11} be?*

A: For the device to be a **matched** device, it must be **zero!**

From the value of S_{11} derived from our odd/even analysis, ICBST (it can be shown that) S_{11} will be equal to zero if the odd and even mode characteristic impedances are related as:

$$Z_0^e Z_0^o = Z_0$$

In other words, we should design our coupled line coupler such that the **geometric mean** of the even and odd mode impedances is **equal to Z_0** .

Now, assuming this design rule has been implemented, we also find (from odd/even mode analysis) that the scattering parameter S_{31} is:

$$S_{31} = \frac{j(Z_0^e - Z_0^o)}{2Z_0 \cot \beta l + j(Z_0^e + Z_0^o)}$$

Thus, we find that **unless** $Z_0^e = Z_0^o$, power must be coupled from port 1 to port 3!

Q: *But what is the value of line **electrical length** βl ?*

A: The **electrical length** of the coupled transmission lines is also a **design parameter**. Assuming that we want to **maximize** the coupling onto port 3, we find from the expression above that this is accomplished if we set βl such that:

$$\cot \beta l = 0$$

Which occurs when the **line length** is set to:

$$\beta l = \pi/2 \quad \Rightarrow \quad l = \lambda/4$$

Once again, our design rule is to set the transmission line length to a value equal to **one-quarter wavelength** (at the design frequency).

$$l = \lambda/4$$

Implementing these **two** design rules, we find that at the design frequency:

$$S_{31} = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

This value is a **very** important one with respect to coupler performance. Specifically, it is the **coupling coefficient** c !

$$c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

Given this definition, we can **rewrite** the scattering parameter S_{31} as:

$$S_{31} = \frac{jc \tan \beta l}{\sqrt{1-c^2} + j \tan \beta l}$$

Continuing with our odd/even mode analysis, we find (given that $Z_0^e Z_0^o = Z_0$):

$$S_{21} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2} \cos \beta l + j \sin \beta l}$$

and so at our **design frequency**, where $\beta l = \pi/2$, we find:

$$S_{21} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2} (0) + j(1)} = -j\sqrt{1-c^2}$$

Finally, our odd/even analysis reveals that at our design frequency:

$$S_{41} = 0$$

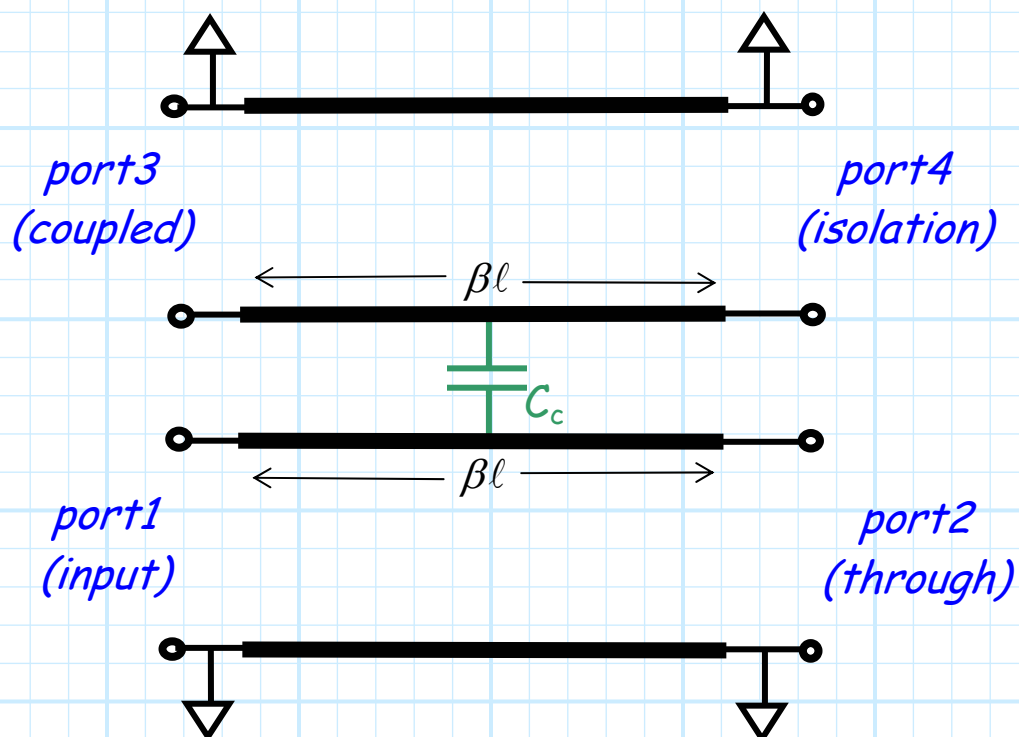
Combining these results, we find that at our design frequency, the **scattering matrix** of our coupled-line coupler is:

$$S = \begin{bmatrix} 0 & -j\sqrt{1-c^2} & c & 0 \\ -j\sqrt{1-c^2} & 0 & 0 & c \\ c & 0 & 0 & -j\sqrt{1-c^2} \\ 0 & c & -j\sqrt{1-c^2} & 0 \end{bmatrix}$$

Q: Hey! Isn't this the same scattering matrix as the *ideal symmetric directional coupler* we studied in the first section of this chapter?

A: The very same! The coupled-line coupler—if our design rules are followed—results in an “ideal” directional coupler.

If the **input** is port 1, then the **through** port is port 2, the **coupled** port is port 3, and the **isolation** port is port 4!



Q: *But, how do we **design** a coupled-line coupler with a **specific coupling coefficient** c ?*

A: Given our **two design constraints**, we know that:

$$Z_0^e Z_0^o = Z_0 \quad \text{and} \quad c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

We can **rearrange** these two expressions to find **solutions** for our odd and even mode impedances:

$$Z_0^e = Z_0 \sqrt{\frac{1+c}{1-c}} \quad Z_0^o = Z_0 \sqrt{\frac{1-c}{1+c}}$$

Thus, **given** the desired values Z_0 and c , we can determine the proper values of Z_0^e and Z_0^o for an ideal directional coupler.

Q: *Yes, but the odd and even mode impedance depends on the **physical structure** of the coupled lines, such as substrate dielectric ϵ_r , substrate thickness (d or b), conductor width W , and separation distance S .*

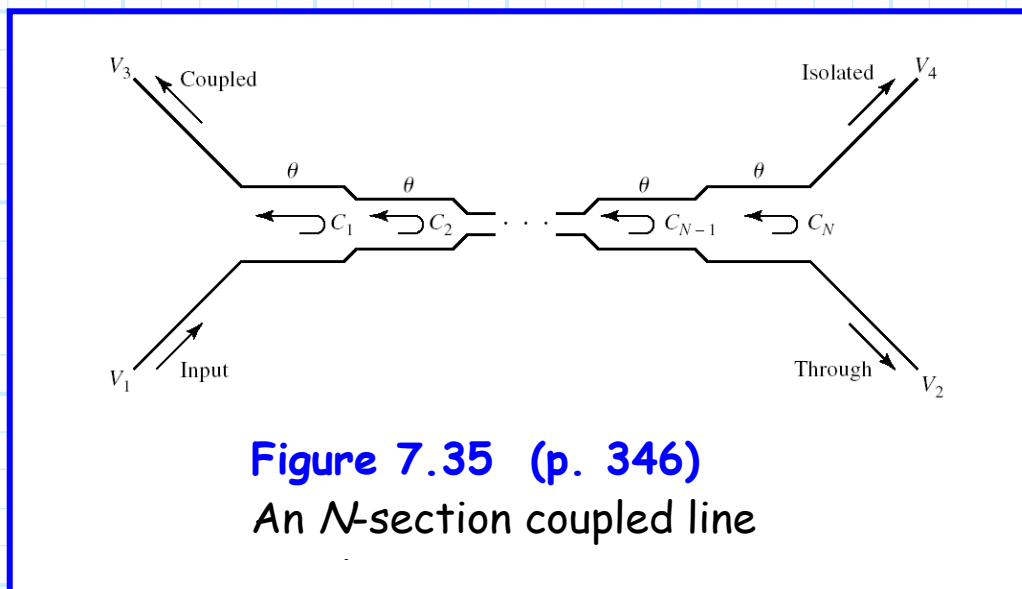
*How do we determine **these** physical design parameters for desired values of Z_0^e and Z_0^o ??*

A: That's a much more difficult question to answer! Recall that there is **no** direct formulation relating microstrip and stripline parameters to **characteristic impedance** (we only have numerically derived **approximations**).

- * So it's no surprise that there is likewise **no direct formulation** relating odd and even mode characteristic impedances to the specific physical parameters of microstrip and stripline coupled lines.
- * Instead, we again have numerically derived **approximations** that allow us to determine (approximately) the required microstrip and stripline parameters, or we can use a **microwave CAD packages** (like ADS!).
- * For example, **figures 7.29 and 7.30** provide **charts** for selecting the required values of W and S , given some ϵ_r and b (or d).
- * Likewise, example 7.7 on page 345 provides a good **demonstration** of the single-section coupled-line coupler **design synthesis**.

Multi-Section Coupled Line Couplers

We can add **multiple** coupled lines in series to increase coupler bandwidth.



We typically design the coupler such that it is **symmetric**, i.e.:

$$C_1 = C_N, C_2 = C_{N-1}, C_3 = C_{N-2}, \text{ etc.}$$

where N is **odd**.

Q: *What is the coupling of this device as a function of frequency?*

A: To analyze this structure, we make an **approximation** similar to that of the theory of small reflections.

First, if c is **small** (i.e., less than 0.3), then we can make the approximation:

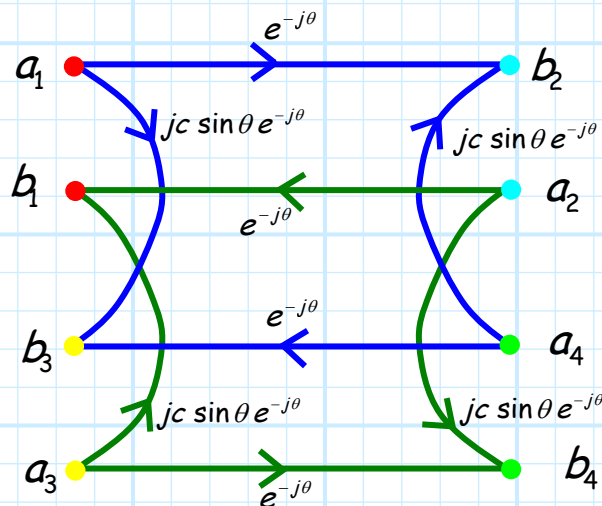
$$\begin{aligned} S_{31}(\theta) &= \frac{j c \tan \theta}{\sqrt{1 - c^2} + j \tan \theta} \\ &\approx \frac{j c \tan \theta}{1 + j \tan \theta} \\ &= j c \sin \theta e^{-j\theta} \end{aligned}$$

Likewise:

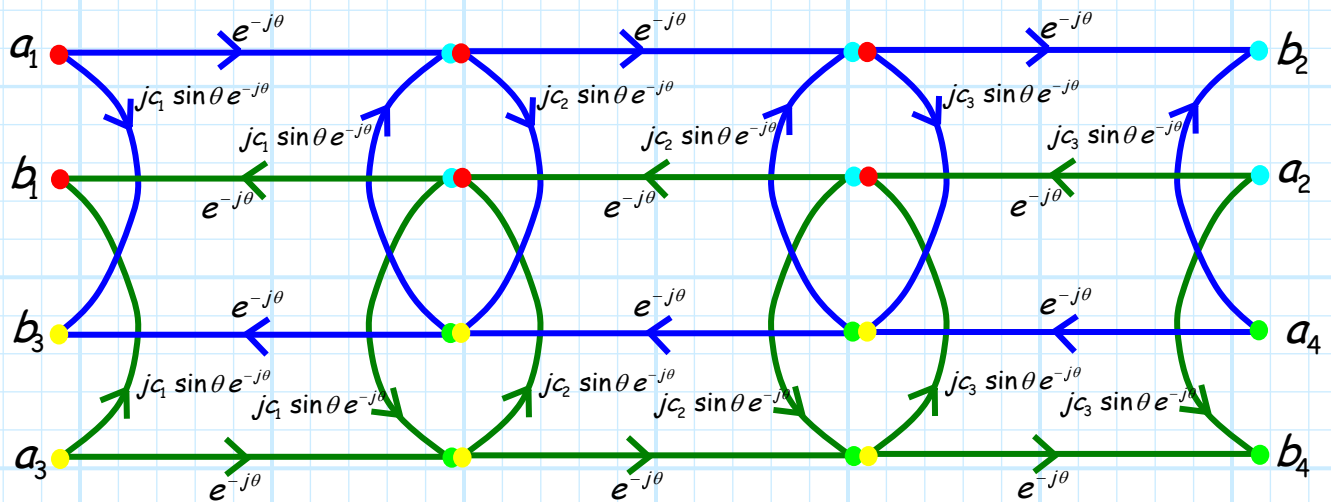
$$\begin{aligned} S_{21}(\theta) &= \frac{\sqrt{1 - c^2}}{\sqrt{1 - c^2} \cos \theta + j \sin \theta} \\ &\approx \frac{1}{\cos \theta + j \sin \theta} \\ &= e^{-j\theta} \end{aligned}$$

where of course $\theta = \beta l = \omega T$, and $T = l/v_p$.

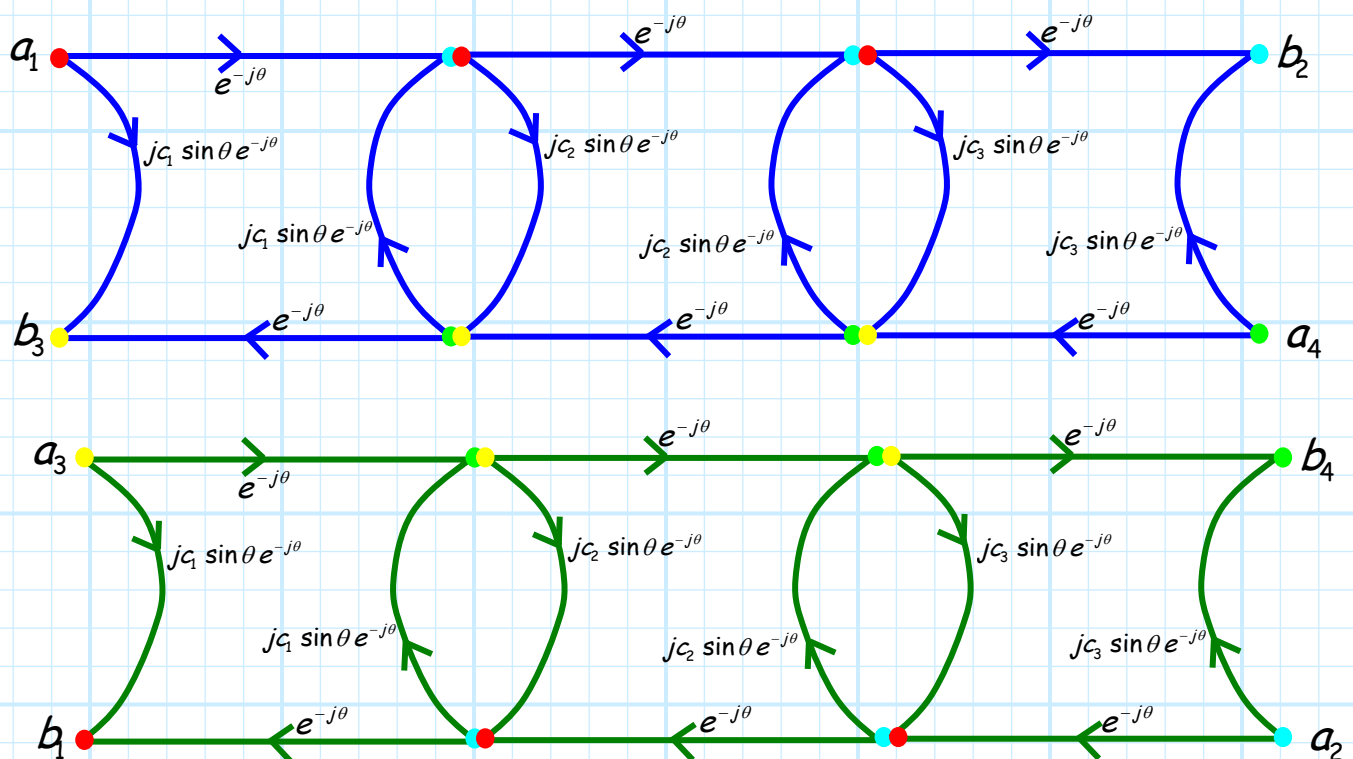
We can use these approximations to construct a **signal flow graph** of a **single-section coupler**:



Now, say we cascade **three** coupled line pairs, to form a **three section** coupled line coupler. The signal flow graph would thus be:

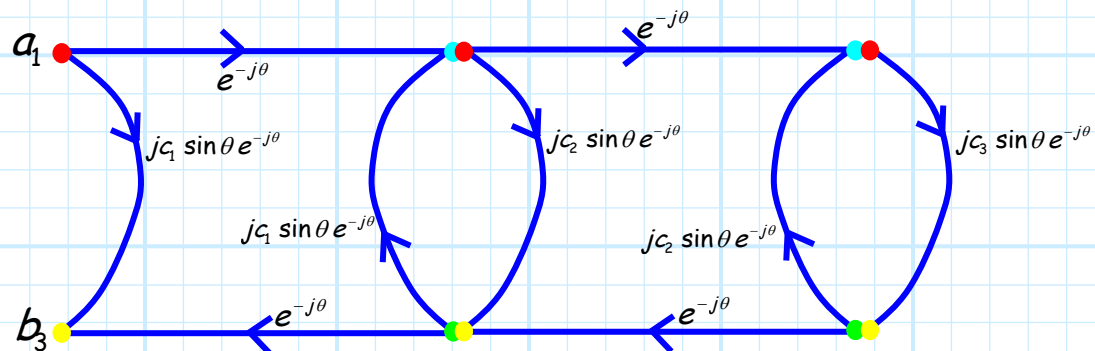


Note that this signal flow graph **decouples** into two separate and graphs (i.e., the **blue** graph and the **green** graph).



Note that these two graphs are essentially **identical**, and emphasize the **symmetric** structure of the coupled-line coupler.

Now, we are interested in describing the **coupled output** (i.e., b_3) in terms of the incident wave (i.e., a_1). Assuming ports 2, 3 and 4 is **matched** (i.e., $a_4 = 0$), we can reduce the graph to simply:



Now, we **could** reduce this signal flow graph even further—or we can use the **multiple reflection viewpoint** to explicitly each propagation term!

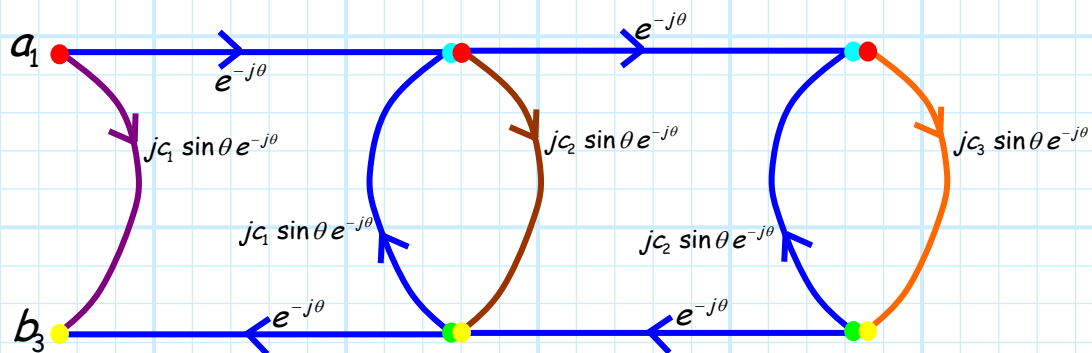
Q: *Multiple reflection viewpoint! I thought you said that this was a particularly **bad** way to perform a network analysis?*

A: Generally speaking it is, as we would have to account for an **infinite** number of terms. However, in **certain** conditions, just a few terms **dominate** this infinite series. If we can correctly **identify** these few terms, we can write an excellent **approximation** to the exact solution!

An example of that was the **theory of small reflections**, where we only considered terms involving a **single** reflection.

Here we can apply a similar methodology, applying a “theory of **small couplings**”. In other words, we consider only the propagation paths where **one coupling** is involved—the signal propagates **across** a coupled-line pair only **once**!

Note from the signal flow graph that there are **three** such mechanisms, corresponding to the coupling across each of the **three** separate coupled line pairs:



$$b_3 \approx \left(j c_1 \sin \theta e^{-j\theta} + e^{-j\theta} j c_2 \sin \theta e^{-j\theta} e^{-j\theta} + e^{-j2\theta} j c_3 \sin \theta e^{-j\theta} e^{-j2\theta} \right) a_1$$

$$= \left(j c_1 \sin \theta e^{-j\theta} + j c_2 \sin \theta e^{-j3\theta} + j c_3 \sin \theta e^{-j5\theta} \right) a_1$$

Note that **all other** terms of the infinite series would involve at least **three** couplings (i.e., three crossings), and thus these terms would be **exceeding** small (i.e., $c^3 \approx 0$).

Therefore, according to this **approximation**:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = j c_1 \sin \theta e^{-j\theta} + j c_2 \sin \theta e^{-j3\theta} + j c_3 \sin \theta e^{-j5\theta}$$

Moreover, for a **multi-section** coupler with N sections, we find:

$$S_{31}(\theta) = j c_1 \sin \theta e^{-j\theta} + j c_2 \sin \theta e^{-j3\theta} + j c_3 \sin \theta e^{-j5\theta} + \dots + j c_N \sin \theta e^{-j(2N-1)\theta}$$

And for **symmetric** couplers with an **odd** value N , we find:

$$S_{31}(\theta) = j 2 \sin \theta e^{-jN\theta} \left[c_1 \cos(N-1)\theta + c_2 \cos(N-3)\theta + c_3 \cos(N-5)\theta + \dots + \frac{1}{2} c_M \right]$$

where $M = (N+1)/2$. Note M is an **even integer**, as N is an **odd** number.

Thus, we find the coupling **magnitude** as a function of frequency is:

$$\begin{aligned} |c(\theta)| &= |S_{31}(\theta)| \\ &= c_1 2 \sin \theta \cos(N-1)\theta + c_2 2 \sin \theta \cos(N-3)\theta \\ &\quad + c_3 2 \sin \theta \cos(N-5)\theta + \dots + c_M \sin \theta \end{aligned}$$

And thus the coupling in **dB** is:

$$C(\theta) = 10 \log_{10} |c(\theta)|^2$$

Now, our design goals are to **select** the coupling values c_1, c_2, \dots, c_N such that:

1. The coupling value $\mathcal{C}(\theta)$ is a specific, **desired** value at our design frequency.
2. The coupling **bandwidth** is as **large** as possible.

For the first condition, recall that the at the **design frequency**:

$$\theta = \beta\ell = \pi/2$$

I.E., the section lengths are a **quarter-wavelength** at our design frequency.

Thus, we find our **first** design equation:

$$\begin{aligned} |c(\theta)|_{\theta=\pi/2} = & c_1 2 \cos(N-1)\pi/2 + c_2 2 \cos(N-3)\pi/2 \\ & + c_3 2 \cos(N-5)\pi/2 + \dots + c_M \end{aligned}$$

where we have used the fact that $\sin(\pi/2) = 1$.

Note the value $|c(\theta)|_{\theta=\pi/2}$ is set to the value necessary to achieve the **desired** coupling value. This equation thus provides **one** design constraint—we have **M-1** degrees of design freedom left to accomplish our **second** goal!

To **maximize bandwidth**, we typically impose the **maximally flat** condition:

$$\left. \frac{d^m |c(\theta)|}{d\theta^m} \right|_{\theta=\pi/2} = 0 \quad m = 1, 2, \dots, M-1$$

Be careful! Remember to perform the derivative **first**, and **then** evaluate the result at $\theta = \pi/2$.

One final note, you may find that this **trig** expression is helpful in **simplifying** your derivatives:

$$\sin \phi \cos \psi = \frac{1}{2} \sin(\phi + \psi) + \frac{1}{2} \sin(\phi - \psi)$$

For **example**, we find that:

$$\begin{aligned} 2 \sin \theta \cos 2\theta &= \sin(\theta + 2\theta) + \sin(\theta - 2\theta) \\ &= \sin(3\theta) + \sin(-\theta) \\ &= \sin(3\theta) - \sin(\theta) \end{aligned}$$