7.6 – Coupled-Line Directional Couplers

Reading Assignment: ***pp. 337-348***

Q: *The Quadrature Hybrid is a 3dB coupler. How do we build couplers with* ***less coupling****, say 10dB, 20dB, or 30 dB?*

A: Directional couplers are typically built using **coupled lines**.

HO: COUPLED LINE COUPLERS

Q: *How can we* ***design*** *a coupled line couplers so that is an* ***ideal*** *directional coupler with a* ***specific*** *coupling value?*

A: HO: ANALYSIS AND DESIGN OF COUPLED-LINE COUPLERS

Q: *Like all devices with quarter-wavelength sections, a coupled line coupler would seem to be inherently* ***narrow band****. Is there some way to* ***increase coupler bandwidth****?*

A: **Yes!** We can add more coupled-line sections, just like with **multi-section** matching transformers.

HO: MULTI-SECTION COUPLED LINE COUPLERS

Q: *How do we* ***design*** *these multi-section couplers?*

A: All the requisite design examples were provided in the last handout, and there are two good design examples on pages 345 and 348 of your textbook!

Coupled-Line Couplers

Two transmission lines in **proximity** to each other will **couple** power from one line into another.

This proximity will **modify** the electromagnetic fields (and thus modify voltages and currents) of the propagating wave, and therefore **alter** the **characteristic impedance** of the transmission line!

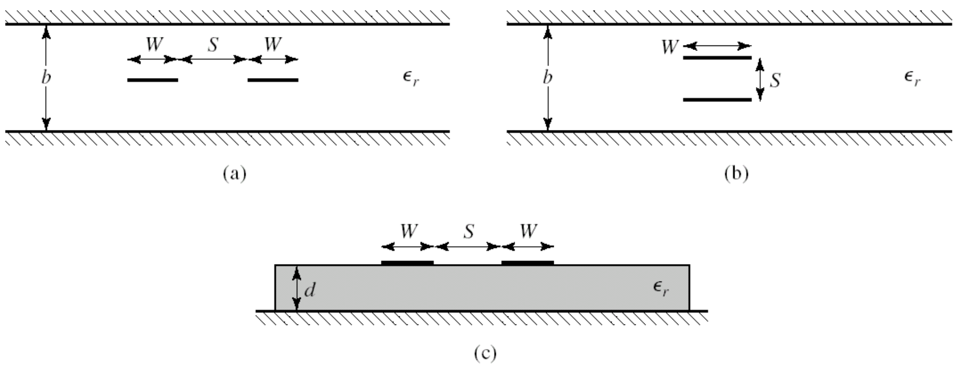


Figure 7.26 (p. 337)
Various coupled transmission line geometries. (a) Coupled stripline (planar, or edge-coupled). (b) Coupled stripline (stacked, or broadside-coupled). (c) Coupled microstrip.

Generally, speaking, we find that this transmission lines are **capacitively coupled** (i.e., it appears that they are connected by a capacitor):

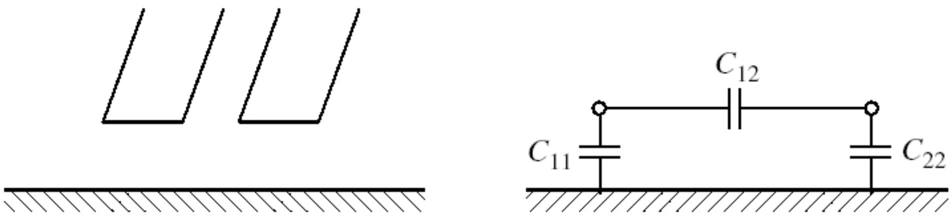
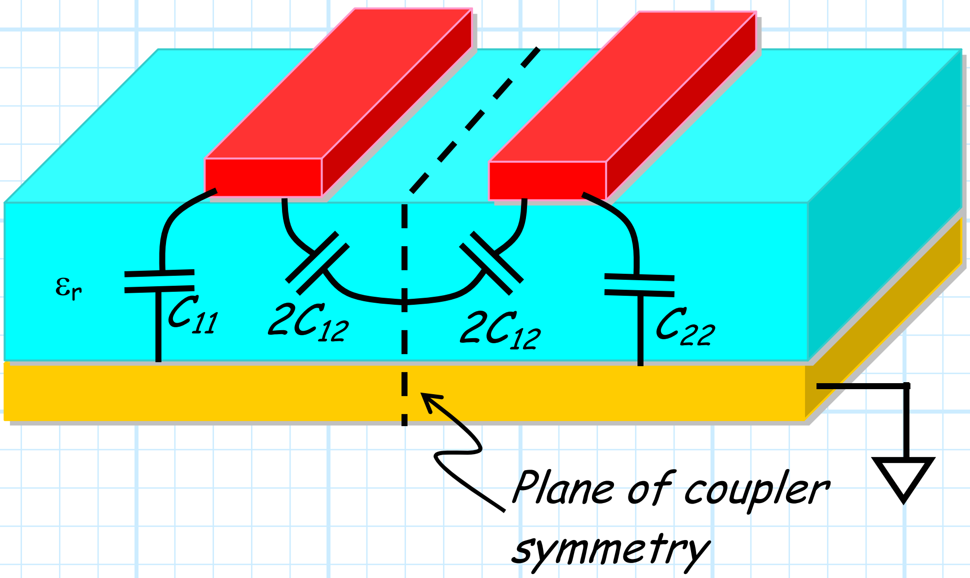


Figure 7.27 (p. 337)
A three-wire coupled transmission line and its equivalent capacitance network.

If the two transmission lines are **identical** (and they typically are), then $C_{11} = C_{22}$.

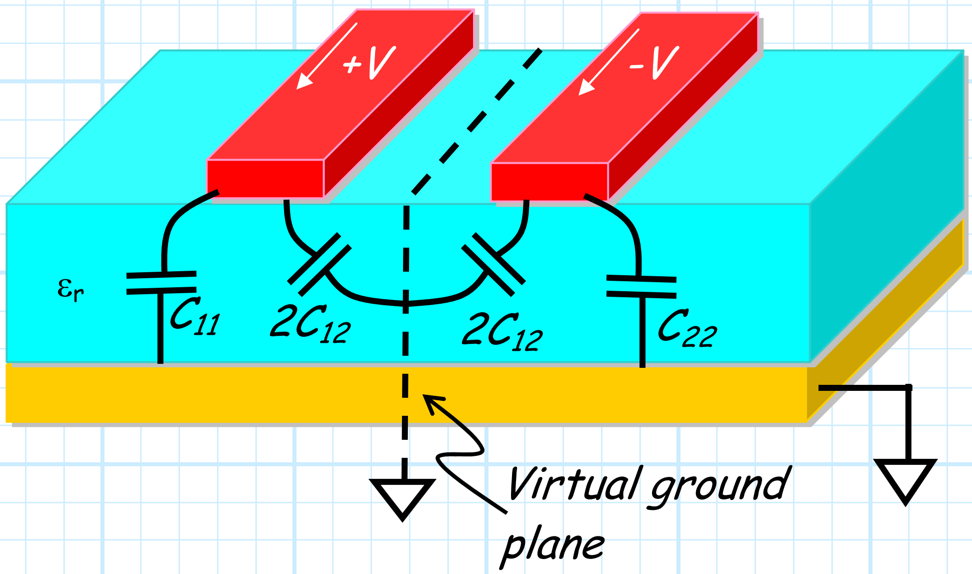
Likewise, if the two transmission lines are identical, then a plane of circuit **symmetry** exists. As a result, we can analyze this circuit using **odd/even mode** analysis!



Note we have divided the C_{12} capacitor into **two series** capacitors, each with at value 2 C_{12}.

Odd Mode

If the incident wave along the two transmission lines are **opposite** (i.e., equal magnitude but 180° out of phase), then a virtual **ground plane** is created at the plane of circuit symmetry.



Thus, the capacitance per unit length of each transmission line, in the **odd** mode, is thus:

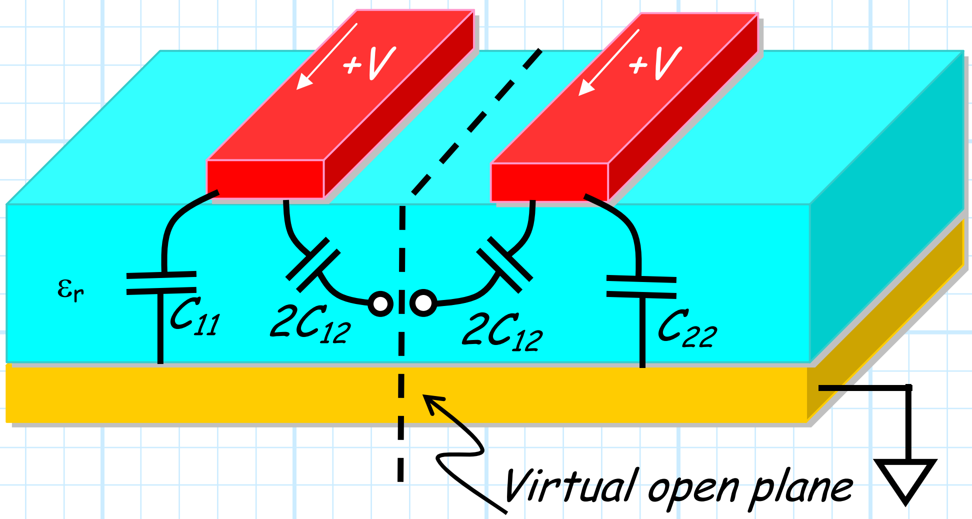
$$C_o = C_{11} + 2C_{12} = C_{22} + 2C_{12}$$

and thus its characteristic impedance is:

$$Z_0^o = \sqrt{\frac{L}{C_o}}$$

Even Mode

If the incident wave along the two transmission lines are **equal** (i.e., equal magnitude and phase), then a **virtual open** plane is created at the plane of circuit symmetry.



Note the $2C_{12}$ capacitors have been "**disconnected**", and thus the capacitance per unit length of each transmission line, in the **even** mode, is thus:

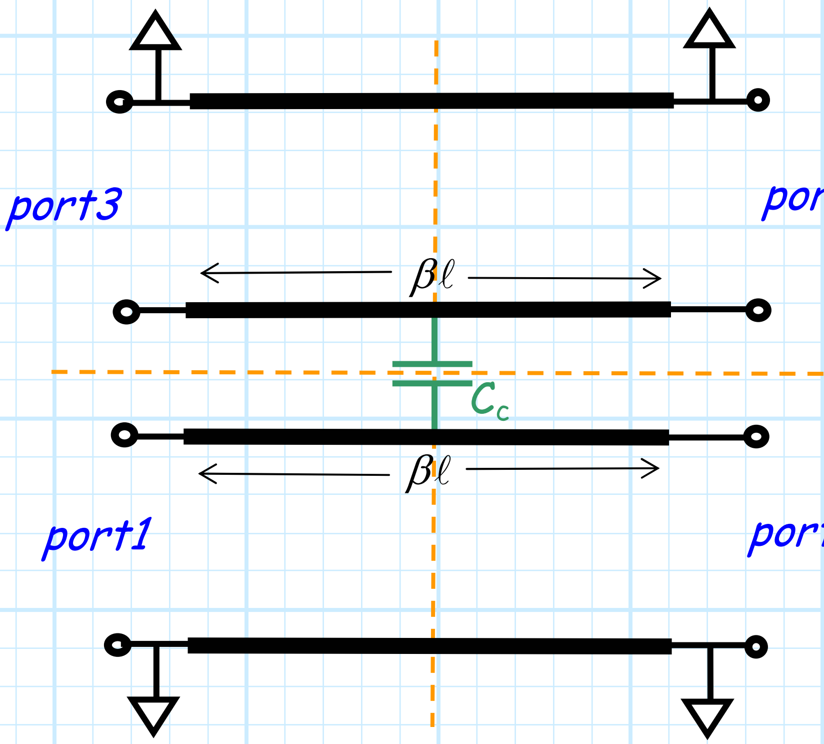
$$C_e = C_{11} = C_{22}$$

and thus its characteristic impedance is:

$$Z_0^e = \sqrt{\frac{L}{C_e}}$$

Analysis and Design of Coupled-Line Couplers

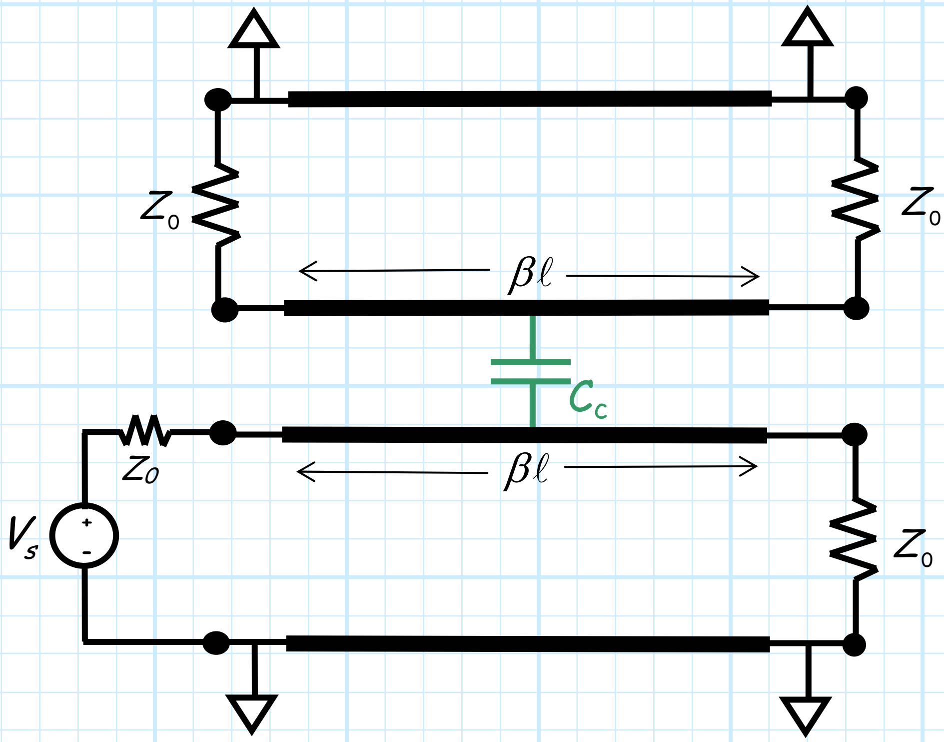
A pair of coupled lines forms a **4-port** device with **two** planes of reflection symmetry—it exhibits **D_4** symmetry.



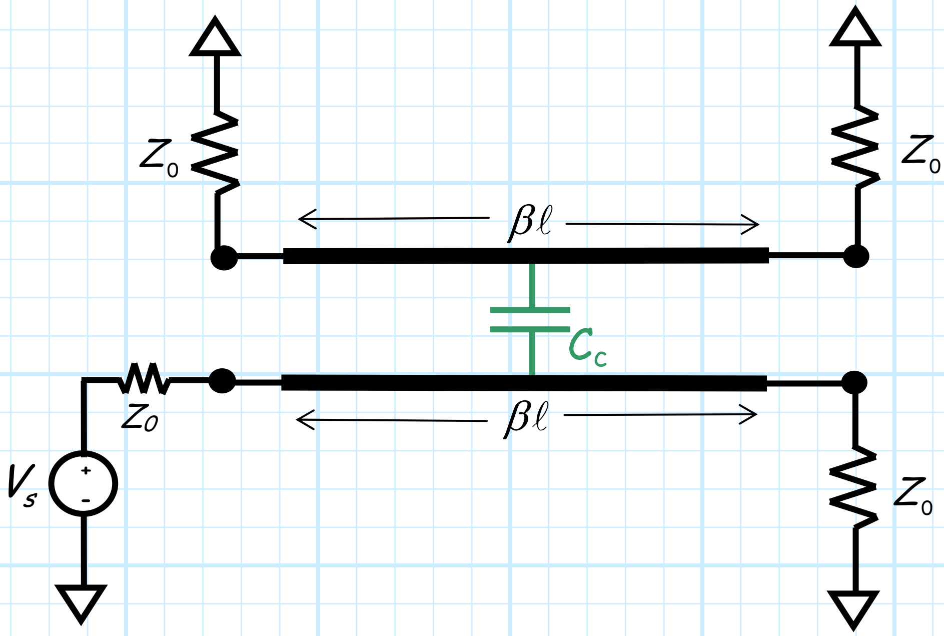
As a result, we know that the **scattering matrix** of this four-port device has just **4 independent** elements:

$$\mathcal{S} = \begin{bmatrix} S_{11} & S_{21} & S_{31} & S_{41} \\ S_{21} & S_{11} & S_{41} & S_{31} \\ S_{31} & S_{41} & S_{11} & S_{21} \\ S_{41} & S_{31} & S_{21} & S_{11} \end{bmatrix}$$

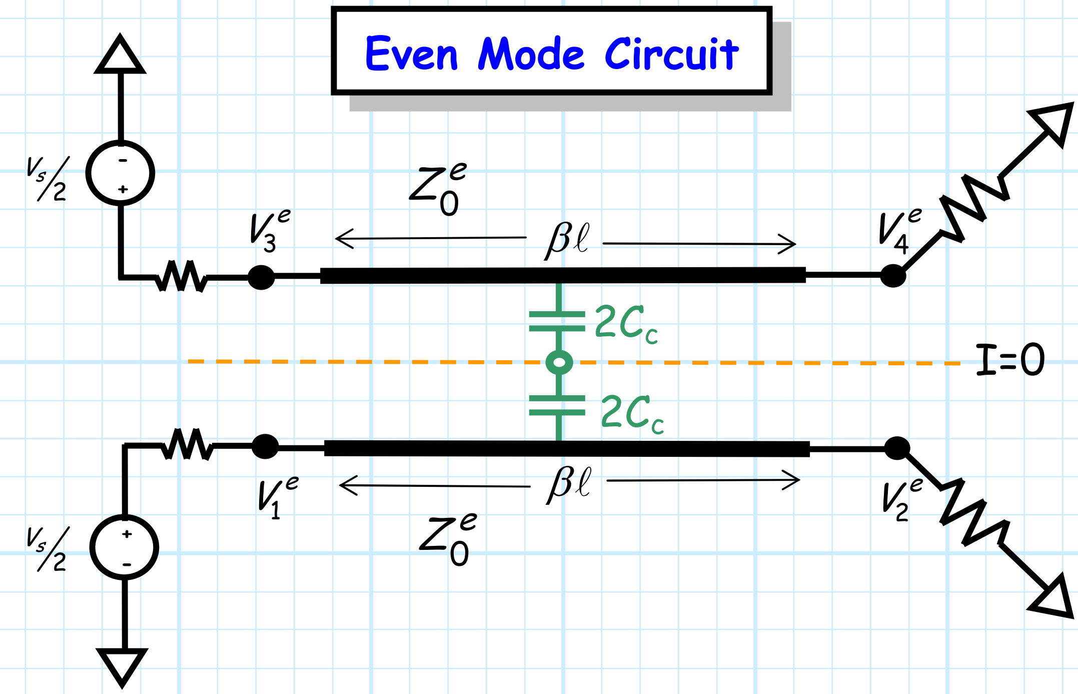
To determine these four elements, we can apply a **source to port 1** and then **terminate** all other ports:

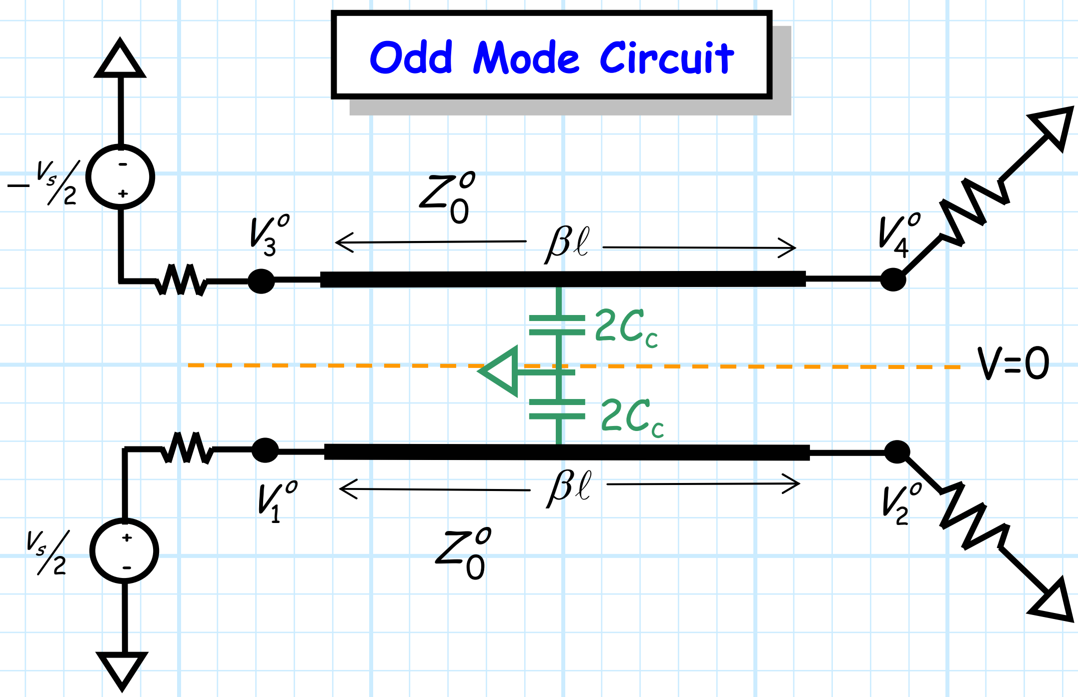


Typically, a coupled-line coupler schematic is drawn **without** explicitly showing the **ground conductors** (i.e., without the ground plane):



To analyze this circuit, we must apply **odd/even mode analysis**. The two circuit analysis modes are:





Note that the **capacitive coupling** associated with these modes are different, resulting in a **different** characteristic impedance of the lines for the two cases (i.e., Z_0^e, Z_0^o).

Q: *So what?*

A: Consider what would happen if the characteristic impedance of each line where **identical** for **each mode**:

$$Z_0^e = Z_0^o = Z_0$$

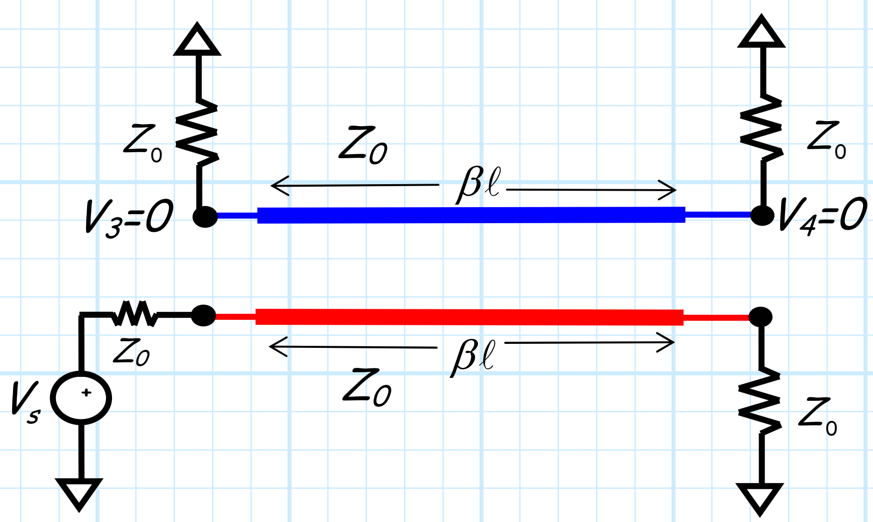
For this situation we would find that:

$$V_3^e = -V_3^o \qquad \text{and} \qquad V_4^e = -V_4^o$$

and thus when applying **superposition**:

$$V_3 = V_3^e + V_3^o = 0 \qquad \text{and} \qquad V_4 = V_4^e + V_4^o = 0$$

indicating that **no power is coupled** from the **"energized"** transmission line onto the **"passive"** transmission line.



This makes sense! After all, **if** no coupling occurs, then the characteristic impedance of each line is **unaltered** by the presence of the other—their characteristic impedance is Z_0 **regardless** of "mode".

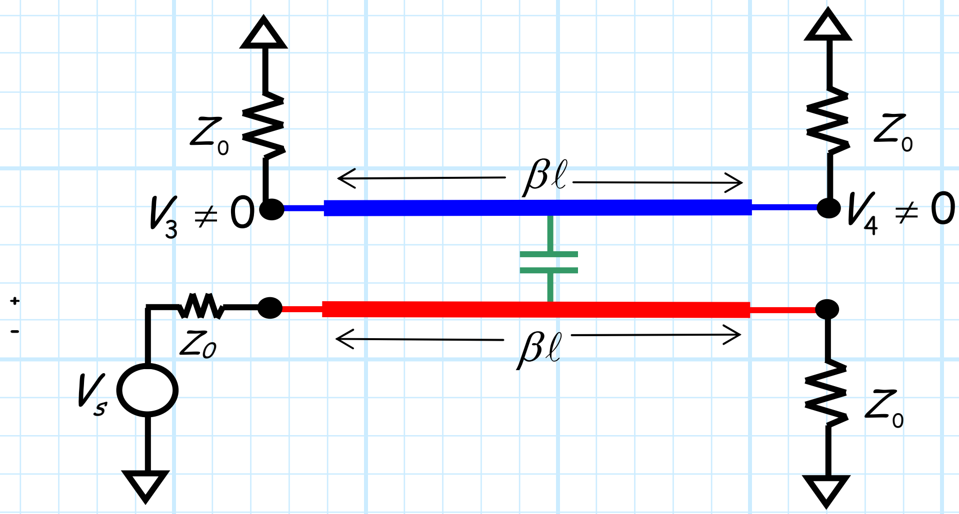
However, if coupling **does** occur, then $Z_0^e \neq Z_0^o$, meaning in general:

$$V_3^e \neq -V_3^o \qquad \text{and} \qquad V_4^e \neq -V_4^o$$

and thus in general:

$$V_3 = V_3^e + V_3^o \neq 0 \qquad \text{and} \qquad V_4 = V_4^e + V_4^o \neq 0$$

The odd/even mode analysis thus reveals the amount of **coupling from** the energized section **onto** the passive section!



Now, our **first step** in performing the odd/even mode analysis will be to determine scattering parameter S_{11}. To accomplish this, we will need to determine voltage V_1:

$$V_1 = V_1^e + V_1^o$$

The result is a bit complicated, so it won't be presented here. However, a question we might ask is, what value **should** S_{11} be?

Q: *What value should S_{11} be?*

A: For the device to be a **matched** device, it must be **zero**!

From the value of S_{11} derived from our odd/even analysis, ICBST (it can be shown that) S_{11} will be equal to zero **if** the odd and even mode characteristic impedances are related as:

$$\boxed{Z_0^e Z_o^o = Z_0^2}$$

In other words, we should design our coupled line coupler such that the **geometric mean** of the even and odd mode impedances is **equal to** Z_0.

Now, assuming this design rule has been implemented, we also find (from odd/even mode analysis) that the scattering parameter S_{31} is:

$$S_{31}(\beta) = \frac{j\left(Z_0^e - Z_0^o\right)}{2Z_0 \cot \beta\ell + j\left(Z_0^e + Z_0^o\right)}$$

Thus, we find that **unless** $Z_0^e = Z_0^o$, power must be coupled from port 1 to port 3!

Q: *But what is the value of line **electrical length** $\beta\ell$?*

A: The **electrical length** of the coupled transmission lines is also a **design parameter**. Assuming that we want to **maximize** the coupling onto port 3 (at design frequency ω_0), we find from the expression above that this is accomplished if we set $\beta_0 \ell$ such that:

$$\cot \beta_0 \ell = 0$$

Which occurs when the **line length** is set to:

$$\beta_0 \ell = \pi/2 \qquad \Rightarrow \qquad \ell = \lambda_0/4$$

Once again, our design rule is to set the transmission line length to a value equal to **one-quarter wavelength** (at the design frequency).

$$\boxed{\ell = \lambda_0/4}$$

Implementing these **two** design rules, we find that at the design frequency:

$$S_{31} = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

This value is a **very** important one with respect to coupler performance. Specifically, it is the **coupling coefficient** c!

$$\boxed{c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}}$$

Given this definition, we can **rewrite** the scattering parameter S_{31} as:

$$S_{31}(\beta) = \frac{jc \;\; tan\,\beta\ell}{\sqrt{1-c^2} + j\,tan\,\beta\ell}$$

Continuing with our odd/even mode analysis, we find (given that $Z_0^e Z_o^o = Z_0^2$:

$$S_{21} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\;cos\,\beta\ell + j\,sin\,\beta\ell}$$

and so at our **design frequency**, where $\beta_0\ell = \pi/2$, we find:

$$S_{21}(\beta)\Big|_{\beta\ell=\pi/2} = \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\,(0) + j\,(1)} = -j\sqrt{1-c^2}$$

Finally, our odd/even analysis reveals that at our design frequency:

$$S_{41} = 0$$

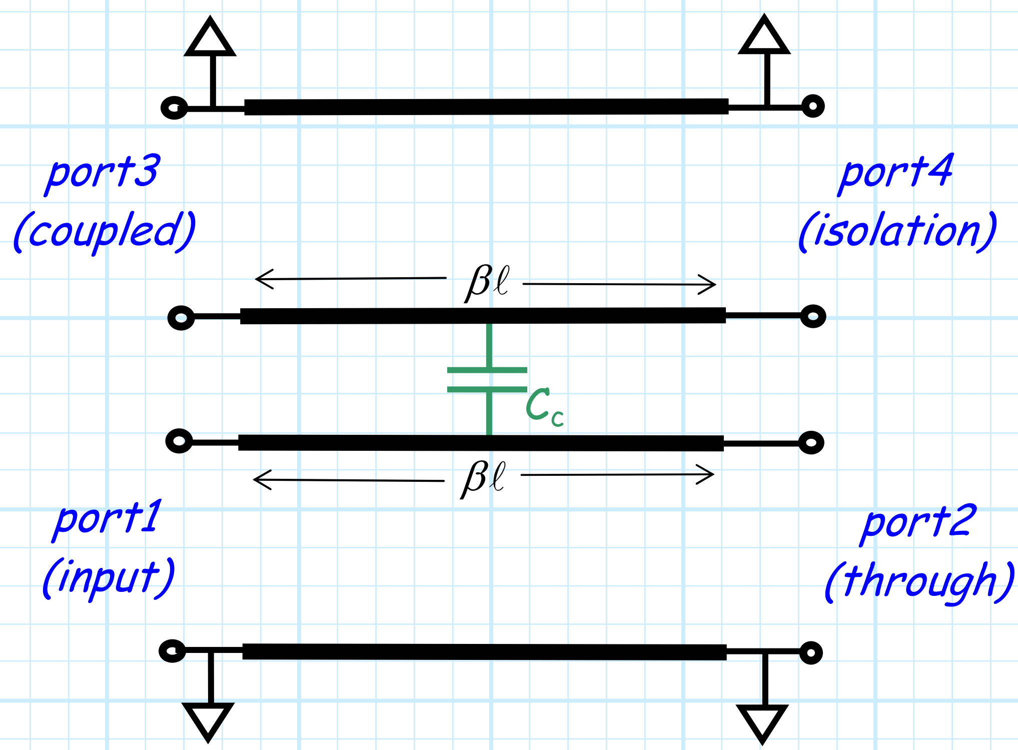
Combining these results, we find that at our **design frequency**, the **scattering matrix** of our coupled-line coupler is:

$$\mathcal{S} = \begin{bmatrix} 0 & -j\sqrt{1-c^2} & c & 0 \\ -j\sqrt{1-c^2} & 0 & 0 & c \\ c & 0 & 0 & -j\sqrt{1-c^2} \\ 0 & c & -j\sqrt{1-c^2} & 0 \end{bmatrix}$$

Q: *Hey! Isn't this the same scattering matrix as the **ideal symmetric directional coupler** we studied in the first section of this chapter?*

A: The very same! The coupled-line coupler—**if** our design rules are followed—results in an "ideal" directional coupler.

If the **input** is port 1, then the **through** port is port 2, the **coupled** port is port 3, and the **isolation** port is port 4!



Q: *But, how do we **design** a coupled-line coupler with a **specific** coupling coefficient c?*

A: Given our **two design constraints**, we know that:

$$Z_0^e Z_o^o = Z_0^2 \qquad \text{and} \qquad c = \frac{Z_0^e - Z_0^o}{Z_0^e + Z_0^o}$$

We can **rearrange** these two expressions to find **solutions** for our odd and even mode impedances:

$$Z_0^e = Z_0 \sqrt{\frac{1+c}{1-c}} \qquad Z_0^o = Z_0 \sqrt{\frac{1-c}{1+c}}$$

Thus, **given** the desired values Z_0 and c, we can determine the proper values of Z_0^e and Z_o^o for an ideal directional coupler.

Q: *Yes, but the odd and even mode impedance depends on the* ***physical structure*** *of the coupled lines, such as substrate dielectric ε_r, substrate thickness (d or b), conductor width W, and separation distance S.*

How do we determine ***these*** *physical design parameters for desired values of Z_0^e and Z_o^o??*

A: That's a much more difficult question to answer! Recall that there is **no** direct formulation relating microstrip and stripline parameters to **characteristic impedance** (we only have numerically derived **approximations**).

* So it's no surprise that there is likewise **no direct formulation** relating odd and even mode characteristic impedances to the specific physical parameters of microstrip and stripline coupled lines.

* Instead, we again have numerically derived **approximations** that allow us to determine (approximately) the required microstrip and stripline parameters, or we can use a **microwave CAD packages** (like ADS!).

* For example, **figures 7.29 and 7.30** provide **charts** for selecting the required values of W and S, given some ε_r and b (or d).

* Likewise, example 7.7 on page 345 provides a good **demonstration** of the single-section coupled-line coupler **design synthesis**.

Multi-Section Coupled Line Couplers

We can add **multiple** coupled lines in series to increase coupler bandwidth.

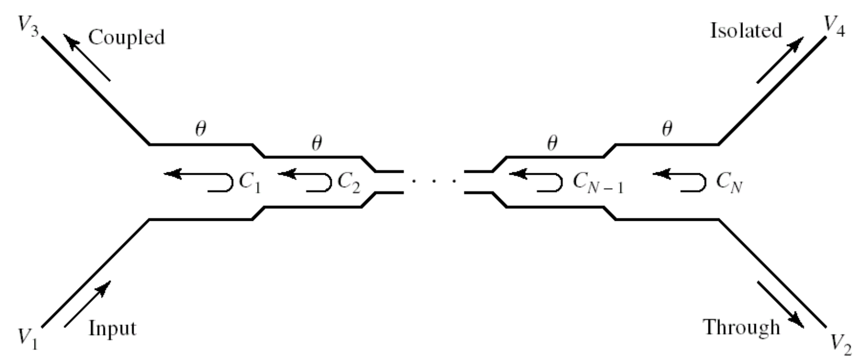


Figure 7.35 (p. 346)
An *N*-section coupled line

We typically design the coupler such that it is **symmetric**, i.e.:

$$C_1 = C_N, \; C_2 = C_{N-1}, \; C_3 = C_{N-2}, \text{ etc.}$$

where N is **odd**.

Q: *What is the coupling of this device as a function of* ***frequency?***

A: To analyze this structure, we make an **approximation** similar to that of the theory of small reflections.

First, if c is **small** (i.e., less than 0.3), then we can make the approximation:

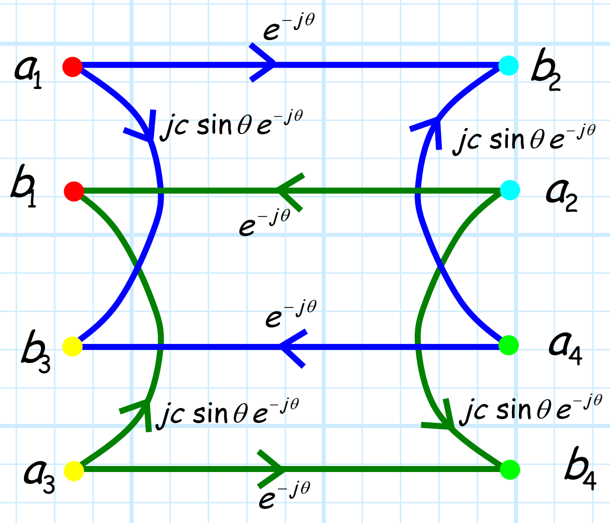
$$\begin{aligned} S_{31}(\theta) &= \frac{jc\tan\theta}{\sqrt{1-c^2}+j\tan\theta} \\ &\approx \frac{jc\tan\theta}{1+j\tan\theta} \\ &= jc\sin\theta\, e^{-j\theta} \end{aligned}$$

Likewise:

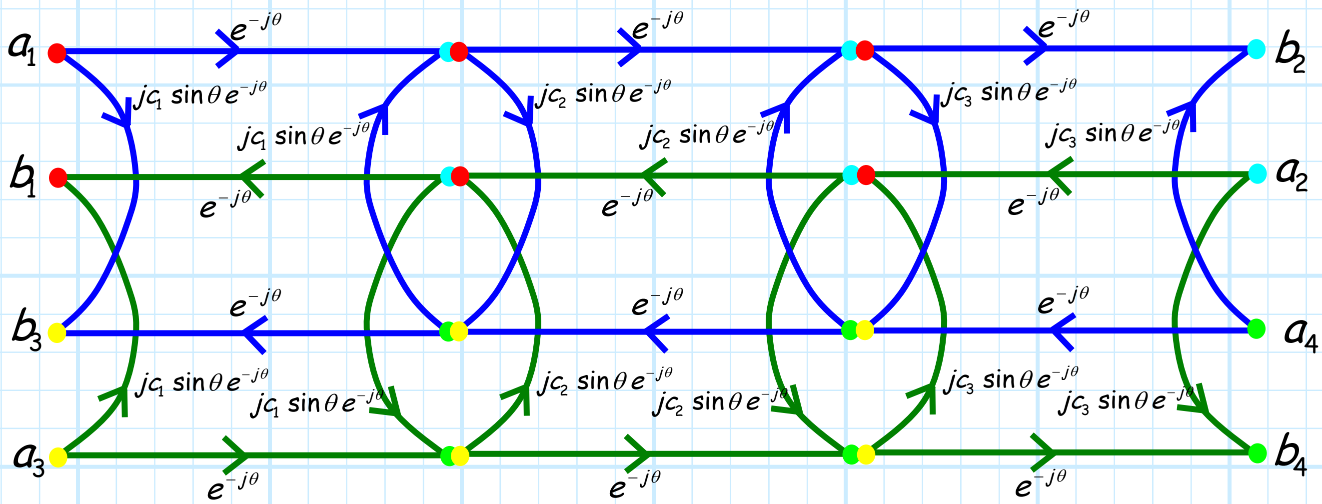
$$\begin{aligned} S_{21}(\theta) &= \frac{\sqrt{1-c^2}}{\sqrt{1-c^2}\cos\theta + j\sin\theta} \\ &\approx \frac{1}{\cos\theta + j\sin\theta} \\ &= e^{-j\theta} \end{aligned}$$

where of course $\theta = \beta\ell = \omega T$, and $T = \ell / v_p$.

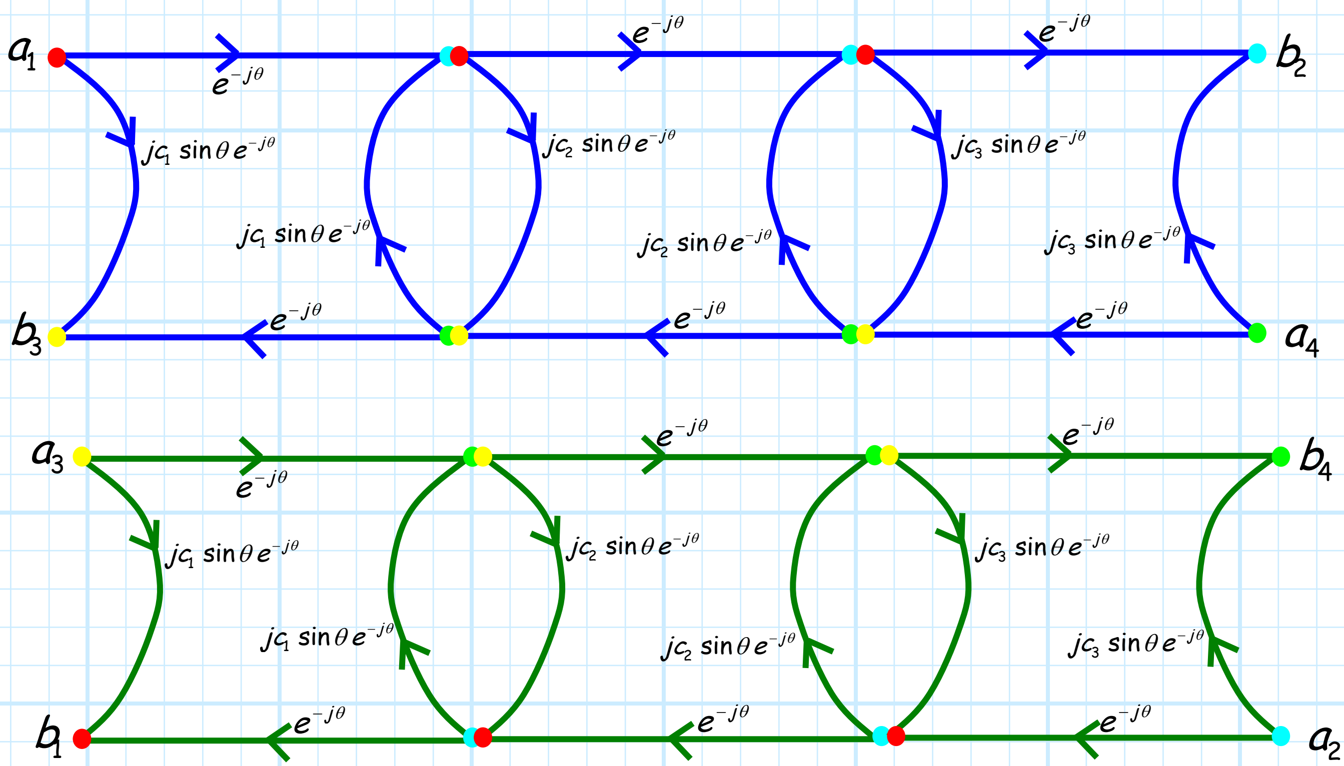
We can use these approximations to construct a **signal flow graph** of a **single**-section coupler:



Now, say we cascade **three** coupled line pairs, to form a **three section** coupled line coupler. The signal flow graph would thus be:

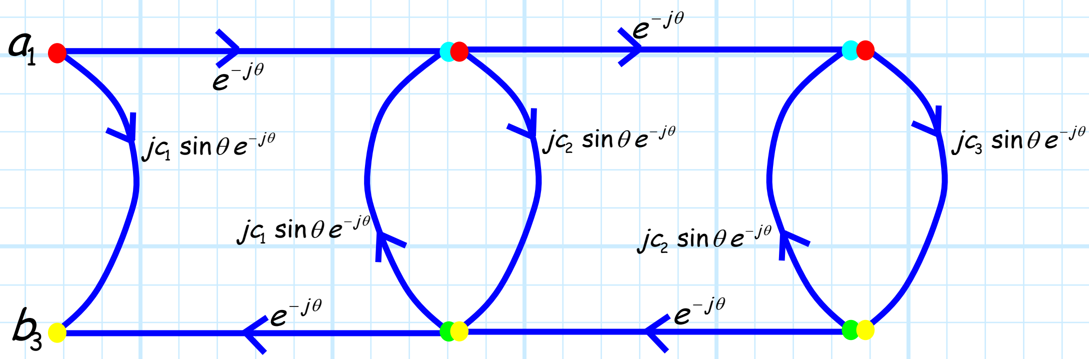


Note that this signal flow graph **decouples** into two separate and graphs (i.e., the blue graph and the green graph).



Note also that these two graphs are essentially **identical**, and emphasize the **symmetric** structure of the coupled-line coupler.

Now, we are interested in describing the **coupled output** (i.e., b_3) in terms of the incident wave (i.e., a_1). Assuming ports 2, 3 and 4 are **matched** (i.e., $a_2 = a_3 = a_4 = 0$), we can reduce the graph to simply:

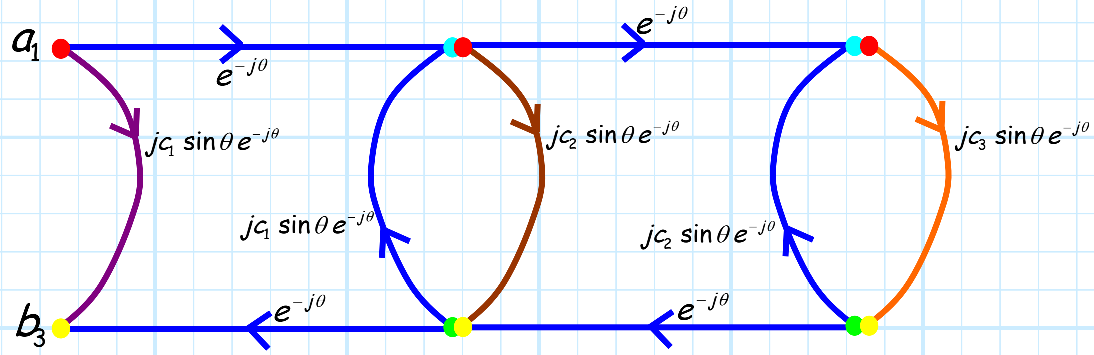


Now, we **could** reduce this signal flow graph even further—**or** we could truncate a **propagation series** by considering only the **direct paths**!

We of course used this idea to analyze multi-section matching networks, an approach dubbed the "theory of small reflections".

Essentially we are now applying a **"theory of small couplings"**. In other words, we consider only the propagation paths where **one coupling** is involved—the signal propagates **across** a coupled-line pair only **once**!

Note from the signal flow graph that there are **three** such mechanisms, corresponding to the coupling across each of the **three** separate coupled line pairs:



$$b_3 \approx \left(jc_1 \sin\theta\, e^{-j\theta} + e^{-j\theta} jc_2 \sin\theta\, e^{-j\theta} e^{-j\theta} + e^{-j2\theta} jc_3 \sin\theta\, e^{-j\theta} e^{-j2\theta} \right) a_1$$

$$= \left(jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + jc_3 \sin\theta\, e^{-j5\theta} \right) a_1$$

Note that **all other** terms of the infinite series would involve at least **three** couplings (i.e., three crossings), and thus these terms would be **exceeding** small (i.e., $c^3 \approx 0$).

Therefore, according to this **approximation**:

$$S_{31}(\theta) = \frac{V_3^-}{V_1^+}(\theta) = \frac{b_3}{a_1}(\theta) = jc_1 \sin\theta\, e^{-j\theta} + jc_2 \sin\theta\, e^{-j3\theta} + jc_3 \sin\theta\, e^{-j5\theta}$$

Moreover, for a **multi**-section coupler with N sections, we find:

$$S_{31}(\theta) = j\, c_1 \sin\theta\, e^{-j\theta} + j\, c_2 \sin\theta\, e^{-j3\theta} + j\, c_3 \sin\theta e^{-j5\theta} + \cdots + j\, c_N \sin\theta\, e^{-j(2N-1)\theta}$$

And for **symmetric** couplers with an **odd** value N, we find:

$$S_{31}(\theta) = j2\sin\theta \, e^{-jN\theta}\left[c_1\cos(N-1)\theta + c_2\cos(N-3)\theta \right.$$
$$\left. + c_3\cos(N-5)\theta + \cdots + \tfrac{1}{2}c_M\right]$$

where $M = (N+1)/2$.

Thus, we find the coupling **magnitude** as a function of frequency is:

$$\begin{aligned} |c(\theta)| &= |S_{31}(\theta)| \\ &= c_1 2\sin\theta\cos(N-1)\theta + c_2 2\sin\theta\cos(N-3)\theta \\ &\quad + c_3 2\sin\theta\cos(N-5)\theta + \cdots + c_M \sin\theta \end{aligned}$$

And thus the **coupling in** ***dB*** is:

$$C(\theta) = -10\log_{10}|c(\theta)|^2$$

Now, our design goals are to **select** the coupling values $c_1, c_2, \cdots c_N$ such that:

1. The coupling value $C(\theta)$ is a specific, **desired** value at our design frequency.

2. The coupling **bandwidth** is as **large** as possible.

For the first condition, recall that the at the **design frequency**:

$$\theta = \beta \ell = \pi/2$$

I.E., the section lengths are a **quarter-wavelength** at our design frequency.

Thus, we find our **first** design equation:

$$\begin{aligned} |c(\theta)|\Big|_{\theta=\pi/2} &= c_1\, 2\cos(N-1)\pi/2 + c_2\, 2\cos(N-3)\pi/2 \\ &\quad + c_3\, 2\cos(N-5)\pi/2 + \cdots + c_M \end{aligned}$$

where we have used the fact that $\sin(\pi/2) = 1$.

Note the value $|c(\theta)|\Big|_{\theta=\pi/2}$ is set to the value necessary to achieve the **desired** coupling value. This equation thus provides **one** design constraint—we have **M-1** degrees of design freedom left to accomplish our **second** goal!

To **maximize bandwidth**, we typically impose the **maximally flat** condition:

$$\left.\frac{d^m |c(\theta)|}{d\theta^m}\right|_{\theta=\pi/2} = 0 \qquad m = 1,2,3\cdots$$

Be careful! Remember to perform the derivative **first**, and **then** evaluate the result at $\theta = \pi/2$.

You will find for a **symmetric** coupler, the **odd**-ordered derivatives (e.g., $d\,|c(\theta)|/d\theta$, $d^3\,|c(\theta)|/d\theta^3$, $d^5\,|c(\theta)|/d\theta^5$)are uniquely **zero**. In other words, they are zero-valued at $\theta = \pi/2$ **regardless** of the values of coupling coefficients $c_1, c_2, c_3, \cdots$!

As a result, these **odd**-order derivatives do **not** impose a maximally flat **design equation**—only the **even**-ordered derivatives do. **Keep taking** these derivatives until your design is **fully** constrained (i.e., the number of design equations **equals** the number of unknown coefficients $c_1, c_2, c_3, \cdots$).

One final note, you may find that this **trig** expression is helpful in **simplifying** your derivatives:

$$\sin\phi\cos\psi = \frac{1}{2}\sin(\phi+\psi) + \frac{1}{2}\sin(\phi-\psi)$$

For **example**, we find that:

$$\begin{aligned} 2\sin\theta\cos 2\theta &= \sin(\theta+2\theta) + \sin(\theta-2\theta) \\ &= \sin(3\theta) + \sin(-\theta) \\ &= \sin(3\theta) - \sin(\theta) \end{aligned}$$