8.3 Filter Design by the Insertion Loss Method

Reading Assignment: ***pp. 389-398***

Chapter **8** cover **microwave filters**.

A microwave **filter** → A two-port microwave network that allows source power to be transferred to a load as an explicit **function of frequency**.

HO: Filters

HO: The Filter Phase Function

Q: *Why do we give a darn about* ***phase*** *function* $\angle S_{21}(\omega)$*? After all, phase* ***doesn't*** *matter.*

A: Phase doesn't matter!?! A typical **rookie mistake**!

HO: Filter Dispersion

HO: The Linear Phase Filter

Q: *So how do we* ***specify*** *a microwave filter? How close to an* ***ideal*** *filter can we build?*

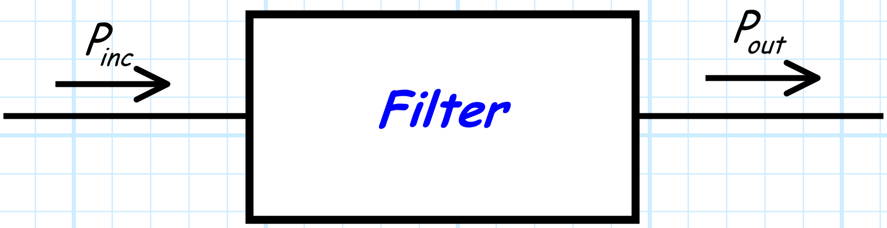
A: HO: The Insertion Loss Method

Q: *So exactly how do* ***construct*** *a microwave filter that exhibits the polynomial function that we choose? How do we "****realize****" a filter polynomial function?*

A: HO: Filter Realizations using Lumped Elements

Filters

A RF/microwave **filter** is (typically) a passive, reciprocal, 2-port linear device.



If port 2 of this device is terminated in a **matched** load, then we can relate the incident and output power as:

$$P_{out} = |S_{21}|^2 \, P_{inc}$$

We define this power transmission through a filter in terms of the **power transmission coefficient T**:

$$\mathbf{T} \doteq \frac{P_{out}}{P_{inc}} = |S_{21}|^2$$

Since microwave filters are typically **passive**, we find that:

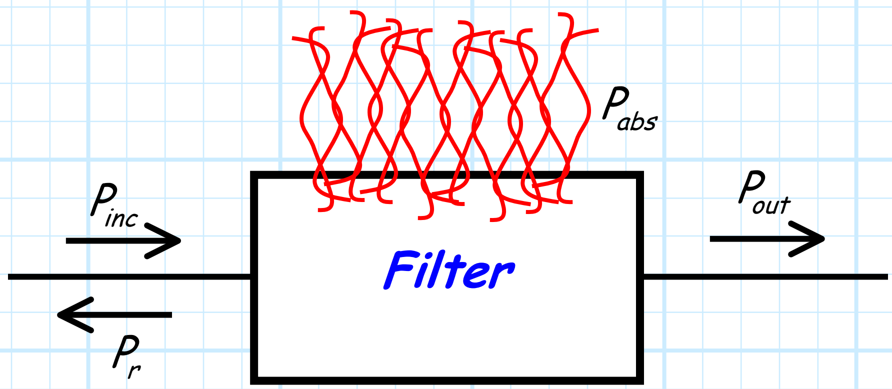
$$0 \leq \mathbf{T} \leq 1$$

in other words, $P_{out} \leq P_{inc}$.

Q: *What happens to the "missing" power $P_{inc} - P_{out}$?*

A: **Two** possibilities: the power is either **absorbed** (P_{abs}) by the filter (converted to heat), or is **reflected** (P_r) at the input port.

I.E.:



Thus, by conservation of energy:

$$P_{inc} = P_r + P_{abs} + P_{out}$$

Now **ideally,** a microwave filter is lossless, therefore $P_{abs} = 0$ and:

$$P_{inc} = P_r + P_{out}$$

which **alternatively** can be written as:

$$\frac{P_{inc}}{P_{inc}} = \frac{P_r + P_{out}}{P_{inc}}$$

$$1 = \frac{P_r}{P_{inc}} + \frac{P_{out}}{P_{inc}}$$

Recall that $P_{out}/P_{inc} = \mathbf{T}$, and we can likewise **define** P_r/P_{inc} as the **power reflection coefficient**:

$$\Gamma \doteq \frac{P_r}{P_{inc}} = |S_{11}|^2$$

We again emphasize that the filter output port is terminated in a **matched** load.

Thus, we can conclude that for a **lossless** filter:

$$1 = \Gamma + \mathbf{T}$$

Which is simply **another** way of saying for a lossless device that $1 = |S_{11}|^2 + |S_{21}|^2$.

Now, **here's** the important part!

For a microwave **filter**, the coefficients Γ and $\mathbf{T}$ are **functions of frequency**! I.E.,:

$$\Gamma(\omega) \quad \text{and} \quad \mathbf{T}(\omega)$$

The **behavior** of a microwave filter is described by these **functions**!

We find that for most signal frequencies ω_s, these functions will have a value equal to one of **two** different **approximate** values.

Either:

$$\Gamma(\omega = \omega_s) \approx 0 \qquad \text{and} \qquad T(\omega = \omega_s) \approx 1$$

or

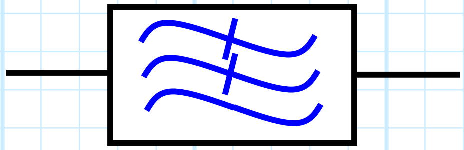
$$\Gamma(\omega = \omega_s) \approx 1 \qquad \text{and} \qquad T(\omega = \omega_s) \approx 0$$

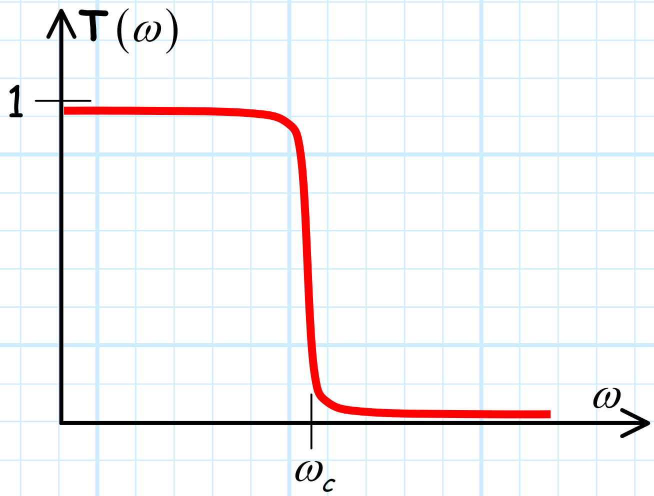
In the **first** case, the signal frequency ω_s is said to lie in the **pass-band** of the filter. Almost all of the incident signal power will **pass through** the filter.

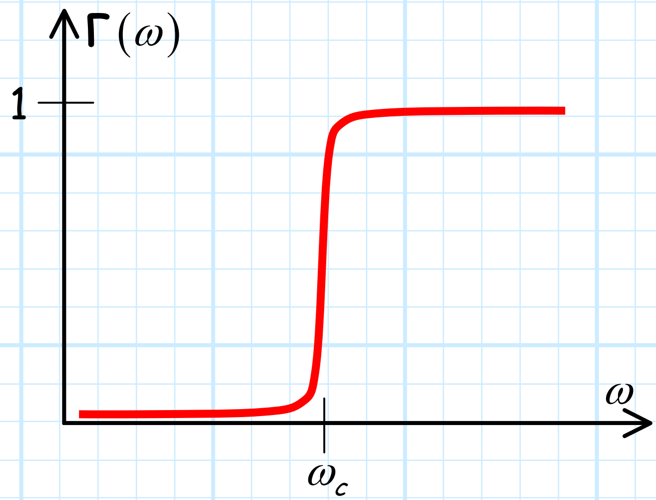
In the **second** case, the signal frequency ω_s is said to lie in the **stop-band** of the filter. Almost all of the incident signal power will be **reflected** at the input—almost no power will appear at the filter output.

Consider then these **four types** of functions of $\Gamma(\omega)$ and $T(\omega)$:

1. Low-Pass Filter







Note for this filter:

$$T(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases}$$

This filter is a **low-pass** type, as it **"passes"** signals with frequencies **less** than ω_c, while **"rejecting"** signals at frequencies **greater** than ω_c.

Q: *This frequency ω_c seems to be very important! What is it?*

A: Frequency ω_c is a filter parameter known as the **cutoff frequency**; a value that **approximately** defines the frequency region where the filter pass-band **transitions** into the filter stop band.

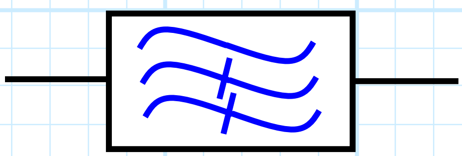
According, this frequency is defined as the frequency where the power **transmission** coefficient is equal to ½:

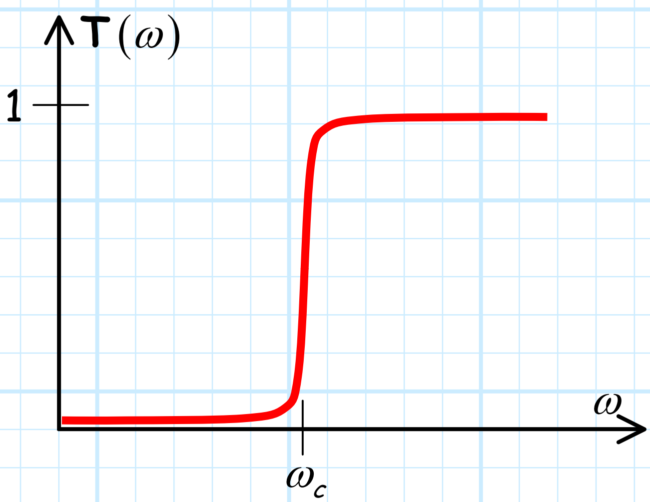
$$\mathrm{T}(\omega = \omega_c) = 0.5$$

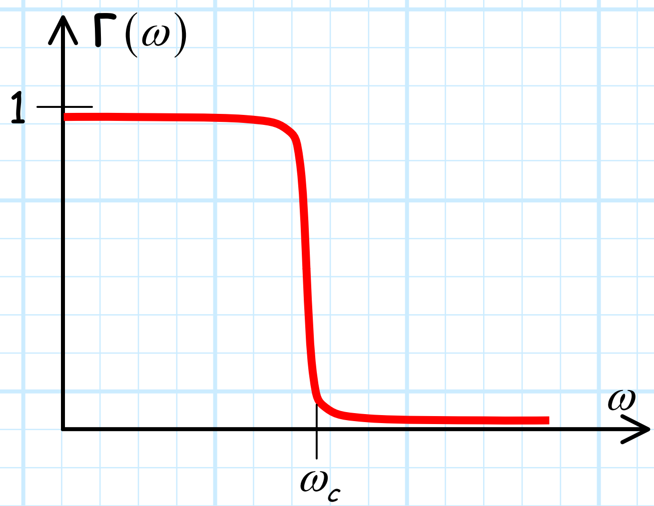
Note for a lossless filter, the cutoff frequency is **likewise** the value where the power **reflection** coefficient is ½:

$$\Gamma(\omega = \omega_c) = 0.5$$

2. High-Pass Filter





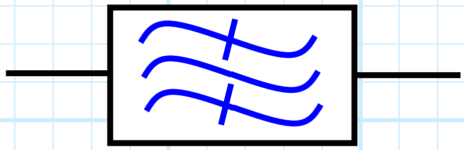


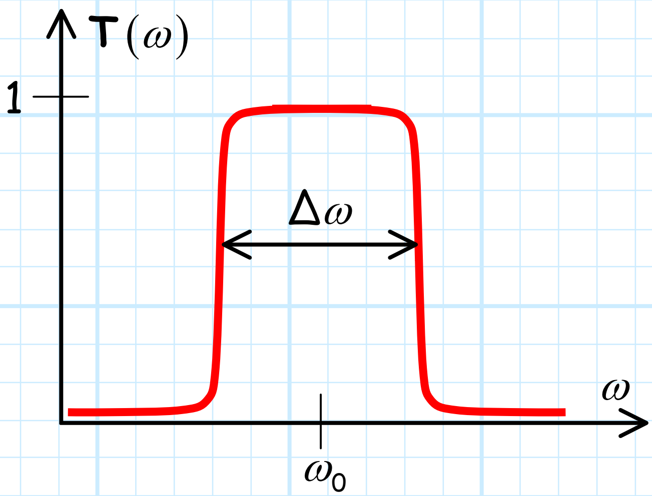
Note for this filter:

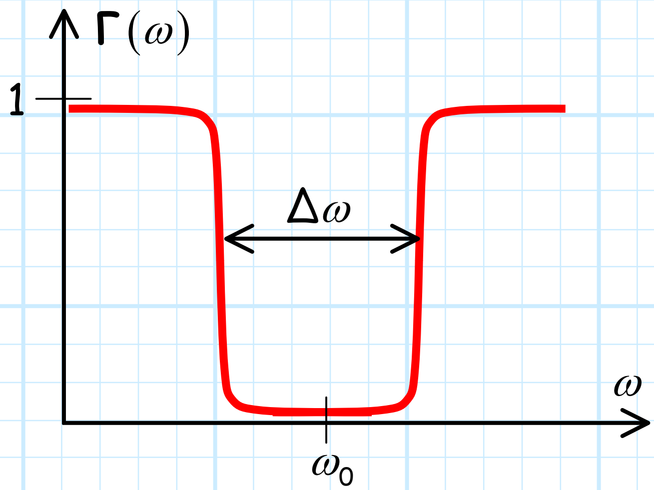
$$\mathsf{T}(\omega) = \begin{cases} \approx 0 & \omega < \omega_c \\ \approx 1 & \omega > \omega_c \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 1 & \omega < \omega_c \\ \approx 0 & \omega > \omega_c \end{cases}$$

This filter is a **high-pass** type, as it **"passes"** signals with frequencies **greater** than ω_c, while **"rejecting"** signals at frequencies **less** than ω_c.

3. Band-Pass Filter







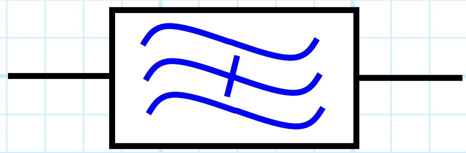
Note for this filter:

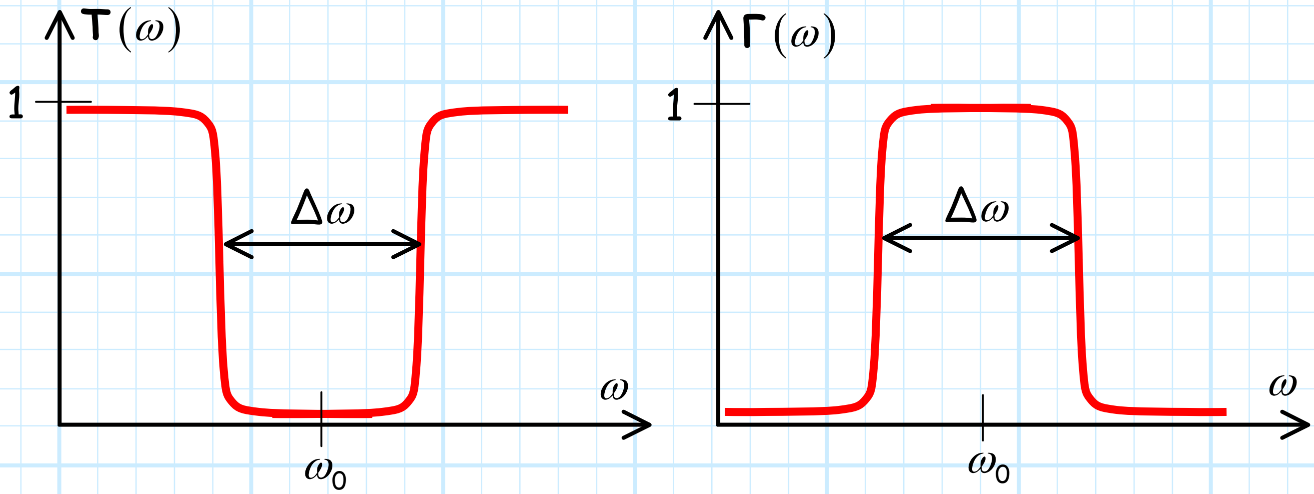
$$\mathsf{T}(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 0 & |\omega - \omega_0| > \Delta\omega/2 \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \end{cases}$$

This filter is a **band-pass** type, as it "**passes**" signals within a frequency bandwidth $\Delta\omega$, while "**rejecting**" signals at all frequencies **outside this bandwidth**.

In addition to filter bandwidth $\Delta\omega$, a fundamental parameter of bandpass filters is ω_0, which defines the **center frequency** of the filter bandwidth.

3. Band-Stop Filter





Note for this filter:

$$\mathbf{T}(\omega) = \begin{cases} \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 1 & |\omega - \omega_0| > \Delta\omega/2 \end{cases} \qquad \Gamma(\omega) = \begin{cases} \approx 1 & |\omega - \omega_0| < \Delta\omega/2 \\ \approx 0 & |\omega - \omega_0| < \Delta\omega/2 \end{cases}$$

This filter is a **band-stop** type, as it "**rejects**" signals within a frequency bandwidth $\Delta\omega$, while "**passing**" signals at all frequencies **outside this bandwidth**.

The Filter Phase Function

Recall that the power transmission coefficient $\mathcal{T}(\omega)$ can be determined from the **scattering parameter** $S_{21}(\omega)$:

$$\mathcal{T}(\omega) = \left|S_{21}(\omega)\right|^2$$

Q: *I see, we only care about the* ***magnitude*** *of complex function $S_{21}(\omega)$ when using microwave filters !?*

A: Hardly! Since $S_{21}(\omega)$ is complex, it can be expressed in terms of its magnitude and **phase**:

$$\begin{aligned} S_{21}(\omega) &= \text{Re}\{S_{21}(\omega)\} + j\,\text{Im}\{S_{21}(\omega)\} \\ &= \left|S_{21}(\omega)\right| e^{j\angle S_{21}(\omega)} \end{aligned}$$

where the phase is denoted as $\angle S_{21}(\omega)$:

$$\angle S_{21}(\omega) = \tan^{-1}\left[\frac{\text{Im}\{S_{21}(\omega)\}}{\text{Re}\{S_{21}(\omega)\}}\right]$$

We likewise care **very** much about this phase function!

Q: *Just what does this phase tell us?*

A: It describes the relative phase **between** the wave incident on the input to the filter, and the wave exiting the output of the filter (given the output port is matched).

In other words, if the **incident** wave is:

$$V_1^+(z_1) = V_{01}^+ \, e^{-j\beta z}$$

Then the exiting (output) wave will be:

$$\begin{aligned} V_2^-(z_2) &= V_{02}^- \, e^{+j\beta z_2} \\ &= S_{21} \, V_{01}^- \, e^{+j\beta z_2} \\ &= \left|S_{21}\right| V_{01}^- \, e^{+j(\beta z + \angle S_{21})} \end{aligned}$$

We say that there has been a "**phase shift**" of $\angle S_{21}$ between the input and output waves.

Q: *What **causes** this phase shift?*

A: Propagation **delay.** It takes some non-zero amount of **time** for signal energy to propagate from the input of the filter to the output.

Q: *Can we tell from $\angle S_{21}(\omega)$ how **long** this delay is?*

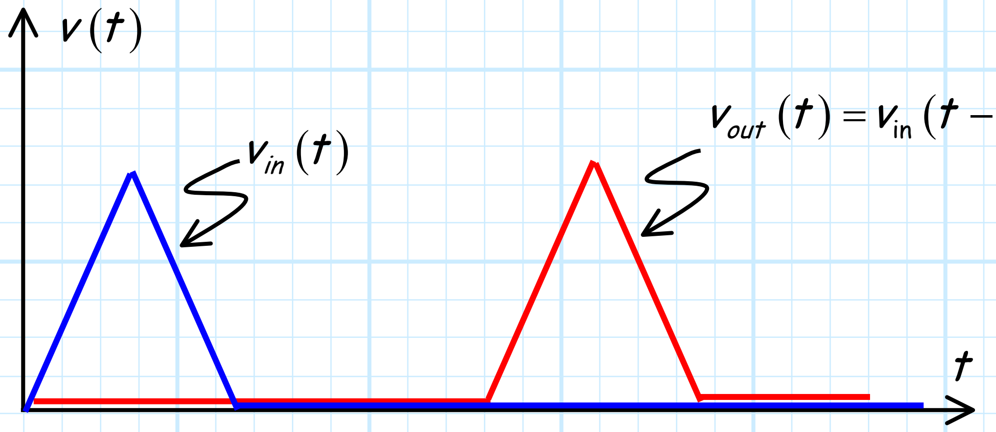
A: Yes!

To see how, consider an **example** two-port network with the impulse response:

$$h(t) = \delta(t - \tau)$$

We determined earlier that this device would merely **delay** and input signal by some amount τ:

$$\begin{aligned} v_{out}(t) &= \int_{-\infty}^{\infty} h(t - t')\, v_{in}(t')\, dt' \\ &= \int_{-\infty}^{\infty} \delta(t - t' - \tau) v_{in}(t')\, dt' \\ &= v_{in}(t' - \tau) \end{aligned}$$



Taking the **Fourier transform** of this impulse response, we find the **frequency response** of this two-port network is:

$$\begin{aligned} H(\omega) &= \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} \delta(t - \tau)\, e^{-j\omega t} dt \\ &= e^{-j\omega\tau} \end{aligned}$$

In other words:

$$|H(\omega)| = 1 \qquad \text{and} \qquad \angle H(\omega) = -\omega\tau$$

The interesting result here is the **phase** $\angle H(\omega)$. The result means that a delay of τ seconds results in an output "phase shift" of $-\omega\tau$ radians!

Note that although the **delay** of device is a **constant** τ, the **phase shift** is a **function** of ω --in fact, it is directly proportional to frequency ω.

Note if the **input** signal for this device was of the form:

$$v_{in}(t) = \cos \omega t$$

Then the output would be:

$$\begin{aligned} v_{out}(t) &= \cos \omega (t - \tau) \\ &= \cos(\omega t - \omega\tau) \\ &= |H(\omega)| \cos\left(\omega t + \angle H(\omega)\right) \end{aligned}$$

Thus, we could **either** view the signal $v_{in}(t) = \cos \omega t$ as being delayed by an amount τ seconds, **or** phase shifted by an amount $-\omega\tau$ radians.

Q: *So, by* ***measuring*** *the output signal phase shift* $\angle H(\omega)$*, we could determine the delay* τ *through the device with the equation:*

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

right?

A: Not exactly. The problem is that we cannot **unambiguously** determine the phase shift $\angle H(\omega) = -\omega\tau$ by **looking** at the output signal!

The reason is that $\cos(\omega t + \angle H(\omega))$ = $\cos(\omega t + \angle H(\omega) + 2\pi)$ = $\cos(\omega t + \angle H(\omega) - 4\pi)$, etc. More specifically:

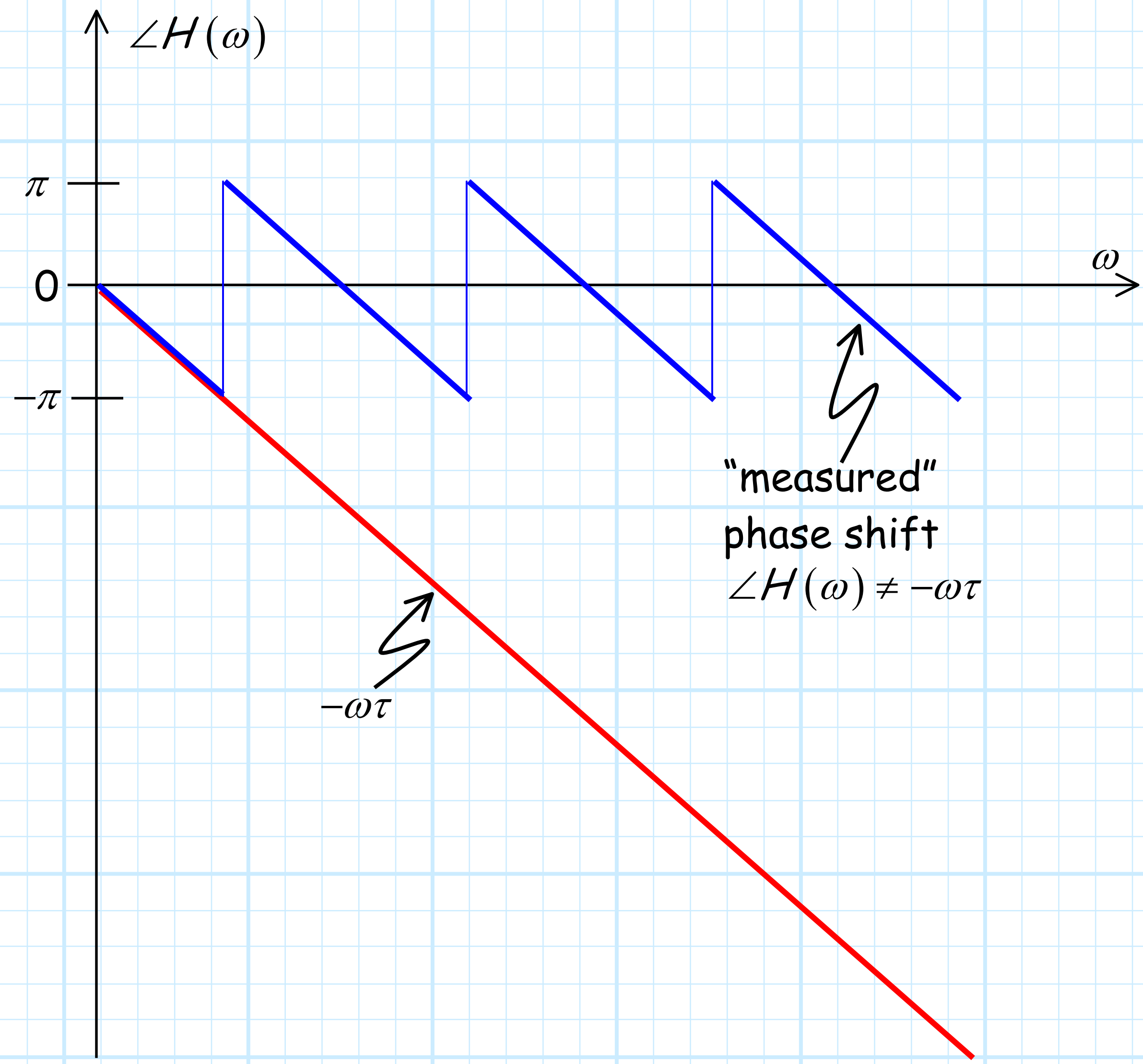
$$\cos(\omega t + \angle H(\omega)) = \cos(\omega t + \angle H(\omega) + n2\pi)$$

where n is **any integer**—positive or negative. We can't tell **which** of these output signal we are looking at!

Thus, any phase shift **measurement** has an inherent **ambiguity**. Typically, we interpret a phase measurement (in radians) such that:

$$-\pi < \angle H(\omega) \leq \pi \qquad \text{or} \qquad 0 \leq \angle H(\omega) < 2\pi$$

But almost certainly the actual value of $\angle H(\omega) = -\omega\tau$ is **nowhere** near these interpretations!



Clearly, using the equation:

$$\tau = -\frac{\angle H(\omega)}{\omega}$$

would **not** get us the correct result in this case—after all, there will be **several** frequencies ω with exactly the **same measured** phase $\angle H(\omega)$!

Q: *So determining the delay τ is **impossible**?*

A: NO! It is **entirely** possible—we simply must find the correct **method**.

Looking at the plot on the previous page, this method should become **apparent**. Not that although the measured phase (blue curve) is definitely **not** equal to the phase function $-\omega\tau$ (red curve), the **slope** of the two are **identical** at every point!

Q: *What good is knowing the **slope** of these functions?*

A: Just look! Recall that we can determine the slope by taking the first **derivative**:

$$\frac{\partial(-\omega\tau)}{\partial\omega} = -\tau$$

The slope directly tells us the **propagation delay**!

Thus, we can determine the propagation delay of this device by:

$$\tau = -\frac{\partial\angle H(\omega)}{\partial\omega}$$

where $\angle H(\omega)$ can be the **measured** phase. Of course, the method requires us to **measure** $\angle H(\omega)$ as a **function** of frequency (i.e., to make measurements at **many** signal frequencies).

Q: ***Now*** *I see! If we wish to* ***determine*** *the propagation delay τ through some* ***filter****, we simply need to take the derivative of $\angle S_{21}(\omega)$ with respect to frequency.* ***Right****?*

A: Well, sort of.

Recall for the **example** case that $h(t) = \delta(t-\tau)$ and $\angle H(\omega) = -\omega\tau$, where τ is a **constant**. For a microwave filter, **neither** of these conditions are true.

Specifically, the phase function $\angle S_{21}(\omega)$ will typically be some arbitrary **function** of frequency ($\angle S_{21}(\omega) \neq -\omega\tau$).

Q: *How could this be true? I thought you said that phase shift was* ***due*** *to filter delay τ!*

A: Phase shift **is** due to device delay, it's just that the propagation delay of most devices (such as filters) is **not a constant**, but instead depends on the **frequency** of the signal propagating through it!

In other words, the propagation delay of a filter is typically some arbitrary **function** of frequency (i.e., $\tau(\omega)$). That's why the phase $\angle S_{21}(\omega)$ is **likewise** an arbitrary function of frequency.

Q: *Yikes! Is there* ***any*** *way to determine the relationship between these two arbitrary functions?*

A: **Yes** there is! Just as before, the two can be related by a first **derivative**:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

This result $\tau(\omega)$ is also know as **phase delay**, and is a **very** important function to consider when designing/specifying/ selecting a microwave **filter**.

Q: *Why; what might happen?*

A: If you get a filter with the wrong $\tau(\omega)$, your **output** signal could be horribly **distorted**—distorted by the evil effects of **signal dispersion**!

Filter Dispersion

Any signal that carries significant **information** must has some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.

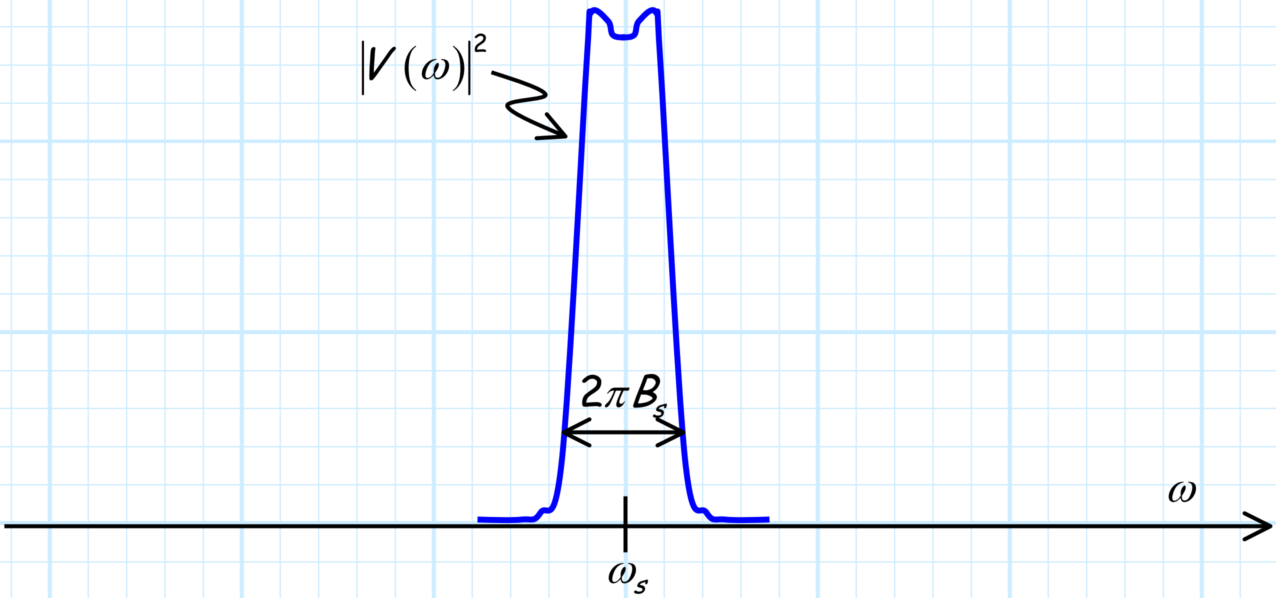
If the different frequencies that comprise a signal propagate at different velocities through a microwave filter (i.e., each signal frequency has a different delay τ), the output signal will be **distorted**. We call this phenomenon signal **dispersion**.

Q: *I see! The phase delay $\tau(\omega)$ of a filter* ***must*** *be a constant with respect to frequency—otherwise signal dispersion (and thus signal distortion) will result. Right?*

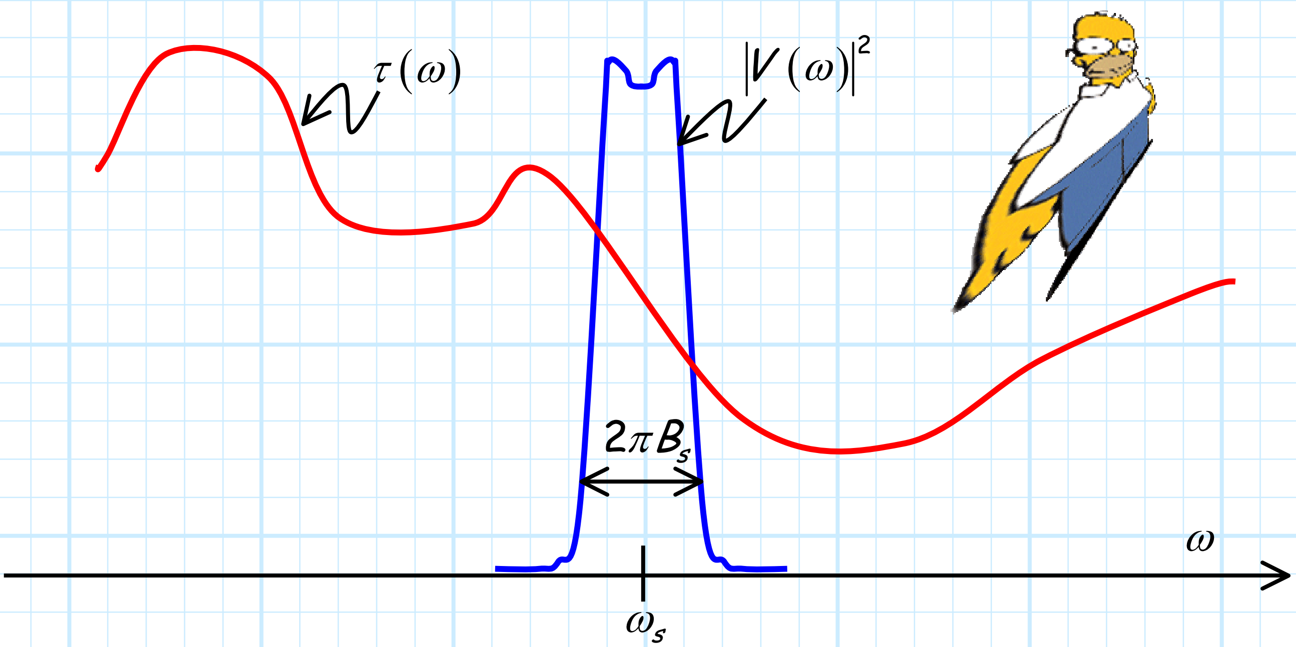
A: Not necessarily! Although a constant phase delay will **insure** that the output signal is not distorted, it is **not** strictly a requirement for that happy event to occur.

This is a **good** thing, for as we shall latter see, building a good filter with a constant phase delay is **very** difficult!

For example, consider a modulated signal with the following frequency **spectrum**, exhibiting a **bandwidth** of B_s Hertz.



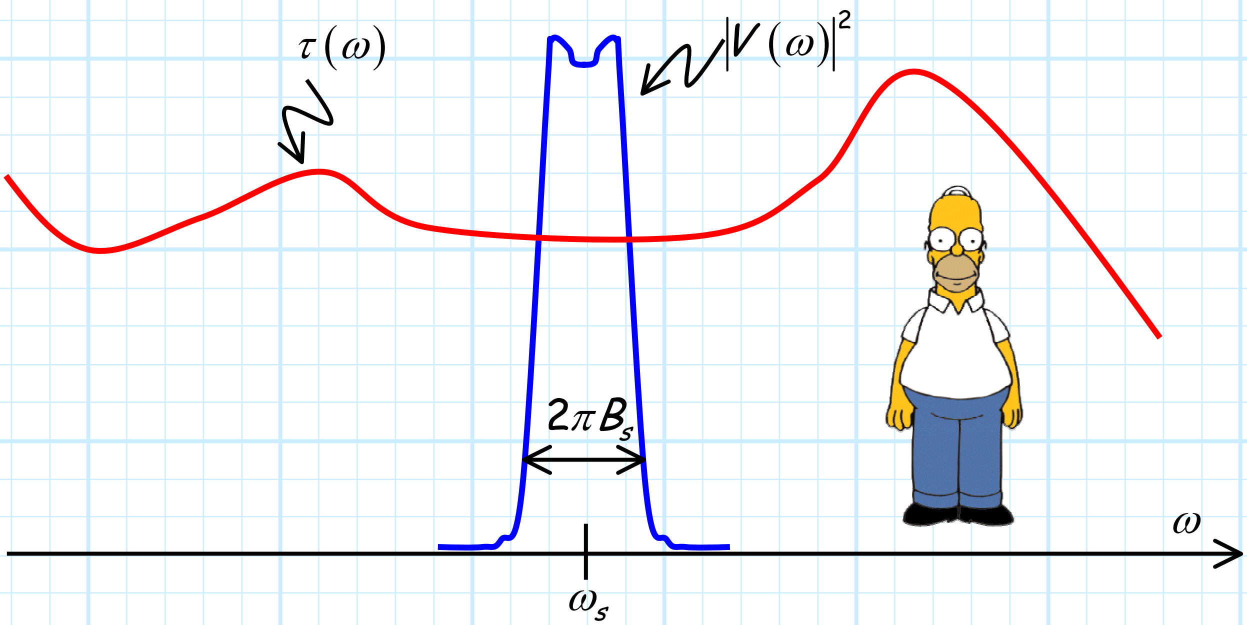
Now, let's likewise plot the **phase delay** function $\tau(\omega)$ of some filter:



Note that for this case the filter phase delay is **nowhere** near a constant with respect to frequency.

However, this fact alone does **not** necessarily mean that our signal would suffer from **dispersion** if it passed through this filter. Indeed, the signal in this case **would** be distorted, but **only** because the phase delay $\tau(\omega)$ changes significantly across the **bandwidth** B_s of the signal.

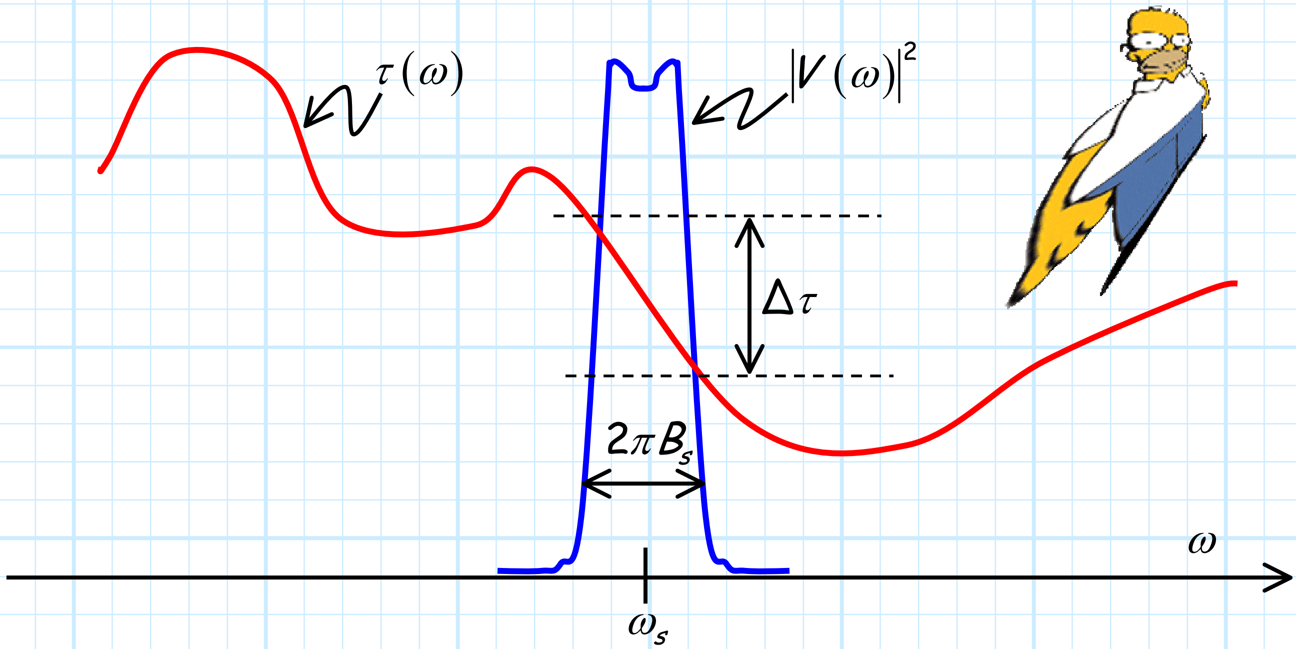
Conversely, consider this **phase delay**:



As with the previous case, the phase delay of the filter is **not** a constant. Yet, if this signal were to pass through this filter, it would **not** be distorted!

The reason for this is that the phase delay across the **signal bandwidth** is approximately constant—each frequency component of the **signal** will be delayed by the **same** amount.

Compare this to the **previous** case, where the phase delay changes by a precipitous value $\Delta\tau$ across signal bandwidth B_s:



Now **this** is a case where dispersion **will** result!

Q: *So does $\Delta\tau$ need to be **precisely** zero for no signal distortion to occur, or is there some **minimum** amount $\Delta\tau$ that is acceptable?*

A: Mathematically, we find that dispersion will be **insignificant** if:

$$\omega_s \, \Delta\tau \ll 1$$

A more specific (but **subjective**) "rule of thumb" is:

$$\omega_s \, \Delta\tau < \frac{\pi}{5}$$

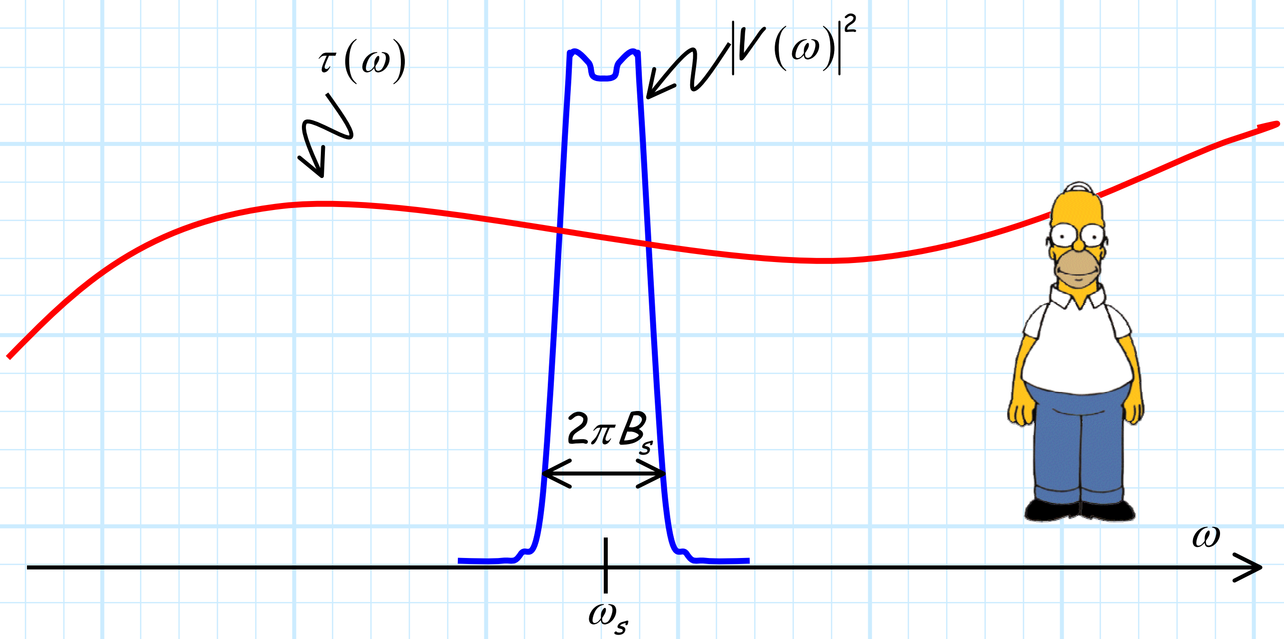
Or, using $\omega_s = 2\pi f_s$:

$$f_s \, \Delta\tau < 0.1$$

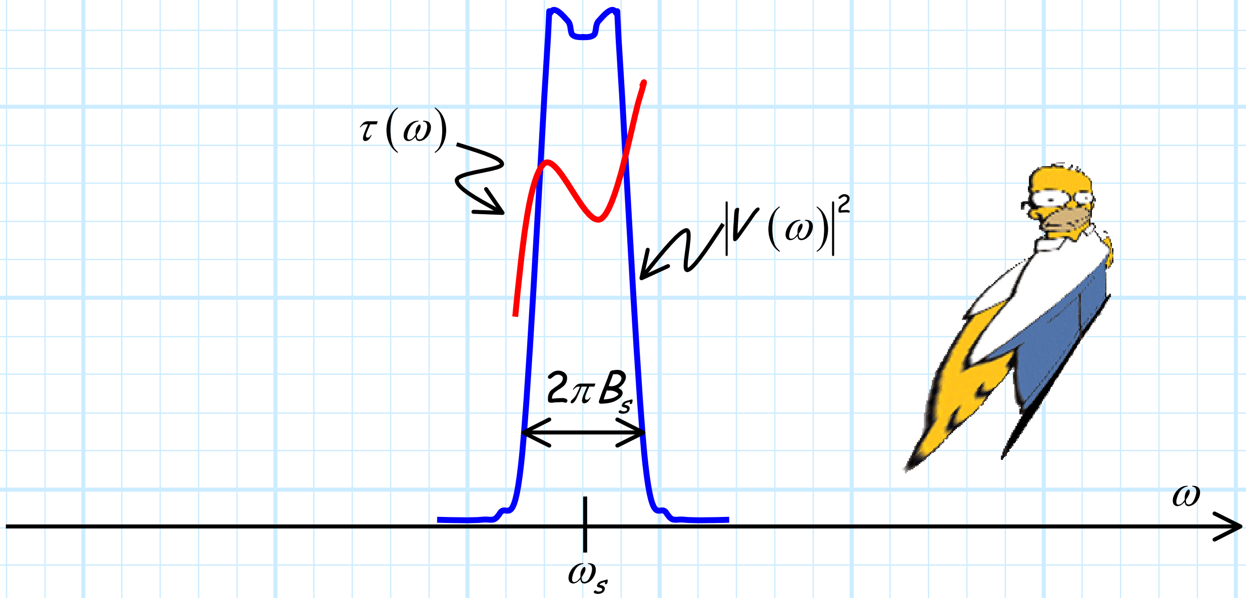
Generally speaking, we find for **wideband** filters—where filter bandwidth B is much greater than the signal bandwidth (i.e., $B \gg B_s$)—the above criteria is **easily** satisfied. In other words, signal dispersion is **not** typically a problem for wide band filters (e.g., preselector filters).

This is **not** to say that $\tau(\omega)$ is a constant for wide band filters. Instead, the phase delay can change **significantly** across the wide **filter** bandwith.

What we typically find however, is that the function $\tau(\omega)$ does not change very **rapidly** across the wide filter bandwidth. As a result, the phase delay will be **approximately** constant across the relatively narrow signal bandwidth B_s.



Conversely, a **narrowband** filter—where filter bandwidth B is approximately **equal** to the signal bandwidth (i.e., $B_s \approx B$)—can (if we're not careful!) exhibit a phase delay which likewise changes **significantly** over **filter** bandwidth B. This means of course that it **also** changes significantly over the **signal** bandwidth B_s!



Thus, a **narrowband** filter (e.g., IF filter) must exhibit a **near constant** phase delay $\tau(\omega)$ in order to **avoid** distortion due to signal dispersion!

The Linear Phase Filter

Q: *So, narrowband filters should exhibit a **constant** phase delay $\tau(\omega)$. What should the phase function $\angle S_{21}(\omega)$ be for this **dispersionless** case?*

A: We can express this problem mathematically as requiring:

$$\tau(\omega) = \tau_c$$

where τ_c is some **constant**.

Recall that the definition of **phase delay** is:

$$\tau(\omega) = -\frac{\partial \angle S_{21}(\omega)}{\partial \omega}$$

and thus **combining** these two equations, we find ourselves with a **differential equation**:

$$-\frac{\partial \angle S_{21}(\omega)}{\partial \omega} = \tau_c$$

The **solution** to this differential equation provides us with the necessary phase function $\angle S_{21}(\omega)$ for a **constant** phase delay τ_c.

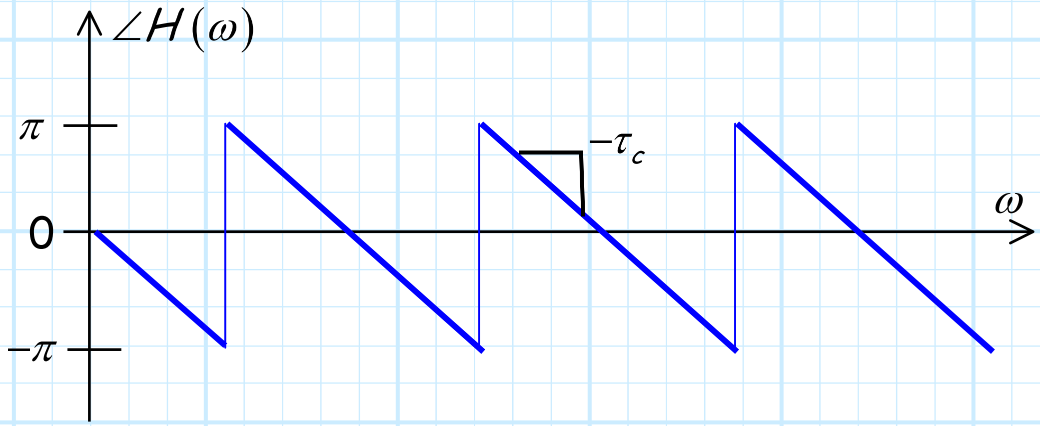
Fortunately, this differential equation is **easily** solved!

The solution is:

$$\angle S_{21}(\omega) = -\omega\,\tau_c + \phi_c$$

where ϕ_c is an arbitrary **constant**.

Plotting this phase function (with $\phi_c = 0$):



As **you** likely expected, this phase function is **linear**, such that it has a **constant slope** ($-\tau_c$).

Filters with this phase response are called **linear phase filters**, and have the desirable trait that they cause **no** dispersion distortion.

The Insertion Loss Method

Recall that a **lossless** filter can be described in terms of either its power transmission coefficient $\mathcal{T}(\omega)$ **or** its power reflection coefficient $\Gamma(\omega)$, as the two values are completely **dependent**:

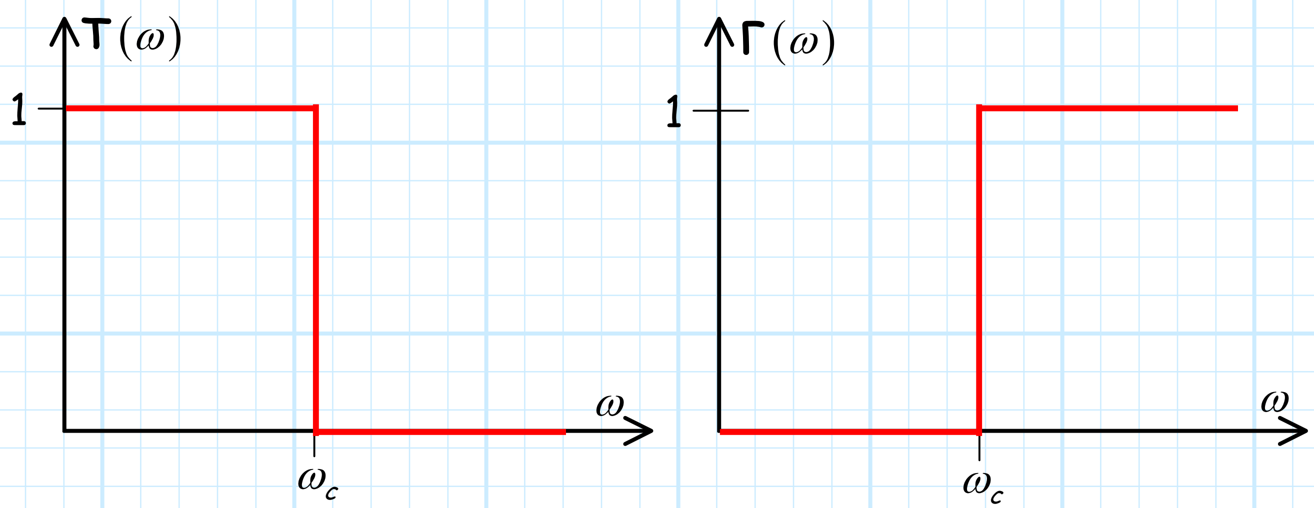
$$\Gamma(\omega) = 1 - \mathcal{T}(\omega)$$

Ideally, these functions would be quite **simple**:

1. $\mathcal{T}(\omega) = 1$ and $\Gamma(\omega) = 0$ for **all** frequencies within the **pass**-band.

2. $\mathcal{T}(\omega) = 0$ and $\Gamma(\omega) = 1$ for **all** frequencies within the **stop**-band.

For example, the **ideal** low-pass filter would be:



Add to this a **linear phase** response, and you have the **perfect** microwave filter!

There's just one small problem with this **perfect** filter→ It's **impossible** to build!

Now, if we consider only possible (i.e., **realizable**) filters, we must limit ourselves to filter functions that can be expressed as **finite polynomials** of the form:

$$\mathbf{T}(\omega) = \frac{a_0 + a_1\,\omega + a_2\,\omega^2 + \cdots}{b_0 + b_1\,\omega + b_2\,\omega^2 + \cdots + b_N\,\omega^{2N}}$$

The **order** N of the (denominator) polynomial is likewise the **order** of the filter.

Instead of the power transmission coefficient, we often use an equivalent function (assuming lossless) called the **power loss ratio** P_{LR}:

$$P_{LR} = \frac{P_1^+}{P_2^-} = \frac{1}{1-\mathbf{\Gamma}(\omega)}$$

Note with this definition, $P_{LR} = \infty$ when $\mathbf{\Gamma}(\omega) = 1$, and $P_{LR} = 0$ when $\mathbf{\Gamma}(\omega) = 0$.

We likewise note that, for a lossless filter:

$$P_{LR} = \frac{1}{1-\mathbf{\Gamma}(\omega)} = \frac{1}{\mathbf{T}(\omega)}$$

Therefore $P_{LR}(dB)$ is :

$$P_{LR}(dB) = 10\, log_{10} P_{LR} = -10\, log_{10} \mathbf{T}(\omega) \doteq \text{Insertion Loss}$$

The power loss ratio in dB is simply the insertion loss of a lossless filter, and thus filter design using the power loss ratio is also called the Insertion Loss Method.

We find that realizable filters will have a power loss ratio of the form:

$$P_{LR}(\omega) = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

where $M(\omega^2)$ and $N(\omega^2)$ are polynomials with terms $\omega^2, \omega^4, \omega^6, etc.$

By specifying these polynomials, we specify the frequency behavior of a realizable filter. Our job is to first choose a desirable polynomial!

There are many different **types** of polynomials that result in good filter responses, and each type has its own set of **characteristics**.

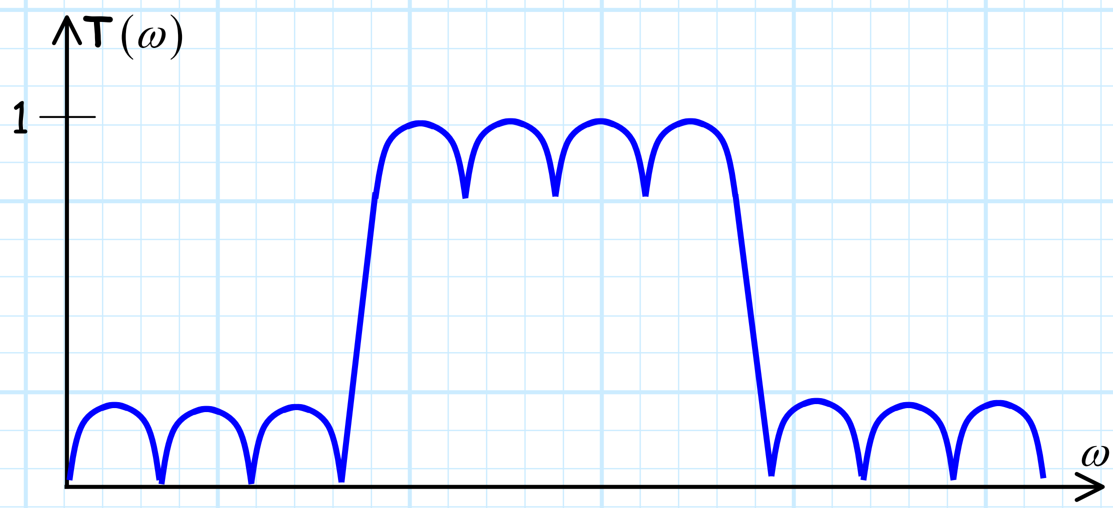
The type of **polynomial** likewise describes the type of microwave **filter**. Let's consider **three** of the most popular types:

1. Elliptical

Elliptical filters have three primary characteristics:

a) They exhibit very **steep** "**roll-off**", meaning that the transition from pass-band to stop-band is very rapid.
b) They exhibit **ripple** in the **pass**-band, meaning that the value of **T** will vary slightly within the pass-band.

c) They exhibit ripple in the **stop**-band, meaning that the value of **T** will vary slightly within the stop-band.



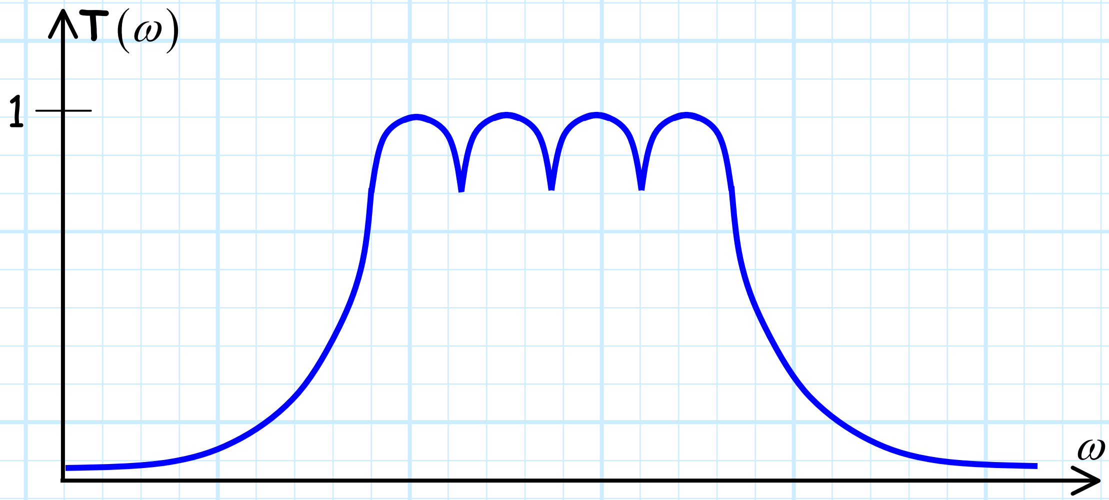
We find that we can make the roll-off **steeper** by accepting more **ripple**.

2. Chebychev

Chebychev filters are also known as **equal-ripple** filters, and have two primary characteristics

a) **Steep** roll-off (but not as steep as Elliptical).

b) Pass-band **ripple** (but not stop-band ripple).



We likewise find that the roll-off can be made steeper by **accepting** more ripple.

We find that Chebychev **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

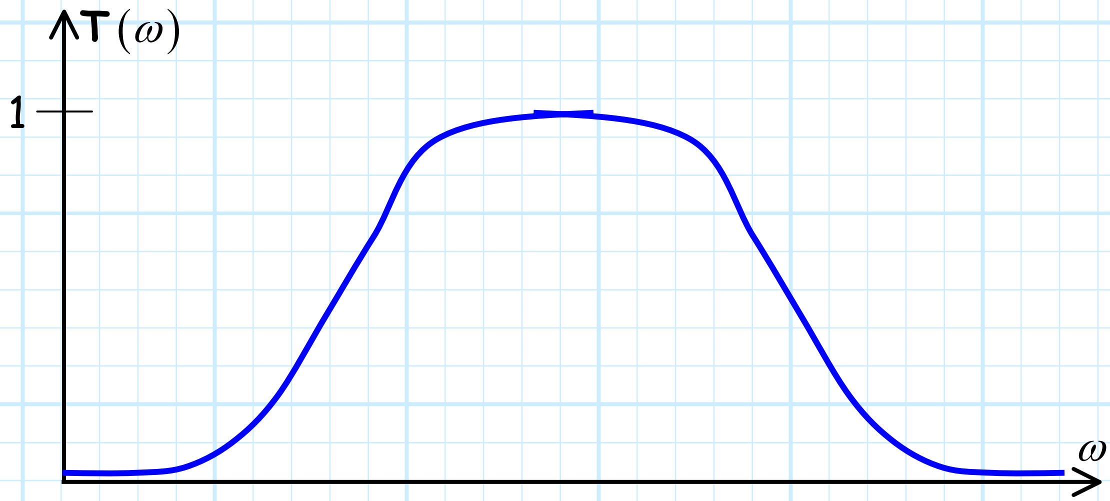
where k specifies the passband **ripple**, $T_N(x)$ is a Chebychev polynomial of **order** N, and ω_c is the low-pass **cutoff frequency**.

3. Butterworth

Also known as **maximally flat** filters, they have two primary characteristics

a) **Gradual** roll-off .

b) **No ripple**—not anywhere.



We find that Butterworth **low-pass** filters have a power loss ratio equal to:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

where ω_c is the low-pass **cutoff frequency**, and N specifies the **order** of the filter.

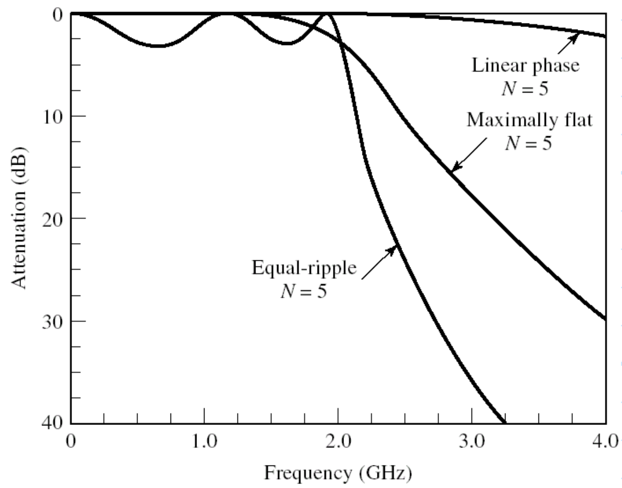
Q: *So we always chose* ***elliptical*** *filters; since they have the steepest roll-off, they are* ***closest*** *to ideal—****right****?*

A: Ooops! I forgot to talk about the **phase response** $\angle S_{21}(\omega)$ of these filters. Let's examine $\angle S_{21}(\omega)$ for each filter type **before** we pass judgment.

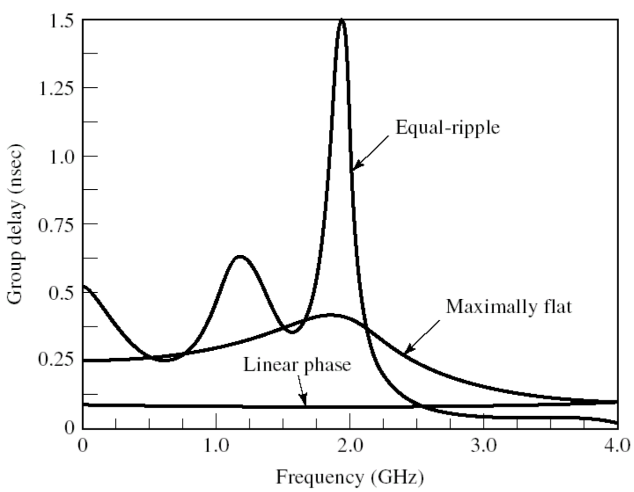
Butterworth $\angle S_{21}(\omega)$ → **Close** to linear phase.

Chebychev $\angle S_{21}(\omega)$ → **Not** very linear.

Elliptical $\angle S_{21}(\omega)$ → A big non-linear **mess**!



(a)



(b)

Thus, it is apparent that as a filter roll-off **improves**, the phase response gets **worse** (watch out for **dispersion!**).

→ There is **no** such thing as the "**best**" filter type!

Q: *So, a filter with **perfectly** linear phase is impossible to construct?*

A: No, it **is** possible to construct a filter with **near** perfect linear phase—but it will exhibit a **horribly** poor roll-off!

Now, for any **type** of filter, we can **improve** roll-off (i.e., increase stop-band attenuation) by **increasing the filter order** N. However, be aware that increasing the filter order likewise has these **deleterious** effects:

1. It makes **phase response** $\angle S_{21}(\omega)$ worse (i.e., more non-linear).

2. It increases filter **cost**, **weight**, and **size**.

3. It increases filter **insertion loss** (this is bad).

4. It makes filter performance more **sensitive** to temperature, aging, etc.

From a **practical** viewpoint, the **order** of a filter should typically be kept to $N < 10$.

Q: *So how do we take these polynomials and make real filters?*

A: Similar to matching networks and couplers, we:

1. Form a general circuit structure with **several** degrees of design freedom.

2. Determine the **general form** of the power loss ratio for these circuits.

3. Use our degrees of design freedom to **equate terms** in the general form to the terms of the **desired** power loss ratio polynomial.

Filter Realizations Using Lumped Elements

Our **first** filter circuit will be "**realized**" with lumped elements.

Lumped elements—we mean inductors *L* and capacitors *C*!

Since each of these elements are (ideally) perfectly **reactive**, the resulting filter will be **lossless** (ideally).

We will first consider two configurations of a **ladder circuit**:

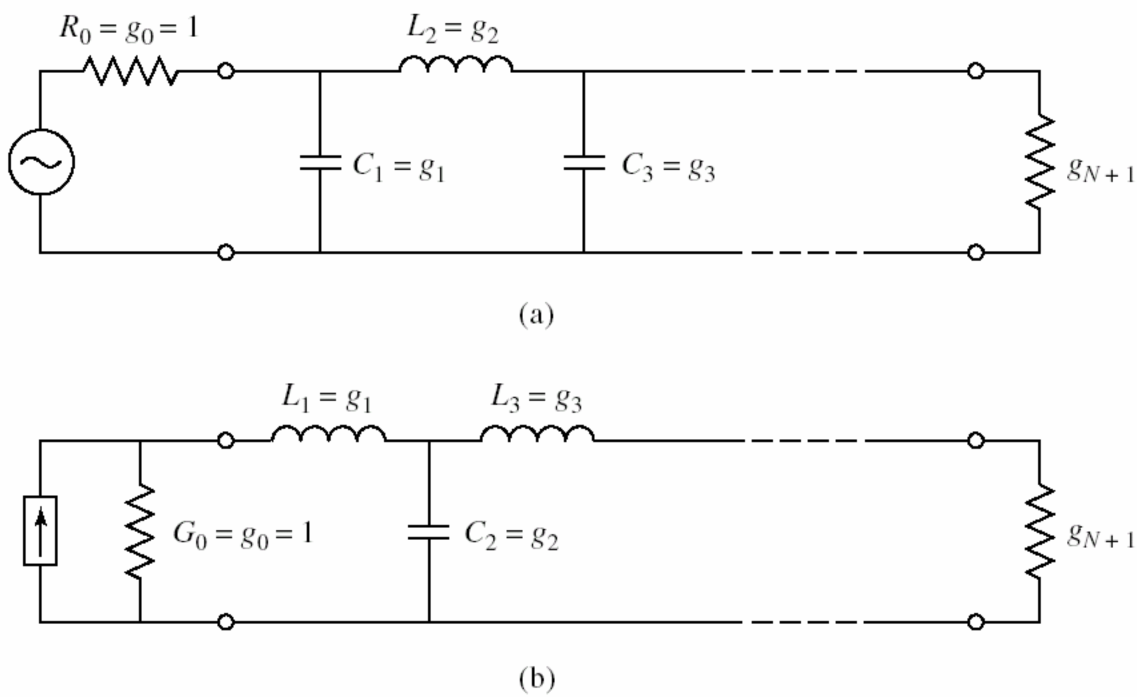


Figure 8.25 (p. 393)
Ladder circuits for low-pass filter prototypes and their element definitions. (*a*) Prototype beginning with a shunt element. (*b*) Prototype beginning with a series element.

Note that these two structures provide a **low-pass** filter response (evaluate the circuits at $\omega = 0$ and $\omega = \infty$!).

Moreover, these structures have N different **reactive elements** (i.e., N degrees of design freedom) and thus can be used to realize an ***N*-order** power loss ratio.

For example, consider the **Butterworth** power loss ratio function:

$$P_{LR}(\omega) = 1 + \left(\frac{\omega}{\omega_c}\right)^{2N}$$

Recall this is a **low-pass** function, as $P_{LR} = 1$ at $\omega = 0$, and $P_{LR} = \infty$ at $\omega = \infty$. Note also that at $\omega = \omega_c$:

$$P_{LR}(\omega = \omega_c) = 1 + \left(\frac{\omega_c}{\omega_c}\right)^{2N} = 1 + 1^{2N} = 2$$

Meaning that:

$$\Gamma(\omega = \omega_c) = \mathbf{T}(\omega = \omega_c) = \frac{1}{2}$$

In other words, ω_c defines the 3dB bandwidth of the low-pass filter.

Likewise, we find that this Butterworth function is **maximally flat** at $\omega = 0$:

$$P_{LR}(\omega = 0) = 1 + \left(\frac{0}{\omega_c}\right)^{2N} = 1$$

and:

$$\left.\frac{d^n\ P_{LR}(\omega)}{d\omega^n}\right|_{\omega=0} = 0 \qquad \text{for all } n$$

Now, we can determine the function $P_{LR}(\omega)$ for a lumped element ladder circuit of N elements using our knowledge of **complex circuit theory**.

Then, we can **equate** the resulting polynomial to the **maximally flat** function above. In this manner, we can determine the appropriate **values** of all inductors L and capacitors C!

An **example** of this method is given on pages 392 and 393 of your book. In this case, the filter is very **simple**—just **one** inductor and **one** capacitor. However, as the book shows, finding the solution requires quite a bit **complex algebra**!

Fortunately, your book likewise provides **tables** of complete Butterworth and Chebychev Low-Pass **solutions** (up to order 10) for the ladder circuits of figure 8.25—**no** complex algebra required!

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

Q: *What?! What the heck do these values g_n mean?*

A: We can use the values g_n to find the values of inductors and capacitors required for a given **cutoff frequency** ω_c and source resistance R_s (Z_0).

Specifically, we use the values of g_n to find ladder circuit **inductor** and **capacitor** values as:

$$L_n = g_n \left(\frac{R_s}{\omega_c} \right) \qquad\qquad C_n = g_n \left(\frac{1}{R_s\, \omega_c} \right)$$

where $n = 1, 2, \cdots, N$

Likewise, the value g_{N+1} describes the **load impedance**. Specifically, we find that **if** the **last** reactive element (i.e., g_N) of the ladder circuit is a **shunt capacitor**, then:

$$R_L = g_{N+1}\, R_s$$

Whereas, **if** the **last** reactive element (i.e., g_N) of the ladder circuit is a **series inductor**, then:

$$R_L = \frac{R_s}{g_{N+1}}$$

Note however for the **Butterworth** solutions (in Table 8.3) we find that $g_{N+1} = 1$ **always**, and therefore:

$$R_L = R_s$$

regardless of the last element.

Moreover, we note (in Table 8.4) that this (i.e., $g_{N+1} = 1$) is likewise true for the Chebychev solutions—provided that ***N* is odd**!

Thus, since we typically desire a filter where:

$$R_L = R_s = Z_0$$

We can use **any** order of **Butterworth** filter, or an **odd** order of **Chebychev**.

→ In other words, avoid **even order Chebychev** filters!