8.5 Filter Implementations

Reading Assignment: ***pp. 405-411***

Q: *So, we now know how to make any and all filters with* ***lumped*** *elements—but this is a* ***microwave*** *engineering course!*

You *said that lumped elements where difficult to make and implement at microwave frequencies.* ***You*** *said that distributed elements were used to make microwave components. So* ***how*** *do we make a filter with* ***distributed elements****!?!*

A: There are **many, many** ways to make microwave filters with distributed elements. Perhaps the most straightforward is to "**realize**" each individual lumped element with transmission line sections, and then insert these **approximations** in our lumped element solutions.

The first of these realizations is:

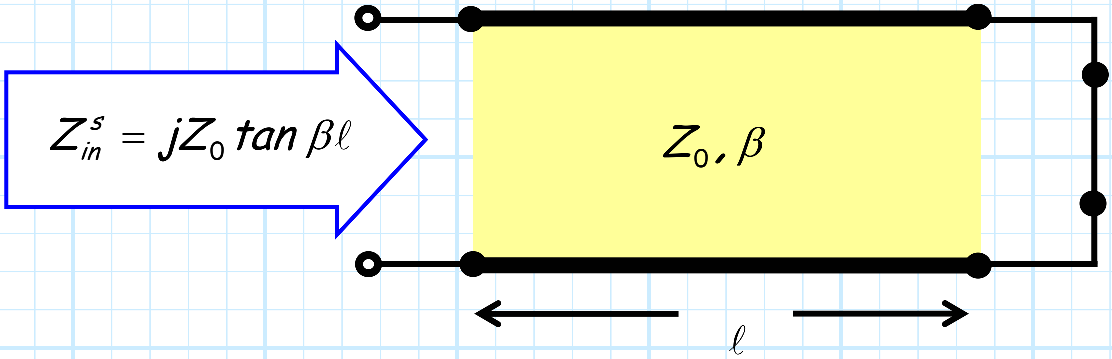
HO: RICHARD'S TRANSFORMATIONS

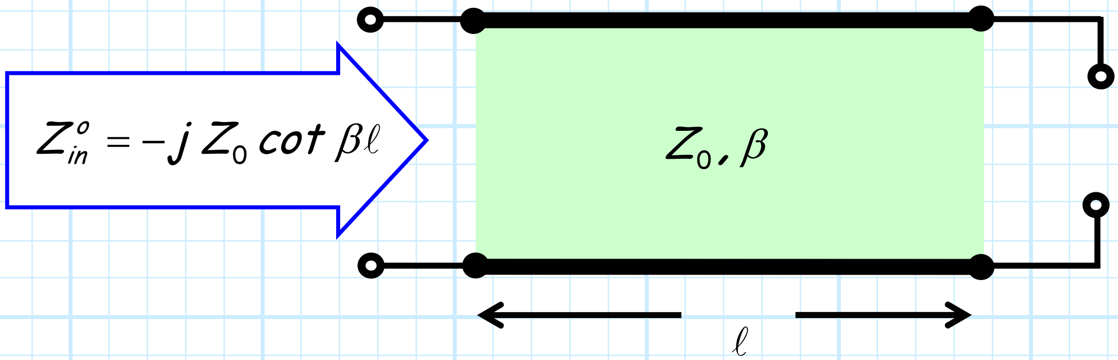
To easily **implement** Richard's Transforms in a microstrip or stripline circuit, we must apply one of **Kuroda's Identities**.

HO: KURODA'S IDENTITIES

Richard's Transformations

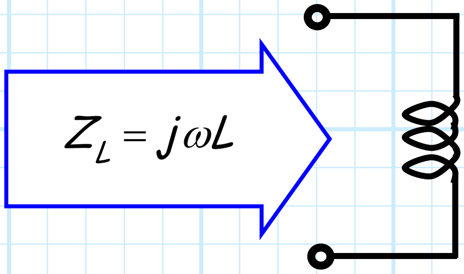
Recall the input impedances of short-circuited and open-circuited transmission line **stubs**.

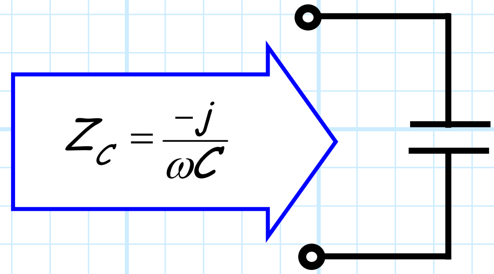




Note that the input impedances are purely **reactive**—just like **lumped** elements!

However, the reactance of lumped inductors and capacitors have a **much** different mathematical form to that of transmission line stubs:





In other words, the impedance of transmission line stubs and lumped elements (capacitors and inductors) are different functions with respect to **frequency**. Therefore, we can say in general that, for example:

$$Z_{in}^{s} \neq Z_L \qquad\qquad Z_{in}^{o} \neq Z_C$$

However, for a given lumped element (L or C) and a given stub (with a given Z_0 and length ℓ) the functions **will** be equal at precisely **one frequency**!

For example, there is one frequency—let's call it ω_c—that satisfies **this** equation for a given $L, Z_0,$ and ℓ:

$$\begin{aligned} j\omega_c L &= j\,Z_0 \tan \beta_c \ell \\ &= j\,Z_0 \tan\left[\frac{\omega_c}{v_p}\ell\right] \end{aligned}$$

or similarly satisfies **this** equation:

$$\begin{aligned} \frac{-j}{\omega_c C} &= -j\,Z_0 \cot \beta_c \ell \\ &= -j\,Z_0 \cot\left[\frac{\omega_c}{v_p}\ell\right] \end{aligned}$$

To make things easier, let's set the **length** of our transmission line stub to $\lambda_c/8$, where:

$$\lambda_c = \frac{v_p}{\omega_c} = \frac{2\pi}{\beta_c}$$

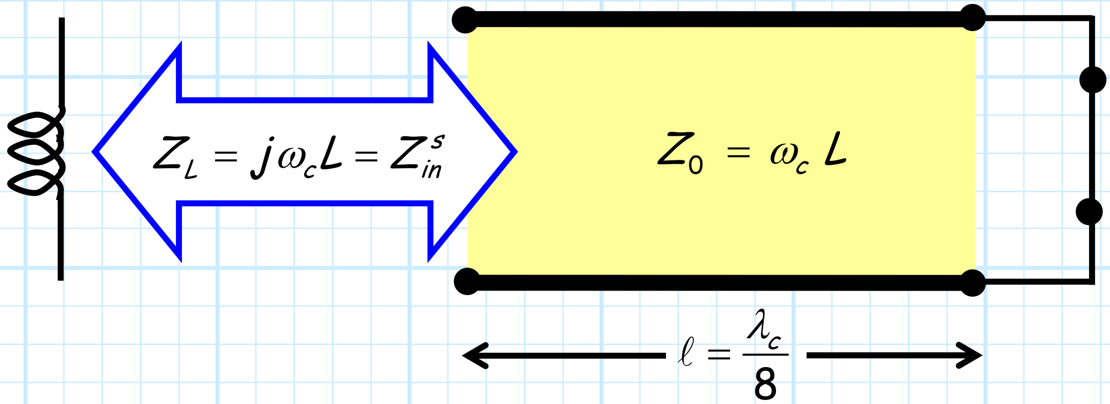
Q: *Why* $\ell = \lambda_c/8$ *?*

A: Well, for **one** reason, $\beta_c \ell = \pi/4$ and therefore $\tan(\pi/4) = 1.0$!

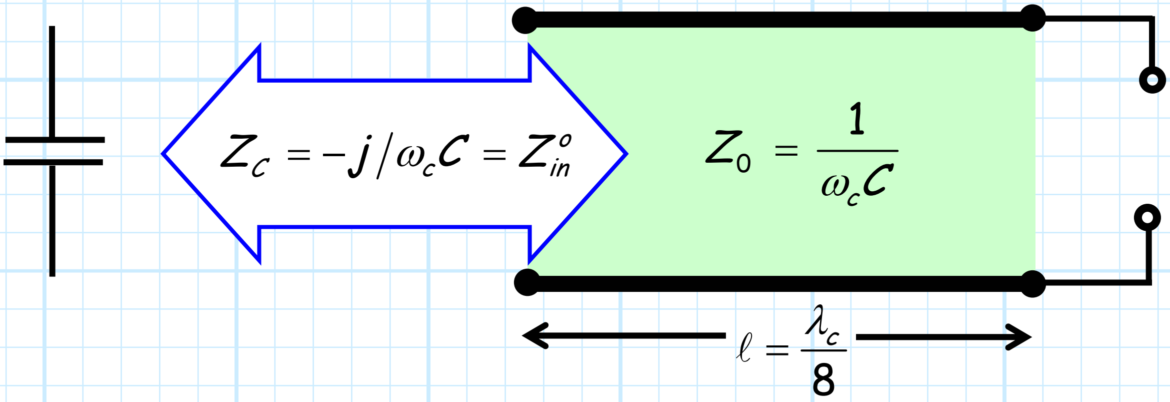
This of course greatly **simplifies** our earlier results:

$$j\omega_c L = j\,Z_0 \tan\left(\frac{\pi}{4}\right) = j\,Z_0 \qquad\qquad \frac{-j}{\omega_c C} = -j\,Z_0 \cot\left(\frac{\pi}{4}\right) = -j\,Z_0$$

Therefore, if we wish to build a **short-circuited** stub with the **same** impedance as an **inductor** L at frequency ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = \omega_c L$:



Likewise, if we wish to build an **open-circuited** stub with the **same** impedance as an **capacitor** C at frequency ω_c, we set the **characteristic impedance** of the stub transmission line to be $Z_0 = 1/\omega_c C$:



We call these two results **Richard's Transformations**.

However, it is important to remember that Richard's Transformations do **not** result in **perfect** replacements for lumped elements—the stubs **do not** behave like capacitors and inductors!

Instead, the transformation is **perfect**—the impedances are **equal**—at **only one frequency** (ω_c).

We can use Richard's transformations to replace the inductors and capacitors of a lumped element filter design. In fact, for **lowpass filter design**, the frequency ω_c is the filter's **cutoff frequency**.

Using these stubs to **replace** inductors and capacitors will result in a filter response **similar** to that of the lumped element design—a low pass filter with cutoff frequency ω_c.

However, the behavior of the filter in the **stopband** will be **very** different from the lumped element design. For example, at the (high) frequencies where the stub length becomes a **multiple** of $\lambda/2$, the filter response will be that of $\omega = 0$—near perfect **transmission**!

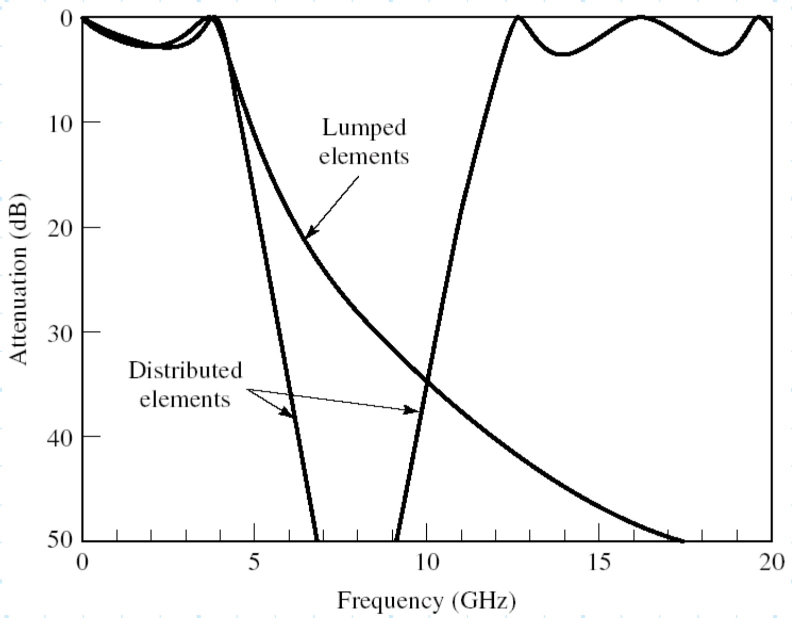


Figure 8.37 (p. 411) *Amplitude responses of lumped-element and distributed-element low-pas filter of Example 8.5.*

Q: *So **why** does the filter response match the lumped element response so **well** in the **passband**?*

A: To see why, we first note that the **Taylor Series approximation** for $tan\,\phi$ and $cot\,\phi$ when ϕ is small (i.e., $\phi \ll 1$) is:

$$tan\,\phi \approx \phi \quad \text{and} \quad cot\,\phi \approx \frac{1}{\phi} \quad \text{for} \quad \phi \ll 1$$

and ϕ is expressed in **radians**.

The **impedance** of our Richard's transformation shorted stub at some **arbitrary frequency** ω is:

$$
\begin{aligned}
Z_{in}^{s}(\omega) &= j\,Z_0 \tan\left(\beta \frac{\lambda_c}{8}\right) \\
&= j\,(\omega_c L)\tan\left(\frac{\omega}{v_p}\frac{\lambda_c}{8}\right) \\
&= j\,(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right)
\end{aligned}
$$

Therefore, when $\omega \ll \omega_c$ (i.e., frequencies in the **passband** of a low-pass filter!), we can **approximate** this impedance as:

$$
\begin{aligned}
Z_{in}^{s}(\omega) &= j\,(\omega_c L)\tan\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \\
&\approx j\,\omega_c L\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \\
&= j\omega L\left(\frac{\pi}{4}\right) \qquad \textit{when } \omega \ll \omega_c
\end{aligned}
$$

Compare this to a **lumped inductor** impedance:

$$
Z_L = j\omega L
$$

Since the value $\pi/4$ is **relatively** close to one, we find that the Richard's Transformation shorted stub has an input impedance **very** close to the lumped element inductor for **all** frequencies **less than** ω_c (i.e., all frequencies of the low-pass filter pass-band)!

Similarly, we find that the Richard's transformation **open-circuit** stub has an input impedance of **approximately**:

$$
\begin{aligned}
Z_{in}^{o}(\omega) &= \frac{-j}{\omega_c C} \cot\left(\frac{\omega}{\omega_c}\frac{\pi}{4}\right) \\
&\approx \frac{-j}{\omega_c C}\left(\frac{\omega_c}{\omega}\frac{4}{\pi}\right) \\
&= \frac{1}{j\omega C}\left(\frac{4}{\pi}\right) \qquad \text{when } \omega \ll \omega_c
\end{aligned}
$$

Again, when compared to the **lumped element capacitor** impedance:

$$
Z_C = \frac{1}{j\omega C}
$$

we find that results are approximately the **same** for all pass-band frequencies (i.e., when $\omega \ll \omega_c$).

Kuroda's Identities

We find that **Kuroda's Identities** can be very useful in making the implementation of Richard's transformations more **practicable.**

Kuroda's Identities essentially provide a list of **equivalent** two port networks. By equivalent, we mean that they have **precisely** the same scattering/impedance/admittance/transmission matrices.

In other words, we can **replace** one two-port network with its equivalent in a circuit, and the behavior and characteristics (e.g., its scattering matrix) of the circuit will **not** change!

Q: *Why would we want to do this?*

A: Because one of the equivalent may be more **practical** to implement!

For example, we can use Kuroda's Identities to:

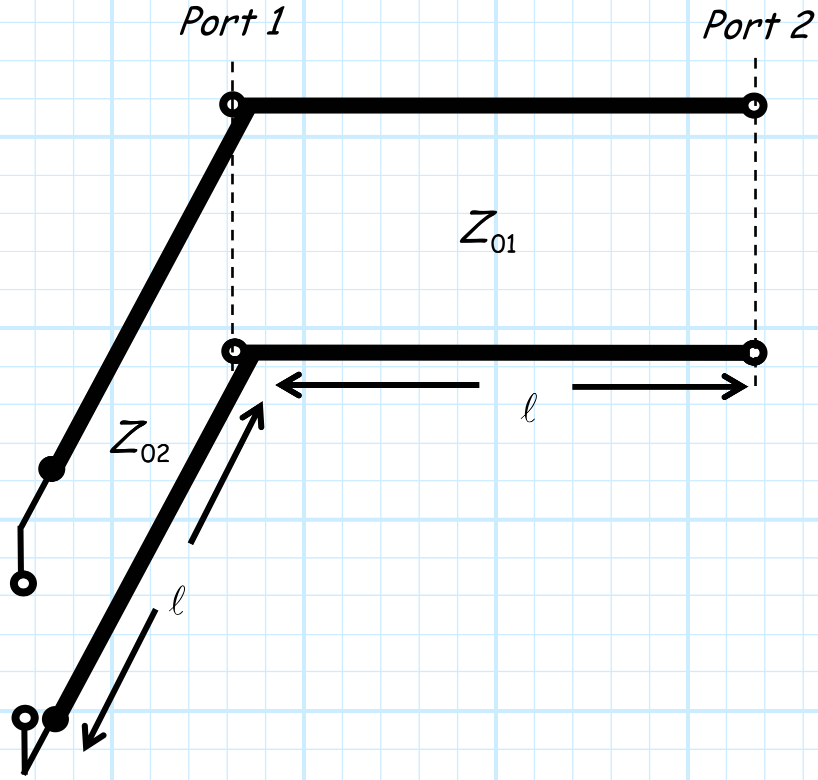
1) Physically **separate** transmission line stubs.

2) Transform series stubs into **shunt** stubs.

3) Change impractical **characteristic impedances** into more realizable ones.

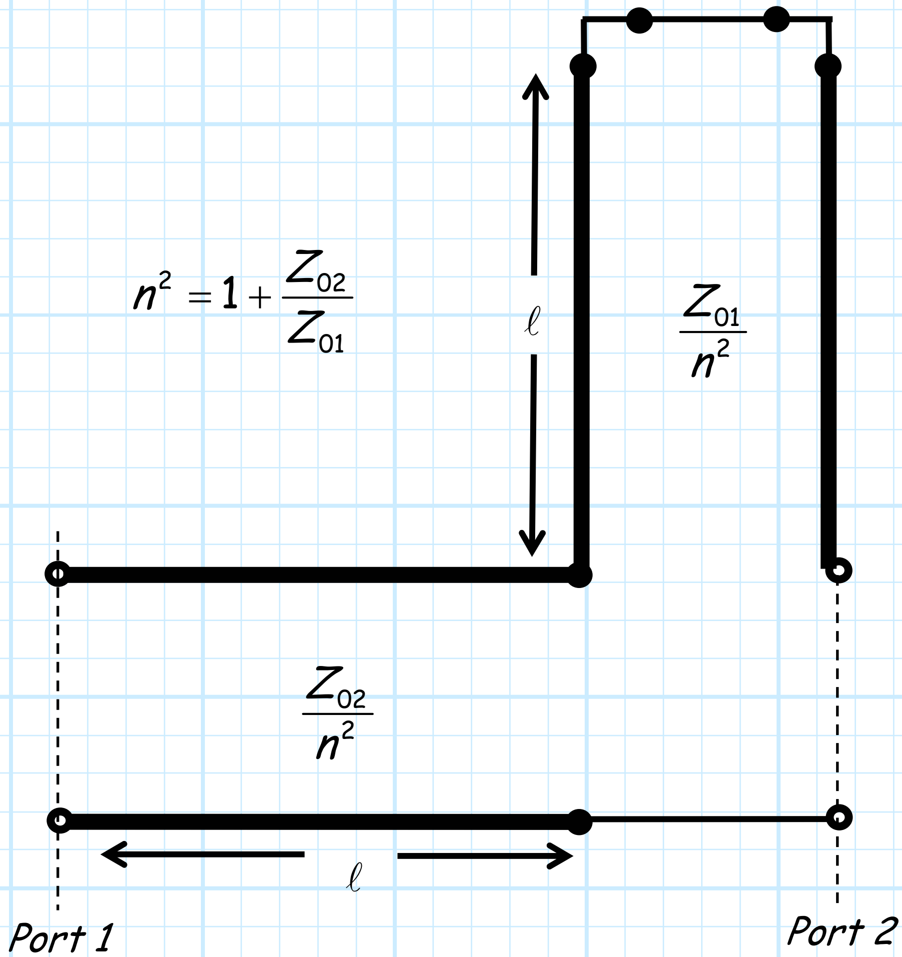
Four Kuroda's identities are provided in a very **ambiguous** and confusing table (Table 8.7) in your **book**. We will find the **first two** identities to be the most useful.

Consider the following two-port network, constructed with a length of transmission line, and an **open**-circuit **shunt** stub:



Note that the **length** of the stub and the transmission line are **identical**, but the characteristic **impedance** of each are **different**.

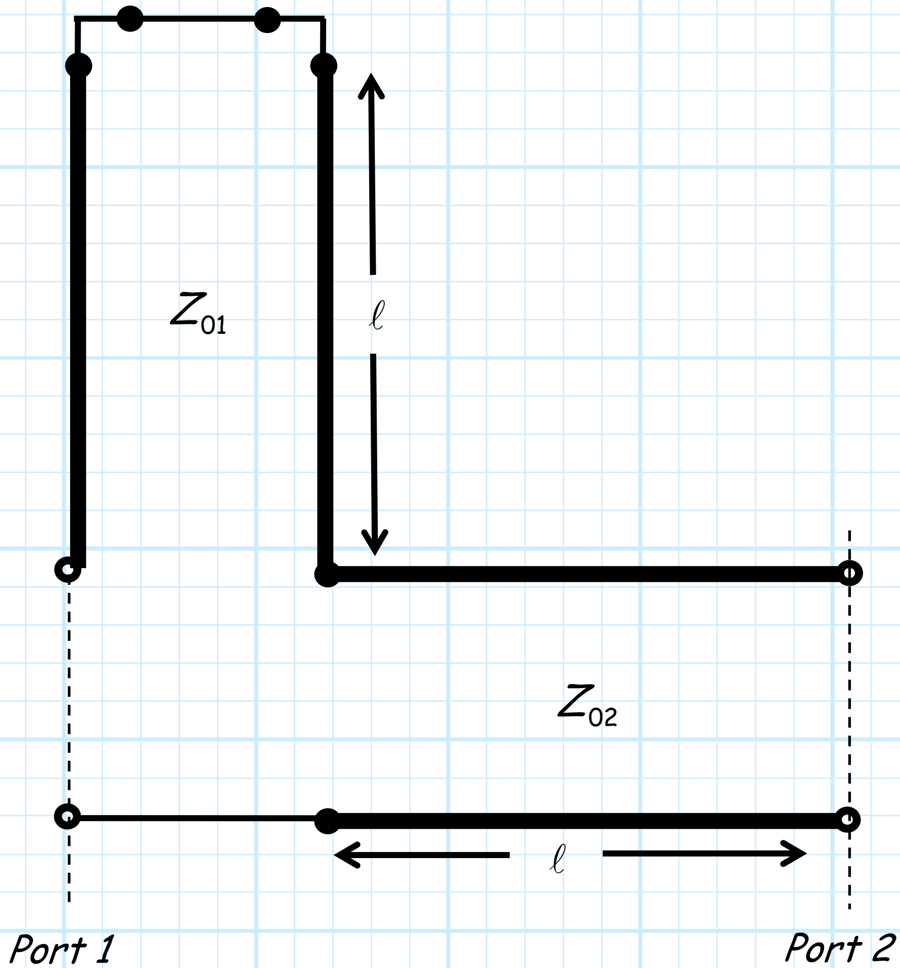
The **first Kuroda identity** states that the two-port network above is **precisely** the same two-port network as **this** one:



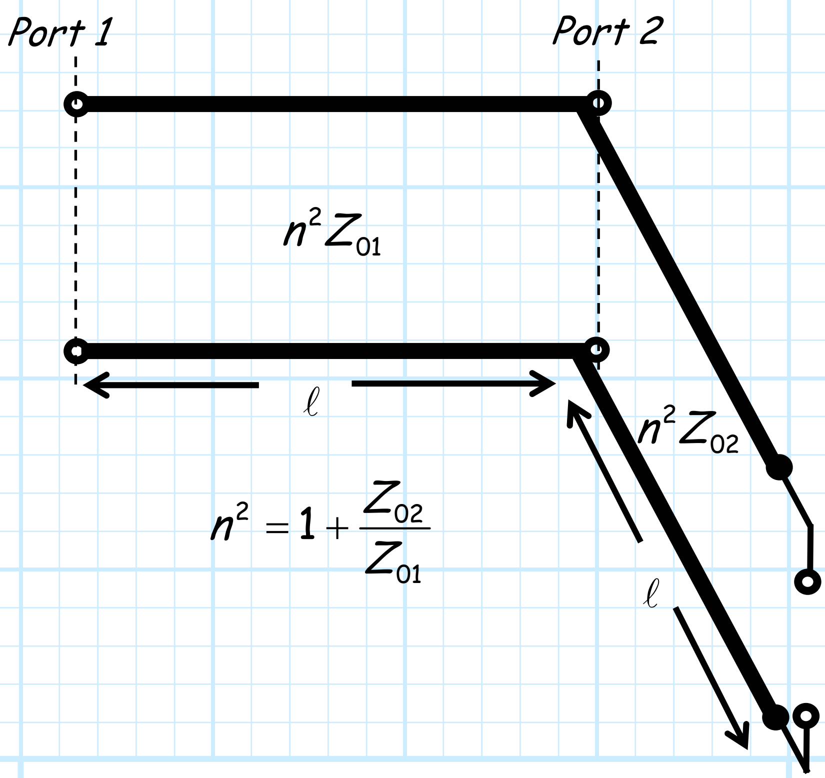
Thus, we can **replace** the first structure in some circuit with the one above, and the behavior that circuit will **not change** in the least!

Note this equivalent circuit uses a **short**-circuited **series** stub.

The **second** of Kuroda's Identities states that this two port network:



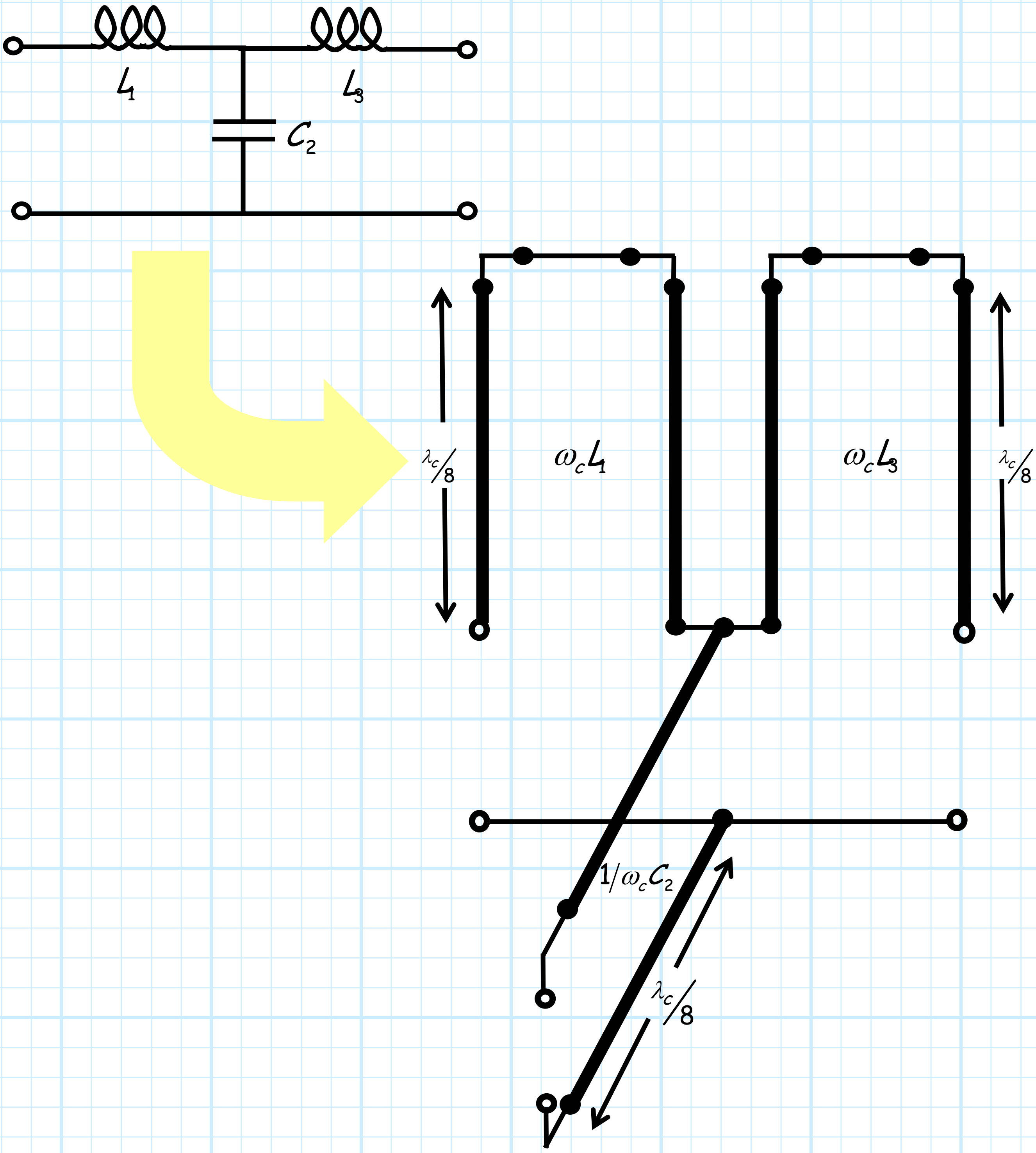
Is **precisely identical** to this two-port network:



$$n^2 = 1 + \frac{Z_{02}}{Z_{01}}$$

With regard to **Richard's Transformation**, these identities are useful when we replace the series **inductors** with **shorted stubs**.

To see **why** this is useful when implementing a **lowpass filter** with distributed elements, consider this third order filter example, realized using Richard's Transformations:

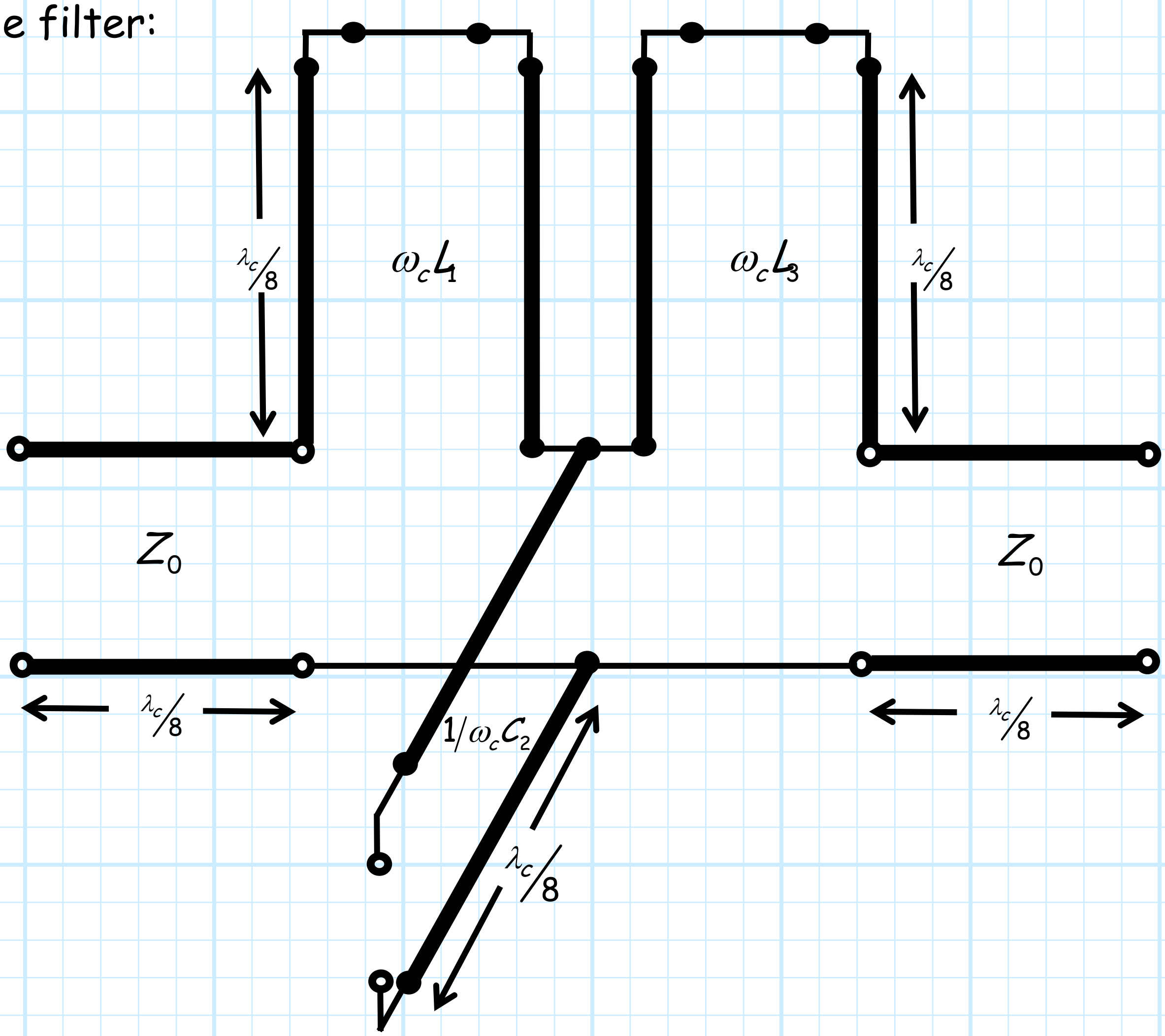


Note that we have a few **problems** in terms of implementing this design!

First of all the stubs are ideally **infinitely close** to each other—how do we build that? We could physically **separate** them, but this would introduce some transmission **line length** between them that would **mess up** our filter response!

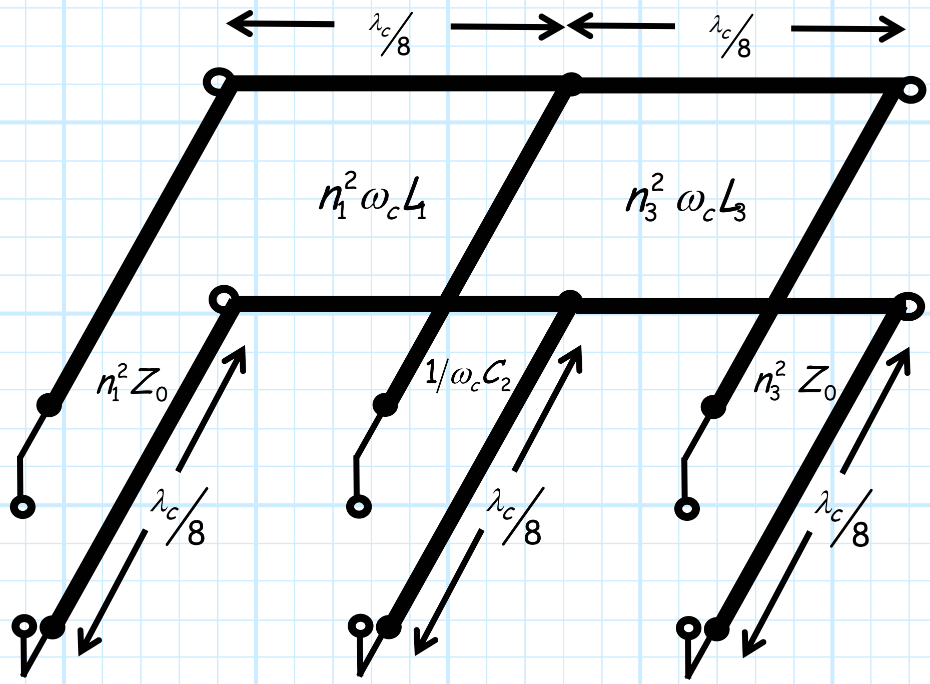
Secondly, **series** stubs are difficult to construct in microstrip/stripline—we like **shunt** stubs **much** better!

To solve these problems, we first **add** a short length of transmission line (Z_0 and $\ell = \lambda_c/8$) to the **beginning** and **end** of the filter:



Note adding these lengths only results in a **phase shift** in the filter response—the transmission and reflection functions will remain **unchanged**.

Now we can use the second of **Kuroda's Identities** to replace the **series** stubs with **shunts**:



where:

$$n_1^2 = 1 + \frac{Z_0}{\omega_c L_1} \qquad\qquad n_3^2 = 1 + \frac{Z_0}{\omega_c L_3}$$

Now **this** is a realizable filter! Note the **three stubs** are separated, and they are all **shunt** stubs.

Note that a specific **numerical** example (example 8.5) of this procedure is given on pp. 409-411 of **your book**.