

8.6 Stepped-Impedance Low-Pass Filters

Reading Assignment: pp. 412-416

Another distributed element realization of a lumped element low-pass filter designs is the **stepped-impedance** low-pass filter.

These filters are also know as “**hi-Z, low-Z**” filters, and we’re about to find out why!

HO: STEPPED-IMPEDANCE LOW-PASS FILTERS

Q: *Are there **other** methods for building microwave filters?*

A: There are a **bundle** of them!

All distributed elements (e.g., transmission lines, coupled lines, resonators, stubs) exhibit **some** frequency dependency. If we are clever, we can construct these structures in a way that their frequency dependency (i.e., $S_{21}(\omega)$) conforms to a desirable function of ω .

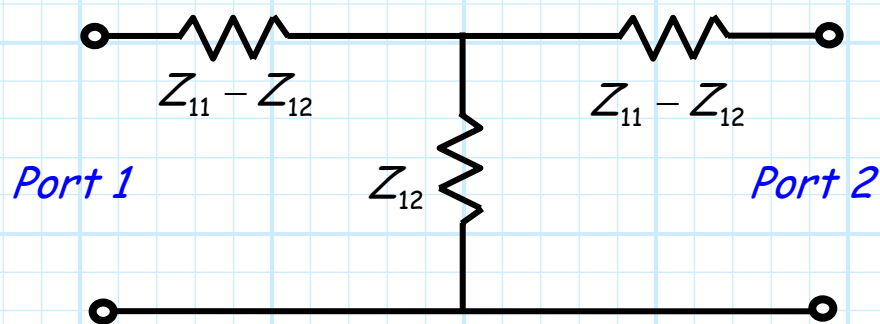
Other examples of filter realizations—ones applicable to **band-pass** filters—are discussed in sections **8.7** and **8.8** of your book.

Stepped-Impedance Low-Pass Filters

Say we know the impedance matrix of a **symmetric** two-port device:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix}$$

Regardless of the construction of this two port device, we can **model** it as a simple "T-circuit", consisting of three impedances:

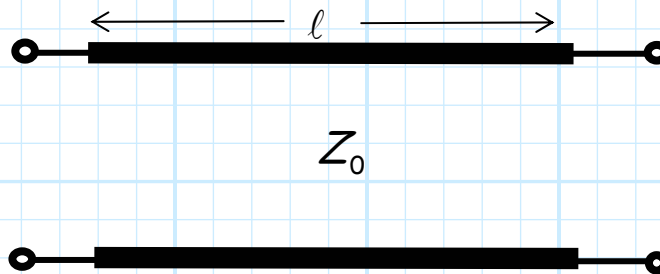


In other words, if the two **series impedances** have an impedance value equal to $Z_{11} - Z_{21}$, and the **shunt impedance** has a value equal to Z_{21} , the impedance matrix of this "T-circuit" is:

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{21} \\ Z_{21} & Z_{11} \end{bmatrix}$$

Thus, **any** symmetric two-port network can be modeled by this "T-circuit"!

For example, consider a length l of **transmission line** (a symmetric two-port network!):



Recall (or determine for yourself!) that the **impedance parameters** of this two port network are:

$$Z_{11} = Z_{22} = -jZ_0 \cot \beta l$$

$$Z_{12} = Z_{21} = -jZ_0 \csc \beta l$$

With a little **trigonometry**, ICBST :

$$Z_{11} - Z_{12} = j Z_0 \tan \left(\frac{\beta l}{2} \right)$$

Furthermore, if βl is **small**:

$$\sin \beta l \approx \beta l \quad \cos \beta l \approx 1 \quad \tan \beta l \approx \beta l$$

where βl is expressed in **radians**. Thus,

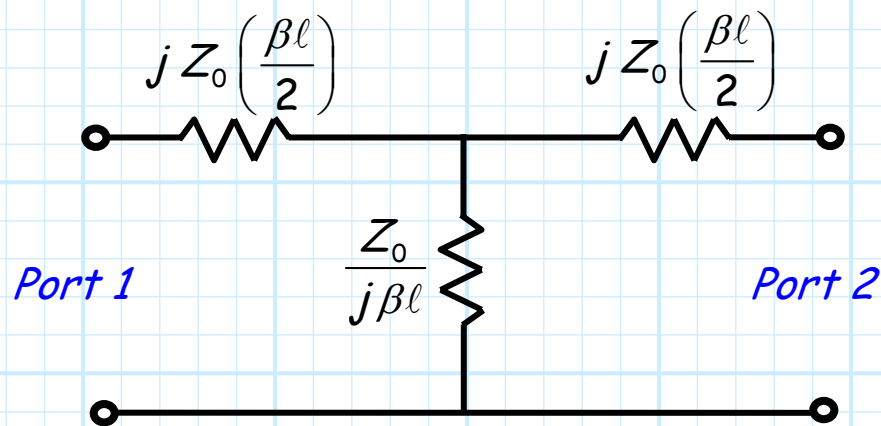
$$Z_{11} - Z_{12} \approx j Z_0 \left(\frac{\beta l}{2} \right)$$

and also:



$$Z_{12} = Z_{21} = -jZ_0 \csc \beta l \approx \frac{Z_0}{j\beta l}$$

Thus, an **electrically short** ($\beta l \ll 1$) transmission line can be **approximately modeled** with a "T-circuit" as:



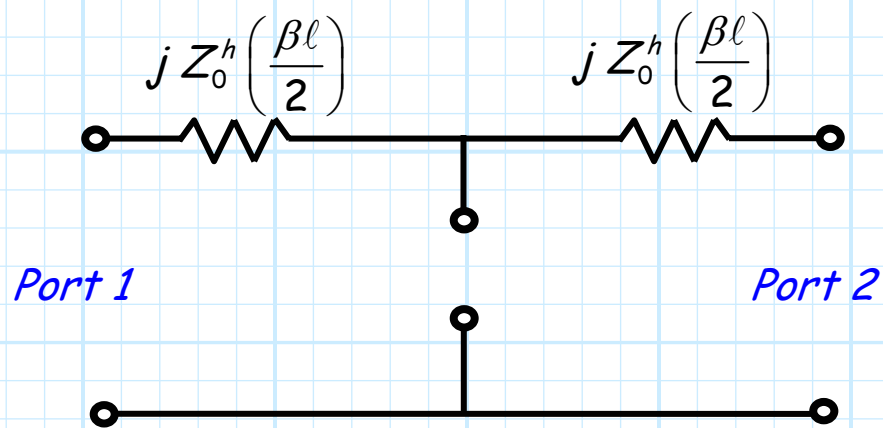
Now, consider also the case where the **characteristic impedance** of the transmission line is **relatively large**. We'll **denote** this large characteristic impedance as Z_0^h .

Note the **shunt impedance**, value $Z_0^h / j\beta l$. Since the **numerator** (Z_0^h) is **relatively large**, and the **denominator** ($j\beta l$) is **small**, the impedance shunt device is **very large**.

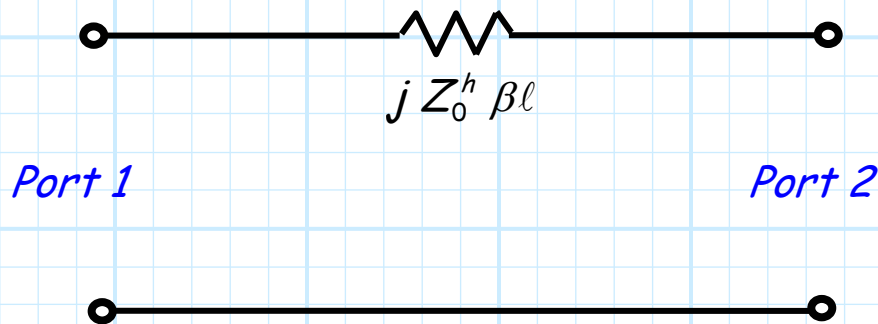
So large, in fact, that we can approximate it as an **open circuit!**

$$\frac{Z_0^h}{j\beta l} \approx \infty \quad \text{for } \beta l \ll 1 \text{ and } Z_0^h \gg Z_0$$

So now we have a further **simplification** of our model:



The remaining impedances are now in **series**, so the circuit can be further simplified to:



The equivalent circuit for transmission line with short electrical length ($\beta l \ll 1$) and large characteristic impedance Z_0^h ($Z_0^h \gg Z_0$).

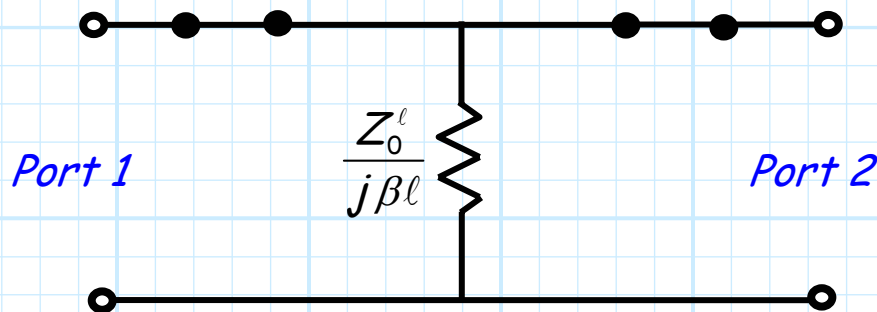
Now, consider the case where the **characteristic impedance** of the transmission line has a relatively **low value**, denoted as Z_0^l , where $Z_0^l \ll Z_0$.

Note the **series** impedance, values $j Z_0^l (\beta l / 2)$. Since both Z_0^l and βl are **small**, the product of the two is **very small**.

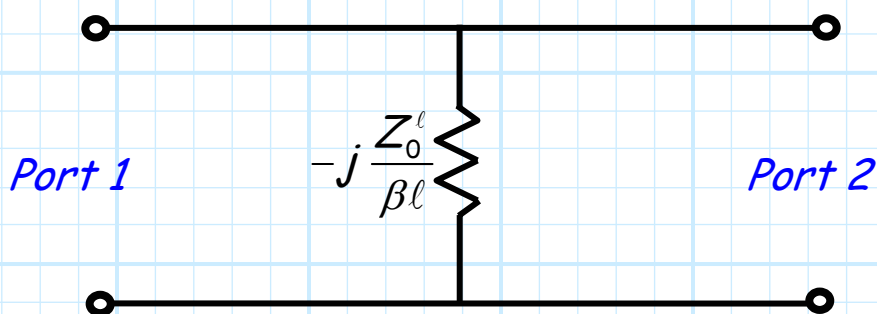
So small, in fact, that we can approximate it as a **short circuit!**

$$jZ_0^\ell \left(\frac{\beta\ell}{2} \right) \approx 0 \quad \text{for } \beta\ell \ll 1 \text{ and } Z_0^\ell \ll Z_0$$

So now we have **another simplification** of our model:



Which of course further simplifies to:



The equivalent circuit for transmission line with short electrical length ($\beta\ell \ll 1$) and small characteristic impedance Z_0^ℓ ($Z_0^\ell \ll Z_0$).

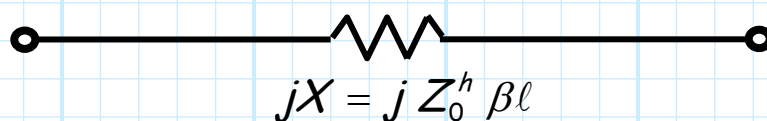
Q: *But, what does all this have to do with constructing a low-pass filter?*

A: **Plenty!** Recall that a lossless low-pass filter constructed with **lumped** elements consists of a "circuit ladder" of **series inductors** and **shunt capacitors!**

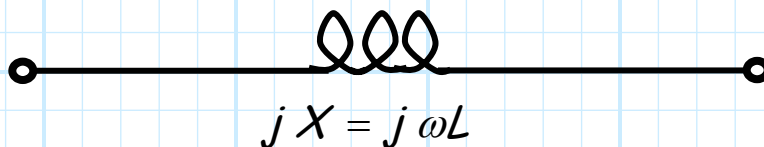
Q: So?

A: Look at the two **equivalent circuits** for an electrically short transmission line. The one with **large** characteristic impedance Z_0^h has the form of a **series inductor**, and the one with **small** characteristic impedance Z_0^l has the form of a **shunt capacitor**!

I.E.:



and:



are identical if:

$$j Z_0^h \beta l = j \omega L \Rightarrow Z_0^h \beta l = \omega L$$

Thus, the "series inductance" of our transmission line length is:

$$L = \frac{Z_0^h \beta \ell}{\omega}$$

Q: *Yikes! Our inductance appears to be a function of frequency ω . I assume we simply set this value to cutoff frequency ω_c , just like we did for Richard's transformation?*

A: **Nope!** We can **simplify** the result a bit more. Recall that $\beta = \omega/v_p$, so that:

$$L = \frac{Z_0^h \beta \ell}{\omega} = \frac{Z_0^h \ell}{v_p}$$

In other words, the **series impedance** resulting from our short transmission line is:

$$Z = j\omega \left(\frac{Z_0^h \ell}{v_p} \right)$$

Q: *Wow! This realization seems to give us a result that **precisely** matches an inductor at **all** frequencies—**right?***

A: **Not quite!** Recall this result was obtained from applying a few **approximations**—the result is **not** exact!

Moreover, one of these approximations was that the **electrical length** of the transmission line be **small**. This obviously **cannot** be true at **all** frequencies. As the signal frequency **increases**, so does the **electrical length**—our **approximate** solution will **no longer** be valid.

hus, this realization is accurate **only** for “**low frequencies**”—recall that was **likewise** true for **Richard's transformations!**

Q: *Low-frequencies? How low is low?*

A: Well, for our filter to provide a response that **accurately** follows the **lumped element** design, our approximation should be valid for frequencies **up to** (and including!) the **filter cutoff frequency** ω_c .

A general “**rule-of-thumb**” is that a **small electrical length** is defined as being **less than** $\pi/4$ radians. Thus, to maintain this small electrical length at frequency ω_c , our realization **must** satisfy the relationship:

$$\beta_c \ell = \frac{\omega_c L}{Z_0^h} < \frac{\pi}{4}$$

Note that this criterion is **difficult** to satisfy if the **filter cutoff frequency** and/or the **inductance value** L that we are trying to realize is **large**.

Our **only** recourse for these challenging conditions is to **increase** the value of **characteristic impedance** Z_0^h .

Q: *Is there some particular difficulty with increasing Z_0^h ?*



A: **Could be!** There is always a **practical** limit to how large (or small) we can make the **characteristic impedance** of a transmission line.

For example, a **large** characteristic impedance implemented in **microstrip/stripline** requires a **very narrow** conductor **width** W . But manufacturing tolerances, power handling capability and/or line loss (line resistance R increases as W decreases) place a **lower bound** on how narrow we can make these conductors!

However, assuming that we can satisfy the above constraint, we can approximately "**realize**" a **lumped inductor** of inductance **value** L by selecting the correct **characteristic impedance** Z_0^h and **line length** ℓ of our short transmission line:

$$L = \frac{Z_0^h \ell}{v_p}$$

Q: *For Richard's Transformation, we first set the stub length to a fixed value (i.e., $\ell = \lambda_c/8$), and then determined the specific characteristic impedance necessary to realize a specific inductor value L . I assume we follow the same procedure here?*

A: **Nope!** When constructing stepped-impedance low-pass filters, we typically do the **opposite!**

1) **First**, we select the value of Z_0^h , making sure that the short electrical length inequality is **satisfied** for the **largest inductance value** L in our lumped element filter:

$$Z_0^h > \frac{4 \omega_c L}{\pi}$$

This characteristic impedance value is typically used to realize **all** inductor values L in our low-pass filter, **regardless** of the actual value of inductance L .

2) Then, we determine the **specific lengths** ℓ_n of the transmission line required to realize **specific filter inductors values** L_n :

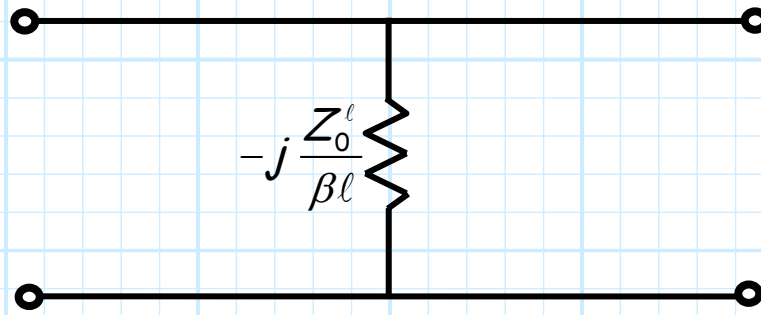
$$\ell_n = \left(\frac{v_p}{Z_0^h} \right) L_n$$

Q: *What about the **shunt capacitors**?*

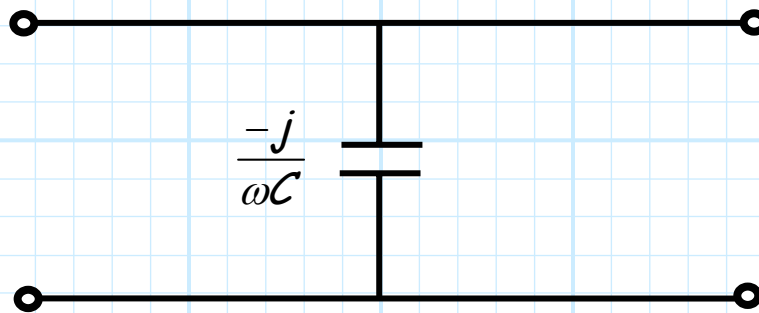
A: Almost forgot!

Recall the **low-impedance** transmission line provided a **shunt impedance** that matched a shunt capacitor:

I.E.:



and:



are identical if:

$$-j \frac{Z_0^l}{\beta l} = -j \frac{1}{\omega C} \Rightarrow \frac{\beta l}{Z_0^l} = \omega C$$

Thus, the "shunt capacitance" of our transmission line length is:

$$C = \frac{\beta l}{\omega Z_0^l}$$

But again using the fact that $\beta = \omega/v_p$:

$$C = \frac{l}{v_p Z_0^l}$$

And thus the **shunt reactance** of our transmission line realization is:

$$Z = \frac{-j}{\omega} \left(\frac{V_p Z_0^\ell}{\ell} \right)$$

Although this again **appears** to provide **exactly** the same behavior as a **capacitor** (as a function of frequency), it is likewise accurate **only** for **low frequencies**, where $\beta\ell < \pi/4$.

Thus from our realization **equality**:

$$\frac{\beta\ell}{Z_0^\ell} = \omega C$$

We can conclude that for our approximations to be valid at all frequencies **up to** the filter **cutoff frequency**, the following inequality **must** be valid:

$$\beta_c \ell = \omega_c C Z_0^\ell < \frac{\pi}{4}$$

Note that for **difficult** design cases where ω_c and/or C is **very large**, the line **characteristic impedance** Z_0^ℓ must be made **very small**.

Q: *I suppose there is likewise a problem with making Z_0^ℓ very small?*



A: Yes! In microstrip and stripline, making Z_0^ℓ **small** means making conductor width W **very large**. In other words, it will take up **lots of space** on our substrate. For most applications the **surface area** of the substrate is both **limited** and precious, and thus there is generally a **practical limit** on how wide we can make width W (i.e., how **low** we can make Z_0^ℓ).

However, assuming that we **can** satisfy the above constraint, we can approximately “**realize**” a **lumped capacitor** of inductance value C by selecting the correct **characteristic impedance** Z_0^ℓ and **line length** ℓ of our short transmission line:

$$C = \frac{\ell}{v_p Z_0^\ell}$$

The **design rules** for **shunt capacitor realization** are thus:

1) **First**, we select the value of Z_0^ℓ , making sure that the short electrical length inequality is **satisfied** for the **largest** capacitance value C in our lumped element filter:

$$Z_0^\ell < \frac{\pi}{4 \omega_c C}$$

This characteristic impedance value is typically used to realize **all** capacitor values C in our low-pass filter, **regardless** of the actual value of capacitance C .

2) Then, we determine the **specific lengths** l_n of the transmission line required to realize **specific** filter capacitor values C_n :

$$l_n = (Z_0^h v_p) C_n$$

An **example** of a low-pass, stepped-impedance filter design is provided on page 414-416 of **your** book (but of course, **you** already knew that—**right?**).

