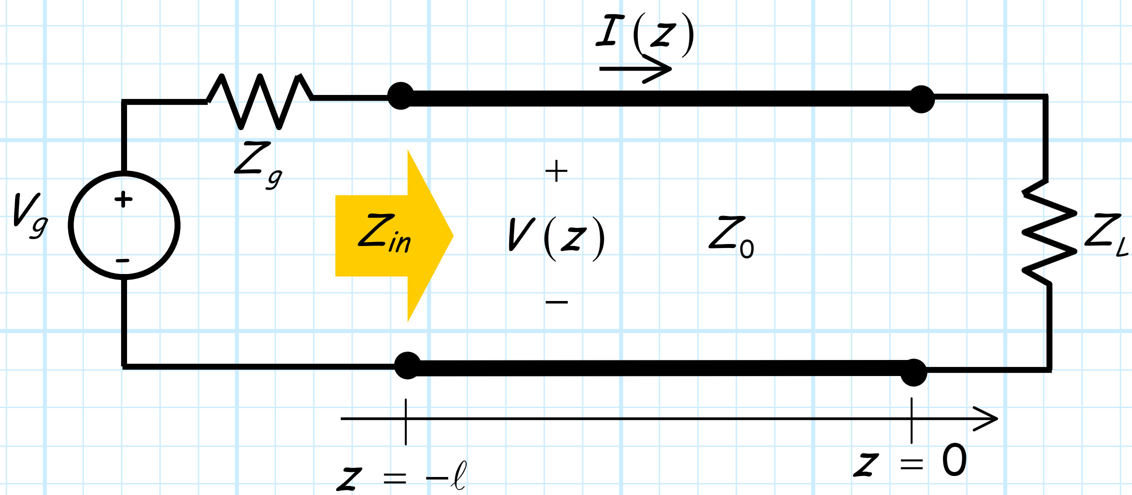
Delivered Power

Q: *If the purpose of a transmission line is to transfer* ***power*** *from a source to a load, then exactly how much power is* ***delivered*** *to Z_L for the circuit shown below ??*



A: We of course **could** determine V_0^+ and V_0^-, and then determine the power absorbed by the load (P_{abs}) as:

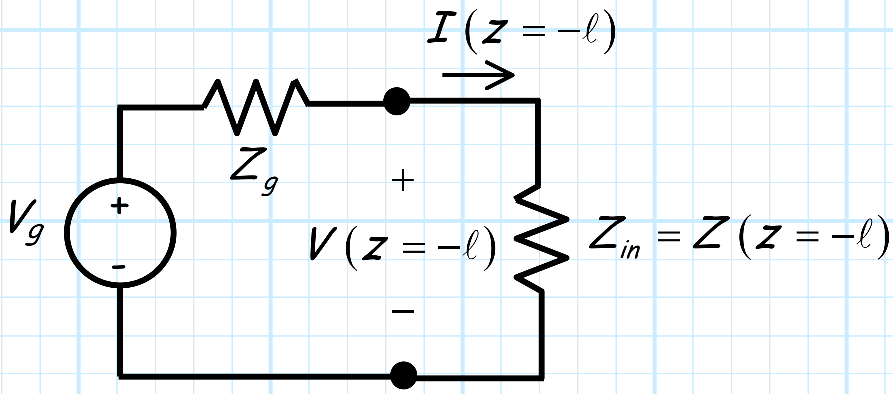
$$P_{abs} = \frac{1}{2}\text{Re}\{V(z=0)\, I^*(z=0)\}$$

However, if the transmission line is **lossless**, then we know that the power delivered to the load must be **equal** to the power "delivered" to the **input** (P_{in}) of the transmission line:

$$P_{abs} = P_{in} = \frac{1}{2}\text{Re}\{V(z=-\ell)\, I^*(z=-\ell)\}$$

However, we can determine this power **without** having to solve for V_0^+ and V_0^- (i.e., $V(z)$ and $I(z)$). We can simply use our knowledge of **circuit theory**!

We can **transform** load Z_L to the beginning of the transmission line (by direct calculation—or with a Smith Chart!), so that we can replace the transmission line with its **input impedance** Z_{in}:



Note by **voltage division** we can determine:

$$V(z=-\ell) = V_g \frac{Z_{in}}{Z_g + Z_{in}}$$

And from **Ohm's Law** we conclude:

$$I(z=-\ell) = \frac{V_g}{Z_g + Z_{in}}$$

And thus, the **power** P_{in} delivered to Z_{in} (and thus the **power** P_{abs} delivered to the load Z_L) is:

$$
\begin{aligned}
P_{abs} = P_{in} &= \frac{1}{2}\text{Re}\left\{V(z=-\ell)\, I^*(z=-\ell)\right\} \\
&= \frac{1}{2}\text{Re}\left\{V_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{V_g^*}{\left(Z_g + Z_{in}\right)^*}\right\} \\
&= \frac{1}{2}\frac{\left|V_g\right|^2}{\left|Z_g + Z_{in}\right|^2}\text{Re}\left\{Z_{in}\right\} \\
&= \frac{1}{2}\left|V_g\right|^2 \frac{\left|Z_{in}\right|^2}{\left|Z_g + Z_{in}\right|^2}\text{Re}\left\{Y_{in}\right\}
\end{aligned}
$$

Note that we could **also** determine P_{abs} from our **earlier** expression:

$$
P_{abs} = \frac{\left|V_0^+\right|^2}{2\, Z_0}\left(1 - \left|\Gamma_L\right|^2\right)
$$

But we would of course have to **first** determine V_0^+ (!):

$$
V_0^+ = V_g\, e^{-j\beta\ell} \frac{Z_0}{Z_0\left(1+\Gamma_{in}\right) + Z_g\left(1+\Gamma_{in}\right)}
$$