I/O-Efficient Algorithms for Problems on Grid-based Terrains

(Extended Abstract)

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Abstract

The potential and use of Geographic Information Systems (GIS) is rapidly increasing due to the increasing availability of massive amounts of geospatial data from projects like NASA's Mission to Planet Earth. However, the use of these massive datasets also exposes scalability problems with existing GIS algorithms. These scalability problems are mainly due to the fact that most GIS algorithms have been designed to minimize internal computation time, while I/O communication often is the bottleneck when processing massive amounts of data.

In this paper, we consider I/O-efficient algorithms for problems on grid-based terrains. Detailed grid-based terrain data is rapidly becoming available for much of the earth's surface. We describe $O(\frac{N}{B}\log_{M/B}\frac{N}{B})$ I/O algorithms for several problems on $\sqrt{N}$ by $\sqrt{N}$ grids for which only $O(N)$ algorithms were previously known. Here M denotes the size of the main memory and B the size of a disk block.

We demonstrate the practical merits of our work by comparing the empirical performance of our new algorithm for the *flow accumulation* problem with that of the previously best known algorithm. Flow accumulation, which models flow of water through a terrain, is one of the most basic hydrologic attributes of a terrain. We present the results of an extensive set of experiments on real-life terrain datasets of different sizes and characteristics. Our experiments show that while our new algorithm scales nicely with dataset size, the previously known algorithm "breaks down" once the size of the dataset becomes bigger than the available main memory. For example, while our algorithm computes the flow accumulation for the Appalachian Mountains in about three hours, the previously known algorithm takes several weeks.

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1 Introduction

Geographic Information Systems (GIS) is emerging as a powerful management and analysis tool in science and engineering, environmental and medical research, and government administration. The potential and use of GIS is rapidly increasing due to the increasing availability of massive amount of new remote sensing data from projects like NASA's Mission to Planet Earth [1]. For example, very detailed terrain data for much of the earth's surface is becoming publicly available in the form of a regular spaced grid with an elevation associated with each grid point. Data at 5 minute resolution (approximately 10 kilometers between grid points) and 30 arc second resolution (approx. 1 kilometer) is available through projects like ETOPO5 [3] and GTOPO30 [4], while data at a few seconds resolution (approx. 10 or 30 meters) is available from USGS [5]. Data at 1 meter resolution exists but is not yet publicly available. Also new projects are aimed at collecting larger amounts of terrain data. For example, NASA's Shuttle Radar Topography Mission [2] (SRTM) to be launched with the space shuttle this year, will acquire 30 meter resolution data for 80% of the earth's land mass (home to about 95% of the world's population). SRTM will collect around 10 terabytes of data!

While the availability of geospatial data at large resolutions increases the potential of GIS, it also exposes scalability problems with existing GIS algorithms. Consider for example a (moderately large) terrain of size 500 kilometers $\times$ 500 kilometers sampled at 10 meter resolution; such a terrain consists of 2.5 billion data points. When sampled at 1 meter resolution, it consists of 250 billion data points. Even if every data point is represented by only one word (4 bytes), such a terrain would be at least 1 terabyte in size. When processing such large amounts of data (bigger than main memory of the machine being used) the Input/Output (or *I/O*) communication between fast internal memory and slow external storage such as disks, rather than internal computation time, becomes the bottleneck in the computation. Unfortunately, most GIS algorithms, including the ones in commercial GIS packages, are designed to minimize internal computation time and consequently they often do not scale to datasets just moderately larger than the available main memory. In fact, the work presented in this paper started when environmental researchers at Duke approached us with the problem of computing the so-called *topographic convergence index* for the Appalachian Mountains. Their dataset contained about 64 million data points (approximately 800 km. $\times$ 785 km. at 100 meter resolution) with 8 bytes per point, totaling approximately 500 megabytes, and even on a fast machine with 512MB of main memory the computation took several weeks!

In this paper, we consider I/O-efficient algorithms for problems on grid-based terrains. We describe theoretically optimal algorithms for several such problems and present experimental results that show that our algorithm for the topographic convergence index problem scales very well with problem size and greatly outperforms the previously known algorithm on large problem instances. Our algorithm solves the problem on the Appalachian dataset in about three hours.

1.1 Digital Elevation Models and GIS algorithms on grid-based terrains

A Digital Elevation Model (DEM) is a discretization of a continuous function of two variables. Typically it represents the spatial distribution of elevations in a terrain, but in general it can represent any of a number of attributes. There are three principal ways of representing a DEM, namely, in a grid-based way as described above, using a so-called *triangulated irregular network* (TIN), or in a contour-based way [28]. Grid-based DEMs are most commonly used in practice, mainly because data are readily available in grid form from remote sensing devices and because the regular grid facilitates the development of very simple algorithms for many problems. On the other hand, TINs often use less space than grid-based DEMs—see e.g [28, 24] for a discussion of advantages and disadvantages of the different representations.

Grid-based DEMs are used in many areas, like topographic and hydrologic analysis, landscape ecology, and wildlife habitat analysis, to compute terrain indices which model processes like susceptibility of terrain to erosion, rainfall infiltration, drainage characteristics, solar radiation distribution, forest and vegetation structure, and species diversity [28]. One of the most basic hydrologic attributes

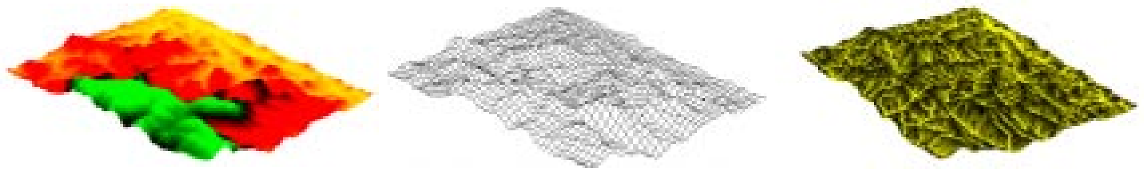


Figure 1: Terrain, its grid-based DEM, and a representation of its topographic convergence index.

of a terrain is *flow accumulation* which models flow of water through the terrain. To compute the flow accumulation of a terrain, one assumes that every grid point initially has one unit of flow (water) and that the flow (initial as well as incoming) at a grid point is distributed to downslope neighbor points proportional to the height difference between the point and the neighbors. The flow accumulation is the total amount of flow through each grid point of the terrain [32, 36].[1] Once flow accumulation has been computed for a terrain, some of the most important hydrologic attributes can be deduced from it. For example, the *drainage network* consists of all grid points for which the flow accumulation is higher than a certain threshold, and the *topographic convergence index*, which quantifies the likelihood of saturation, is defined as the (logarithm of the) ratio of the flow accumulation and the local slope in a point. Figure 1 shows a terrain, its grid-based DEM, and a graphical representation of its topographic convergence index.

There is a natural correspondence between a grid-based DEM and a grid graph where each vertex is labeled with a height. Here we define a grid graph as a graph with vertices on a $\sqrt{N}$ by $\sqrt{N}$ regular grid and where each vertex can only have edges to its eight neighbors. Note that a grid graph is not necessarily planar. The flow accumulation problem can naturally be described as a problem on a directed acyclic grid graph with edges directed from a vertex to its downslope neighbors. It turns out that many other common GIS problems on grid-based terrains correspond to standard, or minor variations of, graph theoretic problems on grid graphs. For example, Arc/Info [9], the most commonly used GIS package, contains functions that correspond to computing depth-first, breadth-first, and minimum spanning trees, as well as shortest paths and connected components on grid graphs.

1.2 Memory model and previous results on I/O-efficient algorithms

We will be working in the standard model for external memory with one (logical) disk [8, 23]:

$$\begin{aligned} N &= \text{number of vertices/edges in the problem instance;} \\ M &= \text{number of vertices/edges that can fit into internal memory;} \\ B &= \text{number of vertices/edges per disk block,} \end{aligned}$$

where $M < N$ and[2] $1 \leq B \leq \sqrt{M}$. An *Input/Output* (or simply *I/O*) involves reading (or writing) a block from disk into (from) internal memory. Our measure of performance of an algorithm is the number of I/Os it performs.

At least $\frac{N}{B}$ I/Os are needed to read N items, and thus the term $\frac{N}{B}$, and not N, corresponds to linear complexity in the I/O model. Aggarwal and Vitter [8] show that external sorting requires $\Theta(\frac{N}{B} \log_{M/B} \frac{N}{B}) = \Theta(\text{sort}(N))$ I/Os. Note that in practice the difference between an algorithm doing N I/Os and one doing $\text{sort}(N)$ I/Os can be very significant: Consider for example a problem involving $N = 256 \times 10^6$ four-byte items (1 GB), and a typical system with block size $B = 8 \times 10^3$ items (approx. 32 KB) and internal memory size $M = 64 \times 10^6$ items (approx. 256 MB). Thus $\frac{M}{B} = 8000$ blocks fit in memory and $\log_{M/B} \frac{N}{B} < 3$, and the speedup is more than three orders of magnitude. If the external

[1]Several definitions of flow accumulation on grids have been proposed; see e.g. [19, 29, 27, 36]. We use the definition proposed by Callaghan and Mark [32] and refined by Wolock [36], which seems to be the most appropriate and widely used definition. Some work has been done on extending the flow accumulation concept to TINs; see e.g. [24, 21, 37].

[2]Often it is only assumed that $B \leq M/2$ but sometimes, as in this paper, the very realistic assumption that the main memory is capable of holding B^2 elements is made.

memory algorithm takes 10 minutes to complete, the internal memory algorithm could use more than 150 hours, or equivalently, about 6 days!

I/O-efficient graph algorithms have been considered by a number of authors [10, 11, 17, 33, 25, 20, 31, 7, 6, 30, 22, 26]. If V is the number of vertices and E the number of edges in a graph, the best known algorithms for depth-first search, depending on the exact relationship between V and E, use $O(\frac{V}{M}\frac{E}{B}+V)$ [17] or $O\left((V+\frac{E}{B})\log\frac{V}{B}+\mathrm{sort}(E)\right)$ [25, 16] I/Os. For breadth-first search an $O(V+\mathrm{sort}(E))$ algorithm has been developed for the undirected case [30], while the best known algorithm for the directed case uses $O\left((V+\frac{E}{B})\log\frac{V}{B}+\mathrm{sort}(E)\right)$ [16]. The best known algorithms for connected components, minimum spanning tree, and single-source shortest path all work on undirected graphs and use $O\left(\mathrm{sort}(E)\log\log\frac{VB}{E}\right)$ [30], $O\left(\mathrm{sort}(E)\log B+\mathrm{scan}(E)\log V\right)$ [25], and $O\left(V+\frac{E}{B}\log_2\frac{V}{B}\right)$ [25] I/Os, respectively. For the special case of grid graphs the connected component and minimum spanning tree algorithms of [17] use $O(\mathrm{sort}(N))$ I/Os. For the other three problems the best known algorithms, even for grid graphs, all use $\Omega(N)$ I/Os. See recent surveys for a complete reference on the field [12, 35].

1.3 Our results

In the first part of this paper (Sections 2 and 3), we develop I/O-efficient algorithms for several standard graph theoretic problems on grid graphs, and thus for many common GIS problems on grid-based terrains, as well as for the problem of computing flow accumulation on a grid-based terrain. Our new algorithms for breadth-first search, single source shortest path, and flow accumulation use $O(\mathrm{sort}(N))$ I/Os. For all of these problems the previously best known algorithm use $O(N)$ I/Os. We also develop a new algorithm for connected components that uses $O(\frac{N}{B}+\mathrm{sort}(N)/\sqrt{M})$ I/Os. The previously best know algorithm for this problem uses $O(\mathrm{sort}(N))$ I/Os.

In the second part of the paper (Section 4), we demonstrate the practical merits of our work by comparing the empirical performance of our new flow accumulation algorithm with that of the previously best known algorithm. We present the results of an extensive set of experiments on real-life terrain data set of different sizes and characteristics. Our experiments show that while our new algorithm scales nicely with dataset size, the previously know algorithm "breaks down" once the size of the dataset becomes bigger than the available main memory.

2 Computing flow accumulation on grids

Recall the definition of flow accumulation; initially one unit of flow is placed on every grid point and then flow is continuously distributed to downslope neighbor points proportional to the height difference. The flow accumulation of a grid point is the total amount of flow through that point [32, 36].

Assume that the grid points are given in a $\sqrt{N}$ by $\sqrt{N}$ elevation matrix H and let H_{ij} denote the height of point (i,j). Let A be a similar matrix such that the after the flow accumulation computation, A_{ij} contains the flow accumulation of grid point (i,j). The standard flow accumulation algorithm works as follows [36]: First a list L is produced by sorting the elevations in H in decreasing order, and every entry in A is initialized to one unit of flow. Then the points are visited in decreasing order of elevation by scanning through L, while flow is distributed to downslope neighbors by updating entries in A. It is easy to see that when point (i,j) is processed, A_{ij} contains the correct final flow accumulation value for (i,j), since all other points (if any) pushing flow into (i,j) have already been processed. Note that conceptually the algorithm corresponds to sweeping the terrain top-down with a horizontal plane, "pushing" flow down the terrain in front of the plane.

The above algorithm uses $\Theta(N\log N)$ time on the initial sort and $\Theta(N)$ time on sweeping the terrain. The space use is linear. However, if the terrain is bigger than the available main memory and assuming virtual memory is used, the number of I/Os performed during the sweep is $\Theta(N)$. The reason for this is that the accesses to H and A exhibit very little temporal locality. At first glance it seems that the computation actually exhibits some sort of locality, because once a point (i,j) is accessed its eight neighbors are also accessed. However, globally the points are accessed in decreasing order of elevation and they are not necessarily well-clustered in this order—refer to Figure 2.

2.1 I/O-Efficient algorithm

The inferior I/O behavior of the standard flow accumulation algorithm is a result of two types of scattered disk access; accesses to A to obtain the flow of the grid point being processed and to update the flow of the neighbor points to which this point distributes flow, and accesses to H to obtain the elevations of the neighbors in order to decide how to distribute flow. The latter accesses can be eliminated simply by augmenting each point in L with the elevations of its eight neighbors. Using external sorting, L can be produced in $O(\text{sort}(N))$ I/Os. Note that we have to sort a dataset which is 9 times bigger than the original set.

The key observation that allows us to eliminate the scattered access to A is that when point (i, j) is processed and we want to update the flow of its neighbors, we already know at what "time" the neighbors are going to be processed, that is, when we will need to know how much flow (i, j) distributed to them. This happens when the sweep-plane reaches their elevation and we know their elevation from the information in L. The key idea is therefore to send the needed information (flow) "forward in time" by inserting it in an I/O-efficient priority queue. This idea is similar to *time forward processing* [17, 10].

Our new sweep algorithm works as follows. We maintain an external priority queue [10, 15] on grid points (i, j), with primary key equal to the elevation H_{ij} and secondary key equal to (i, j). Each priority queue element also contains a flow value (the flow "sent forward" to (i, j) from one of its neighbors). When we process a grid point (i, j) during the sweep (scan of L), its flow is distributed to downslope neighbors by inserting an element for each of them into the queue. The accumulated flow of (i, j) is found by performing *extract_max* operations on the priority queue. As a grid point can receive flow from multiple neighbors, we may need to perform *extract_max* operations several times to obtain its total flow. Flow sent to (i, j) from its upslope neighbors will be returned by consecutive *extract_max* operations as they all have the same key. If *extract_max* does not return flow for (i, j) (an element with key (H_{ij}, i, j)), it means that (i, j) does not have any upslope neighbors and we only distribute its initial one unit of flow. The processing of a grid point is illustrated in Figure 3. As the total number of priority queue operations performed is $O(N)$, and as external priority queues with $O(\frac{1}{B} \log_{M/B} \frac{N}{B})$ I/O bound per operation exist [10, 15], the sweep uses $O(\text{sort}(N))$ I/Os.

The above sweep algorithm does not directly output the flow accumulation matrix A. Instead, the flow accumulation of grid points are written to a list as they are calculated during the sweep. After the sweep, A can then be obtained by sorting this list in an appropriate way. We have the following.

Theorem 1 *The flow accumulation of a terrain stored as a $\sqrt{N}$ by $\sqrt{N}$ grid can be computed using $O(sort(N))$ I/Os, $O(N \log N)$ time, and linear space.*

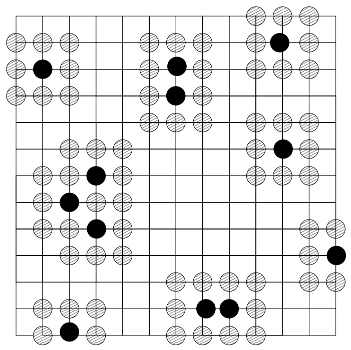


Figure 2: Points with the same height (black circles) are distributed over the terrain: Processing them after each other might result in loading most of the disk blocks storing the terrain.

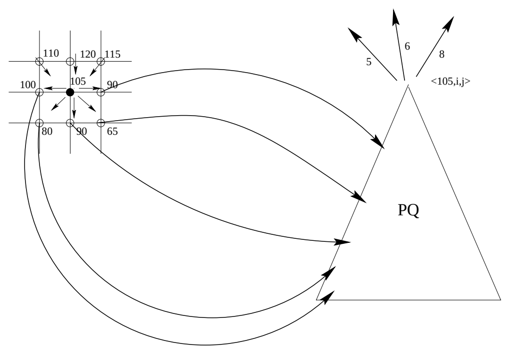


Figure 3: Point (i, j) receives flow $5 + 6 + 8 = 19$ from its three upslope neighbors and distributes it to its 5 downslope neighbors using a priority queue. (Point with height 90 gets $\frac{105-90}{100} \cdot 20$ units of flow).

3 I/O-Efficient algorithms on grid graphs

In the previous section we designed an $O(\text{sort}(N))$ algorithm for the flow accumulation problem. In this section we develop $O(\text{sort}(N))$ algorithms for breadth-first search (BFS) and single source shortest path (SSSP) on grid graphs. The previous best known algorithms use $\Omega(N)$ I/Os. We also develop an $O(\frac{N}{B} + \frac{\text{sort}(N)}{\sqrt{M}})$ algorithm for computing connected components on a grid graphs, compared to the previous $O(\text{sort}(N))$ algorithm. As discussed in the introduction, all these problems correspond to important GIS problems on grids.

In Section 3.1 we describe our new SSSP algorithm in detail. In Section 3.2 we then briefly discuss the two other algorithms. Further details will appear in the full paper.

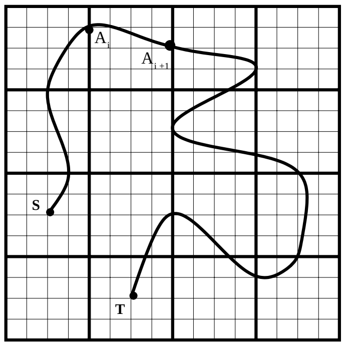
3.1 Single Source Shortest Paths

Consider a grid of size $\sqrt{N}$ by $\sqrt{N}$ divided into $O(N/M)$ subgrids of size $\sqrt{M}$ by $\sqrt{M}$ each. Our algorithm will rely on the following key lemma (proof omitted)—refer to Figure 4.

Lemma 1 *Consider a shortest path $\delta(s,t)$ between two vertices s and t in the grid. Let $\{s = A_0, A_1, A_2, ..., A_{k-1}, A_k = t\}$ denote the intersections of the path with the boundaries of the subgrids. Then the paths $A_i \rightarrow A_{i+1}$ induced by $\delta(s,t)$ in a subgrid is the shortest paths between A_i and A_{i+1} in that subgrid.*

Using lemma 1, our first idea for improving the general $O\left(V + \frac{E}{B}\log_2 \frac{V}{B}\right) = O\left(N + \frac{N}{B}\log_2 \frac{N}{B}\right)$ SSSP algorithm [25] in the case of a grid graph is via sparcification: We first replace each $\sqrt{M} \times \sqrt{M}$ subgrid with a full graph on the "boundary vertices". The resulting graph R has $\Theta(N/\sqrt{M})$ vertices and $\Theta(N)$ edges. The weight of an edge in R is the shortest path between the corresponding two boundary vertices, among all the paths within the subgrid—refer to Figure 4. The new edges and weights of R can easily be computed in $O(N/B)$ I/Os, simply by loading each subgraph in turn and use an internal memory all-pair-shortest-paths (APSP) algorithm to compute the weights of the new edges. Next we use the general SSSP algorithm on R to compute the length of the shortest paths from the source vertex s to all the boundary vertices. Using Lemma 1, we know that these paths are identical to the shortest paths in the original grid graph. Finally, we compute the shortest paths from s to all the vertices we removed, simply by loading each subgraph in turn (together with the shortest path lengths we found for the corresponding boundary vertices) and use an internal memory algorithm to compute for each vertex t in the subgrid the shortest path from s using the formula $\delta(s,t) = \min_v\{\delta(s,v) + \delta_{\text{subgrid}}(v,t)\}$, where v ranges over all the boundary vertices in t's subgrid, and where δ_{subgrid} denotes the shortest path within a subgrid.

It is easy to see that the above algorithm uses $O(\frac{N}{B}) + O(\frac{N}{\sqrt{M}} + \frac{N}{B}\log_2 \frac{N}{\sqrt{MB}}) + O(\frac{N}{B}) = O(\frac{N}{B}\log_2 \frac{N}{\sqrt{MB}})$ I/Os. We would like to improve this bound to $O(\text{sort}(N)) = O(\frac{N}{B}\log_{M/B} \frac{N}{B})$, that is, the base of the logarithm should be M/B instead of 2. The binary logarithm in our bound comes from the general SSSP algorithm by Kumar and Schwabe [25]. Their algorithm is a variant of Dijkstra's



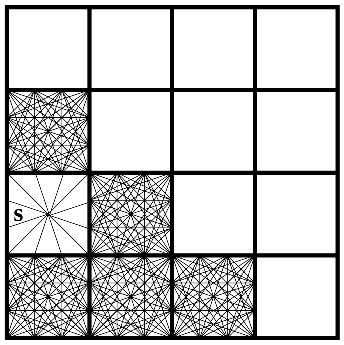


Figure 4: Dividing a grid in subgrids which fit in memory

algorithm [18], modified so that when processing a vertex v, no lookup is needed in order to determine the length of the currently known shortest path to neighbors of v (used to determine if a new shorter path has been found and thus if a *decrease_key* operation should be performed on the priority queue that controls the order in which vertices are processed). Such lookups would result in an E-term in the I/O bound. The priority queue used in their algorithm is an external version of a tournament tree, which supports *insert*, *delete_min*, and *decrease_key* in $O(\frac{1}{B}\log_2 \frac{V}{B})$ I/Os amortized.

The idea in the improvement of our algorithm is to avoid using *decrease_key* operations and thus to be able to use one of the $O(\frac{1}{B}\log_{M/B}\frac{N}{B})$-I/O-per-operation priority queues discussed earlier [10, 15]. In order to do so, we use Dijkstra's algorithm directly, instead of the algorithm by Kumar and Schwabe, and take advantage of the special structure of R to avoid the E term Kumar and Schwabe avoids using the external tournament tree. The main idea is to store R so that when we process a vertex we can access its $\Theta(\sqrt{M})$ neighbors in $O(\sqrt{M}/B)$ I/Os. To see how to do so, recall that the $\Theta(N/\sqrt{M})$ vertices of R consist of every $\sqrt{M}$th column and $\sqrt{M}$th row of the original grid graph. We store all $\Theta(N/M)$ "corner vertices" (vertices which are both on one of the columns and one of the rows) consecutively in $\Theta(N/(MB))$ blocks. Then we store the remaining vertices row by row and column by column, such that consecutive vertices are consecutive on disk. To obtain the neighbors of a vertex in R, we then just need to access six corner vertices and access the disk in five different places, reading $\Theta(\sqrt{M}/B)$ blocks in each of these places, using $O(\sqrt{M}/B + 11) = O(\sqrt{M}/B)$ I/Os in total.

Our SSSP algorithm on R now works as follows: Together with each vertex in the above representation we store a distance, which we initially set to ∞. We then use a slightly modified version of Dijkstra's algorithm where we maintain the invariant that for every vertex the distance in the above representation is identical to the distance stored in the priority queue controlling the algorithm (the shortest distance seen so far). We repeatedly perform a *delete_min* on the priority queue to obtain the next vertex v to process, and then we load all $\Theta(\sqrt{M})$ edges incident to v and all the $\Theta(\sqrt{M})$ neighbor vertices (and their current distances) into main memory using $O(\sqrt{M}/B)$ I/Os. Without any further I/O, we then compute which vertices need to have their distance updated. Finally, we write the updated distances back to disk and perform the corresponding updates (*decrease_key*) on the priority queue. One *decrease_key* operation is performed using a *delete* and an *insert* operation—we can do so since we now know the key (distance) of the vertex that we update. In total we perform $O(E) = O(N)$ operations on the queue and use $O(\sqrt{M}/B)$ on each of the $\Theta(N\sqrt{M})$ vertices, and thus our algorithm uses $O(\text{sort}(N))$ I/Os.

Theorem 2 *The single source shortest path problem on a $\sqrt{N}$ by $\sqrt{N}$ grid graph can be solved in $O(sort(N))$ I/Os.*

So far we have only described how to compute the lengths of the shortest paths. By adding an extra step to our algorithm, we can also produce the shortest path tree. Using a recent result due to Hutchinson, Maheshwari and Zeh [22], we can then construct a structure such that for any vertex t, the actual shortest path between s and t can be returned in ℓ/B I/Os, where ℓ is the length (number of vertices) of the path.

3.2 Other problems

Given our $O(\text{sort}(N))$ SSSP algorithm, we can also develop an $O(\text{sort}(N))$ BFS algorithm. We simply use the SSSP algorithm on the graph where all edges have weight one. The BFS numbering of the vertices can then be found in a few sorting steps. Details will appear in the full paper.

The problem of finding connected components (CC) of a grid graph can be solved efficiently in a way similar to the way we solved SSSP; we divide the grid into subgrids of size $\sqrt{M}$ by $\sqrt{M}$ and find connected components in each of them. We then replace each subgraph with a graph on the boundary vertices, where we connect boundary edges in the same connected component in a way such that $l-1$ edges are used to connect l vertices. The graph constructed in this way has size $N' = O(\frac{N}{\sqrt{M}})$ and because of its special structure we can find connected components in it in $O(\text{sort}(N'))$ I/Os. Using this information we can easily solve CC. Details will appear in the full paper.

Theorem 3 *The breadth-first search tree and the connected components of a $\sqrt{N}$ by $\sqrt{N}$ grid graph can be found in $O(sort(N))$ and $O(\frac{N}{B} + \frac{sort(N)}{\sqrt{M}})$ I/Os, respectively.*

4 Experimental results

4.1 Implementation

Our implementations are based on the TPIE (Transparent Parallel I/O Environment) system developed at Duke University [13, 34]. TPIE is a collection of templated functions and classes designed to facilitate easy and portable implementation of external memory algorithms. The basic data structure in TPIE is a *stream*, representing a list of objects of the same type. The system contains I/O-efficient implementations of algorithms for scanning, merging, distributing, and sorting streams. TPIE facilitated the implementation of the two flow accumulation algorithms discussed in section 2, as it contains all the building blocks we needed, except for the external priority queue. We implemented a simplified version of the external priority queue of Brodal and Katajainen [15], very similar to the implementation discussed in [14].

We implemented the flow accumulation algorithms described in Section 2 with one minor modification. In order to more accurately model water flow through horizontal portions of the terrain (where there are no downslope neighbors), as well as through small depressions of the terrain, our algorithms take as input not only the elevation grid, but also an auxiliary grid of the same dimensions which is used during the sweep. The latter grid contains values output from another terrain application and the use of these values does not affect our analysis of the algorithms. The modification is a result of the fact that our implementations are being used by global change researchers at Duke in real applications. Details will appear in the full version of this paper.

In our implementation of the standard algorithm (in the following called the *internal* algorithm), we first read the elevation and auxiliary grids from disk into a two-dimensional array in internal memory. For a grid of size N, the array is of size $8N$ bytes. Then we sort the elevations (along with information about the position of a given elevation in the grid) in decreasing order using the standard system quicksort implementation and store them in a file on disk; in the following we refer to this file as the *sweep file*. The sweep file is of size $16N$ bytes. We then allocate a flow accumulation array of size equal to the elevation array and sweep the terrain by reading the elevations from the sweep file, while accessing the elevation and flow arrays as described in Section 2. The amount of memory needed during both the sorting and sweeping phases of the algorithm is roughly $16N$ bytes and if this is more than the available main memory, we rely on the underlying virtual memory system to perform the necessary I/Os.

Our implementation of the I/O-efficient algorithm (in the following called the *external* algorithm) differs from the internal algorithm in that it uses TPIE's I/O-efficient sorting algorithm [8] to sort the sweep file and utilizes an external priority queue during the sweep. It also has to perform an extra sort after the sweep to produce the flow accumulation grid in the right format. As discussed in Section 2, each elevation in the sweep file is augmented with the elevations of its eight neighbors. Thus in the external algorithm the sweep file is of size $64N$ bytes. All the I/O performed during the external algorithm is controlled explicitly by TPIE and the virtual memory system is never in use. The external memory priority queue was implemented such that no I/O is performed if enough memory is available for it to be in main memory.

4.2 Experiments

Experimental setup. In order to investigate the efficiency of our algorithms we performed an extensive set of experiments on real-life terrain data of different sizes and characteristics, as well as with different main memory sizes. Our test data consisted of three 100 meter resolution datasets of *Puerto Rico*, *Hawaii*, and the *Appalachian* Mountains, as well as two 30 meter resolution datasets of the *Kaweah* River (in the foothills the Sierra Nevada Range of California), and a part of the Sequoia/Kings Canyon National Park in the *Sierra* Nevada region. The datasets vary in size from 1.6

million points or 12.8 MB (Kaweah) to 63.5 million points or 508 MB (Appalachian), and they also vary greatly in terms of elevation distribution. Intuitively, the latter should affect the performance of the sweep phase of both the internal and the external algorithms; especially of the external memory algorithm, because the size of the priority queue is determined by the elevation distribution. The characteristics of the five terrains are summarized in Table 1, which also contains the maximal size of the priority queue during the sweep. We performed experiments with main memory sizes of 512, 256, 128, 96, and 64 MB. We performed experiments with relatively low main memory sizes, as well as with more realistic sizes, in order to simulate flow accumulation computations on terrains of much larger size than the ones that were available to us. Such much bigger datasets will soon be available. If for example the Appalachian dataset was sampled at 30 instead of 100 meter resolution, it would be of size 5.5 gigabytes. Sampled at 10 meter it would be 50 gigabytes, and at 1 meter a mind-blowing 5 terabytes.

Dataset	**Surface Coverage**	**Grid size**	**Approx. size**	**Priority queue size**
Kaweah	34 x 42 kilometers	1163 x 1424	1.6 $\times 10^6$, 13MB	2 $\times 10^4$, 0.4MB
Puerto Rico	445 x 137 kilometers	4452 x 1378	5.9 $\times 10^6$, 47MB	14 $\times 10^4$, 2.8MB
Sierra	112 x 80 kilometers	3750 x 2672	9.5 $\times 10^6$, 76MB	19 $\times 10^4$, 3.8MB
Hawaii	678 x 4369 kilometers	6784 x 4369	28.2 $\times 10^6$, 225MB	9 $\times 10^4$, 1.8MB
Appalachian	847 x 785 kilometers	8479 x 7850	63.5 $\times 10^6$, 508MB	2.8 $\times 10^6$, 56MB

Table 1: Characteristics of terrain data sets.

We performed our experiments on a 450MHz Xeon Intel PII with 512 MB of main memory running FreeBSD 4.0, and with an external local (striped) disk array composed of 4 17GB Maxtor 91728D8 IDE drives. For each experiment the machine was rebooted with the indicated amount of main memory. In the case of the external algorithm, the TPIE limit on main memory use was set (somewhat conservatively) to 25, 50, 95, 200, and 350MB, in the experiments with 64, 96, 128, 256, and 512MB of main memory. The rest of the memory was reserved for the operating system.

Experimental results. The main results of our experiments are shown in Figure 5. As it can be seen, the external algorithm scales nicely for all main memory sizes. The internal algorithm on the other hand "breaks down" (or *thrashes*) when the memory use exceeds the main memory size; At 512MB the internal algorithm can handle all datasets except Appalachian. At 256MB the picture is the same, but the algorithm starts thrashing on the Hawaii dataset. At 96MB the algorithm thrashes on the Sierra dataset, while Hawaii slows down further. The algorithm can still handle Hawaii because of the "nice" distribution of elevations in that dataset (part of the grid is over water). At 64MB the algorithm can only handle the relatively small Kaweah and Puerto Rico datasets. It should be noted that on the small datasets the internal algorithm is slightly faster than the external algorithm. However, the external algorithm could easily be made "adaptive" so that it runs the internal algorithm if enough memory is available. This would result in the external algorithm always being at least as fast as the internal.

An interesting observation one can make from Table 1 is that in most of the experiments discussed so far, the priority queue used in the external algorithm actually fits in main memory. External memory is only really needed when processing the Appalachian dataset using only 64MB of main memory. This suggests that in most cases one could greatly simplify the implementation by just using an internal memory priority queue. One could even suspect that in practice an internal priority queue implementation would perform reasonably well, since the memory accesses incurred when operating on the queue are relatively local. In order to investigate this we performed experiments with the external algorithm modified to use an internal priority queue in the sweep while letting the OS handle the I/O. We ran the algorithm on the Appalachian dataset using 64 and 48MB of main memory and compared its performance to that of the unmodified external algorithm (that is, using the external

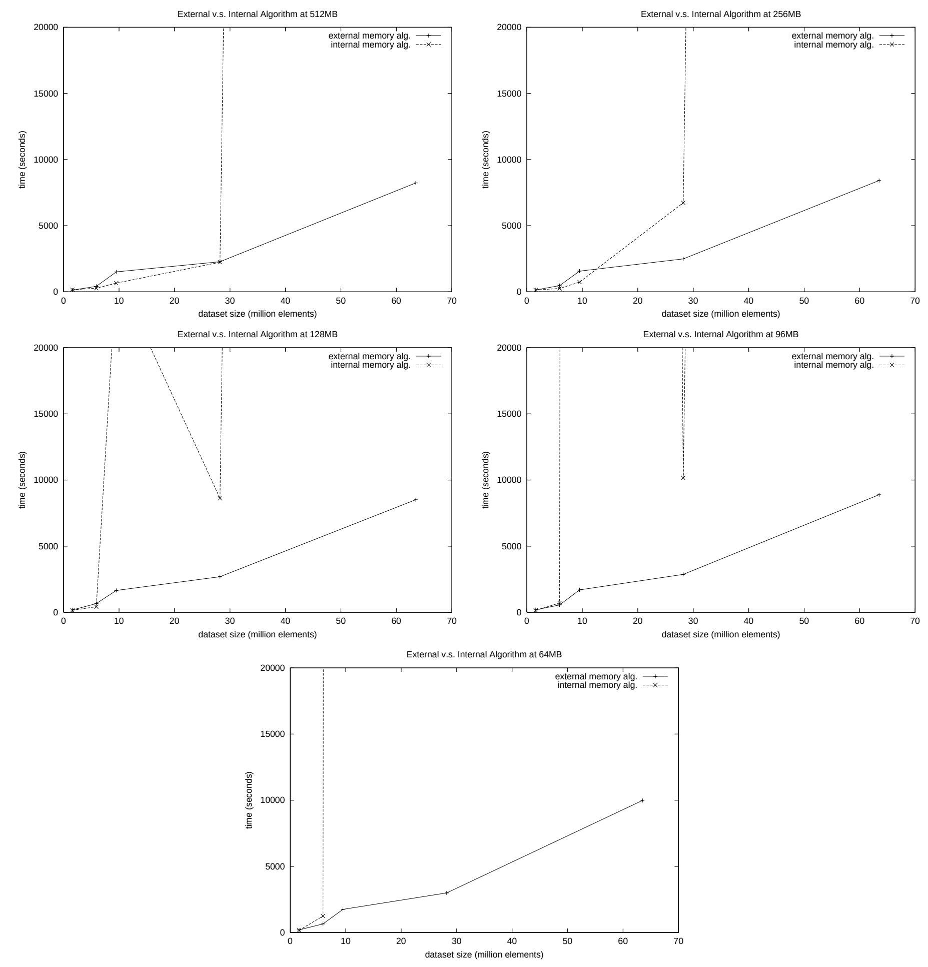


Figure 5: Comparison of standard (*internal*) and I/O-efficient (*external*) algorithms at different main memory size. Data size is in million elements and running time is in seconds.

priority queue). Our experiments showed that the external priority queue is needed on large terrains. Using 64MB of main memory the internal priority queue algorithm performance was only slightly worse (5844 seconds) than the external algorithms (4738 seconds). However, using only 48MB of main memory the external algorithm finished the sweep in 5056 seconds, only slightly worse than when using 64MB, while the internal priority queue implementation only finished 70% of the sweep in 13 hours.

Detailed running time analysis. Figure 5 only illustrates the overall results of a set of much more detailed experiments we performed with the two algorithms and the five data sets. The detailed results are included in Tables 5 and 6 in Appendix A. In these tables, the running time of each experiment is split into three parts; *input* time, *sweep* time and *output* time, which is further divided into CPU time and idle (I/O) time. For the internal algorithm, the input time is the time used to load the elevation grid into memory and to create and sort the sweep file. The sweep time is the time used to perform the sweep, and the output time is the time used to write the flow accumulation matrix to disk. For the external algorithm, the input time is the time used to read the elevation grid and create and sort the sweep file. The sweep time is the time used to perform the sweep (scan the sweep file performing *insert* and *extract_max* on the priority queue as needed), as well as to write the flow accumulation values to a stream and sort this stream once the sweep is completed. The output time is the time used to scan the sorted stream and write the flow accumulation to disk. As already discussed, the internal algorithm thrashes in several of the experiments on large datasets and we had to stop the program before it finished. The corresponding entries in the table are marked with ∞. Below we discuss the results in a little more detail.

First consider the internal algorithm (Table 5). The Kaweah dataset is small enough to fit in internal memory at all memory sizes and the algorithm consistently takes 60 seconds to complete. Its CPU utilization is around 85%. The Puerto Rico dataset fits in memory at 512MB and 256MB and completes in 100 seconds. When memory is reduced to 64MB, the CPU utilization drops from 87% to 19% and the performance is 5 times slower (480 seconds). Similarly, the Sierra Nevada dataset completes in 4 (5) minutes and 79% (66%) CPU utilization using 512MB (256MB) internal memory where it fits in memory. At 128MB, the running time jumps from 4 minutes to 4 hours, and the algorithm spends 98% of the time waiting for I/O. When the memory is reduced even more the algorithm thrashes—after 13 hours only half of the sweep had been completed. The Hawaii dataset fits in memory only at 512MB. As main memory gets smaller, the CPU utilization falls from 49% at 512MB to 2% at 64MB, and the running time increases from 15 minutes at 512MB to almost 8 hours at 64MB. The Appalachian dataset is beyond the limits of the internal algorithm—with 512MB of main memory we let it run for 4 days without finishing. Note that the thrashing of the internal algorithm is mainly due to sweeping. For instance, sorting the 425MB in the Hawaii dataset sweep file using 512MB of main memory takes 462 seconds (64% CPU), while at 64MB memory it takes 2668 seconds (7% CPU). Sweeping the same file takes 299 seconds using 512MB of main memory, but 24347 seconds using 64MB of main memory. The sorting performance is better due to the relatively good locality of quicksort.

Unlike the internal algorithm, the external algorithm scales (almost) linearly with memory decrease and maintains its CPU utilization constant (Table 6). For instance, the flow accumulation computation for the Appalachian dataset (which the internal algorithm thrashes on) takes 2 hours in total (79% CPU) using 512MB of main memory. Using 64MB of main memory is uses only 40 minutes more (68% CPU).

Finally, as already discussed briefly, it should be noted how the sweeping time depends not only on the size of the dataset but also on intrinsic terrain properties (quantified by the size of the priority queue in the external algorithm). Consider for example the performance of the internal algorithm on the Sierra and Hawaii dataset. With 512 and 256MB of main memory, both datasets fit in memory but the sweep of the Sierra dataset is significantly faster than the sweep of the Hawaii dataset, the reason being that the Hawaii sweep file is around six times bigger than the sierra sweep file. Using 128MB of main memory the algorithm thrashes on the Sierra dataset even though it is smaller than the Hawaii dataset. The reason for this is that that the Hawaii dataset is relatively easy to sweep (small priority queue size). This can also be seen from the CPU utilization during sweeping in the external algorithm.

5 Conclusions and open problems

In this paper we have developed I/O-efficient algorithms for several graph problems on grid graphs with applications to GIS problems on grid-based terrains. We have also shown that while the standard algorithm for the flow accumulation problem is severely I/O bound when the datasets get larger than the available main memory, our new I/O-efficient algorithm scales very well.

A number of interesting problems on grid graphs remain open. For example, if it is possible to develop an $O(\text{sort}(N))$ depth-first search algorithm and if connected components can be computed in $O(N/B)$ I/Os. For general graphs it remains an intriguing open problem if *any* graph problem on a general graph can be solved in $O(\text{sort}(E))$ I/Os.

In terms of computing flow accumulation, it would be interesting to develop more realistic models of flow of water over a terrain than the one used in current algorithms. Other interesting problems include developing models and algorithms for flow accumulation on DEMs stored as TINs.

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Appendix A

Mem	Time	Kaweah			Puerto Rico			Sierra Nevada			Hawaii			Appalachian		
MB	secs	cpu	idle	total	cpu	idle	total	cpu	idle	total	cpu	idle	total	cpu	idle	total
512	input	34	3	37	155	60	215	257	199	456	786	580	1366	1778	1289	3067
	sweep	76	1	77	97	10	107	872	14	886	303	45	348	3513	215	3728
	output	5	1	6	22	1	23	31	1	32	111	3	114	231	4	235
	TOTAL	115	5	120	274	71	345	1160	214	1374	1200	628	1828	5522	1508	7030
	%	96	4		79	21		84	16		66	34		79	21	
256	input	35	2	37	156	134	290	256	240	496	803	637	1440	1825	1344	3174
	sweep	77	11	88	98	14	112	870	17	887	324	97	421	3517	266	3783
	output	5	0	5	22	1	23	31	1	32	111	2	113	231	4	235
	TOTAL	117	13	130	276	149	425	1157	258	1415	1238	736	1974	5573	1619	7192
	%	90	10		65	35		82	18		63	37		77	23	
128	input	42	18	60	159	209	368	263	280	543	829	752	1581	1888	1337	3225
	sweep	77	11	88	98	15	113	882	25	907	329	120	449	3531	267	3798
	output	5	0	5	22	1	23	32	1	33	111	3	114	231	4	235
	TOTAL	124	29	153	279	225	504	1177	306	1483	1269	875	2144	5650	1608	7258
	%	81	19		55	45		79	21		59	41		78	22	
96	input	41	28	69	164	151	315	468	87	555	845	924	1769	1917	2185	4102
	sweep	77	10	87	104	14	118	885	35	920	331	131	462	3784	292	4076
	output	5	1	6	22	1	23	32	1	33	111	2	113	232	5	237
	TOTAL	123	39	162	290	166	456	1385	123	1508	1287	1057	2344	5933	2482	8415
	%	76	24		64	36		92	8		55	45		71	29	
64	input	43	32	75	169	226	395	277	293	570	873	950	1823	1978	2561	4539
	sweep	77	14	91	105	19	124	887	64	951	339	143	482	4228	510	4738
	output	5	1	6	22	1	23	32	1	33	111	2	113	231	5	236
	TOTAL	125	47	172	296	246	542	1196	358	1554	1323	1095	2418	6437	3076	9513
	%	73	27		55	45		77	23		55	45		68	32	

Figure 6: External memory algorithm experiments.

Mem	Time	Kaweah			Puerto Rico			Sierra Nevada			Hawaii			Apallachian		
MB	secs	cpu	idle	total	cpu	idle	total	cpu	idle	total	cpu	idle	total	cpu	idle	total
512	input	35	4	39	57	7	64	99	24	123	296	166	462		∞	∞
	sweep	7	6	13	9	6	15	78	32	110	24	275	299		∞	∞
	output	5	0	5	21	0	21	29	0	29	103	4	107			
	TOTAL	47	10	57	87	13	100	206	56	262	423	445	868		∞	∞
	%	82	18		87	13		79	21		49	51				
256	input	35	1	36	57	5	62	101	9	110	306	1527	1833			
	sweep	6	3	9	8	2	10	71	94	165	25	424	449		∞	∞
	output	5	0	5	21	0	21	29	1	30	104	81	185			
	TOTAL	46	4	50	86	7	93	201	104	305	435	2032	2467		∞	∞
	%	92	8		92	8		66	34		18	82				
128	input	35	4	39	60	50	110	103	323	426	307	2076	2383			
	sweep	7	3	10	8	5	13	124	11949	12073	27	343	370		∞	∞
	output	5	1	6	21	2	23	29	131	160	104	208	312			
	TOTAL	47	8	55	89	57	146	256	12403	12659	438	2627	3065		∞	∞
	%	85	15		61	39		2	98		14	86				
96	input	35	3	38	58	129	187	107	413	520	325	2052	2377			
	sweep	7	4	11	9	20	29		∞	∞	30	1268	1298		∞	∞
	output	5	0	5	21	12	33				104	181	285			
	TOTAL	47	7	54	88	161	249		∞	∞	459	3501	3960		∞	∞
	%	87	13		35	65		1	99		12	88				
64	input	36	6	42	62	234	296	109	493	602	199	2469	2668			
	sweep	7	3	10	9	101	110		∞	∞	101	24246	24347		∞	∞
	output	5	1	6	21	53	74				110	138	248			
	TOTAL	48	10	58	92	388	480		∞	∞	410	26853	27263		∞	∞
	%	83	17		19	81					2	98				

Figure 7: Internal memory algorithm experiments.