Tables, Priority Queues, Heaps

- Table ADT
  - purpose, implementations
- Priority Queue ADT
  - variation on Table ADT
- Heaps
  - purpose, implementation
  - heapsort

Table ADT

- A table in generic terms has M columns and N rows
  - each row contains a separate record
  - each column contains a different component, or field, of the same record
- Each table, or set of data, is also generally sorted, or accessed, by a key record component
  - a single set of data can be organized into several different tables, sorted according to different keys
- Another common terms is a dictionary, whose entries are records, inserted and accessed according to a *key value*
  - key may be a field in the record or not
  - may also be used as frontends for data base access

ADT Table – Example

- The ADT table, or dictionary
  - Uses a search key to identify its items
  - Its items are records that contain several pieces of data

City	Country	Population
Athens	Greece	2,500,000
Barcelona	Spain	1,800,000
Cairo	Egypt	9,500,000
London	England	9,400,000
New York	U.S.A.	7,300,000
Paris	France	2,200,000
Rome	Italy	2,800,000
Toronto	Canada	3,200,000
Venice	Italy	300,000

ADT Table – Operations

- A simple and obvious set of operations can be used for a wide range of program activities
  - Create and Destroy Table instance
  - Determine the number of items including zero
  - Insert an item in a table using a key value
  - Delete an item with a given key value
  - Retrieve an item with a given key value
  - Retrieve the items in the table (sorted or unsorted)
- Entries with identical key values maybe forbidden, but can be handled with a little imagination

The ADT Table

- **void tableInsert(ItemType& item)**:
  - store item under its key
- **boolean tableDelete(KeyType key_value)**:
  - delete item with key == key_value, if present
- **ItemType* tableRetrieve(KeyType key_value)**:
  - return pointer to item with key==key_value
- **void traverseTable(Functor visitor):**
  - Functor: a function-object, much like a fn pointer
  - visitor is executed for each node in table

Table
items
createTable() *destroyTable()* *tableIsEmpty()* *tableLength()* *tableInsert()* *tableDelete()* *tableRetrieve()* *traverseTable()*

The ADT Table

- Our table assumes distinct search keys
  - other tables could allow duplicate search keys
- The `traverseTable` operation visits table items in a specified order
  - one common order is by sorted search key
  - a client-defined visit function is supplied as an argument to the traversal
    - called once for each item in the table

Selecting an Implementation

- Linear implementations: Four categories
  - Unsorted: array based or pointer based
  - Sorted (by search key): array based or pointer based

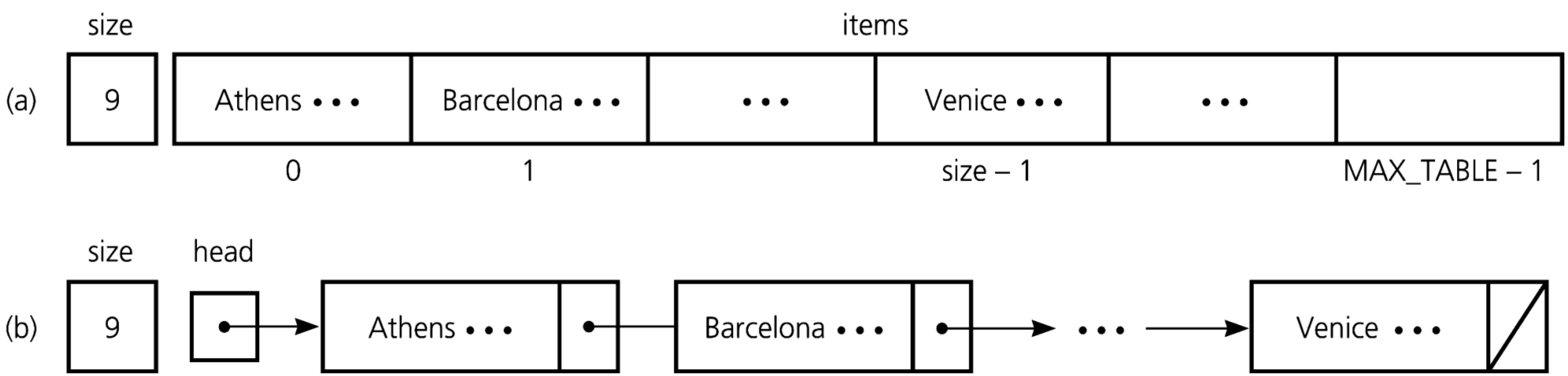


Figure 11-3 The data members for two sorted linear implementations of the ADT table for the data in Figure 11-1: (a) array based; (b) pointer based

Selecting an Implementation

- Nonlinear implementations
  - Binary search tree implementation
    - Offers several advantages over linear implementations

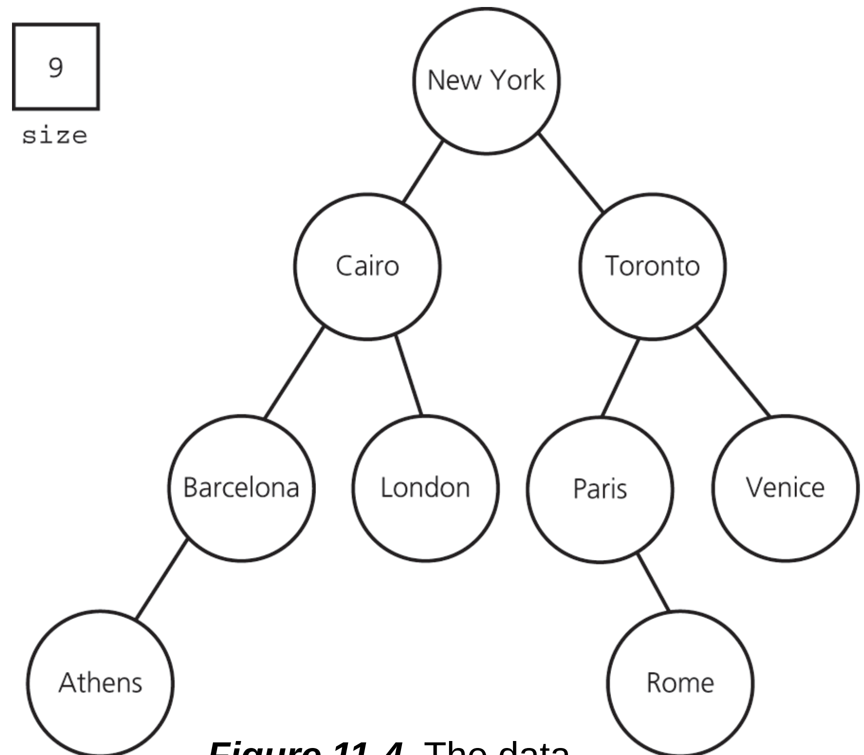


Figure 11-4 The data members for a binary search tree implementation of the ADT table for the data in Figure 11-1

Selecting an Implementation

- The requirements of a particular application influence the selection of an implementation
  - Questions to be considered about an application before choosing an implementation
    - What operations are needed?
    - How often is each operation required?
    - Are frequently used operations efficient given a particular implementation?

Comparing Linear Implementations

- Unsorted array-based implementation
  - Insertion is made efficiently after the last table item in an array
  - Deletion usually requires shifting data
  - Retrieval requires a sequential search

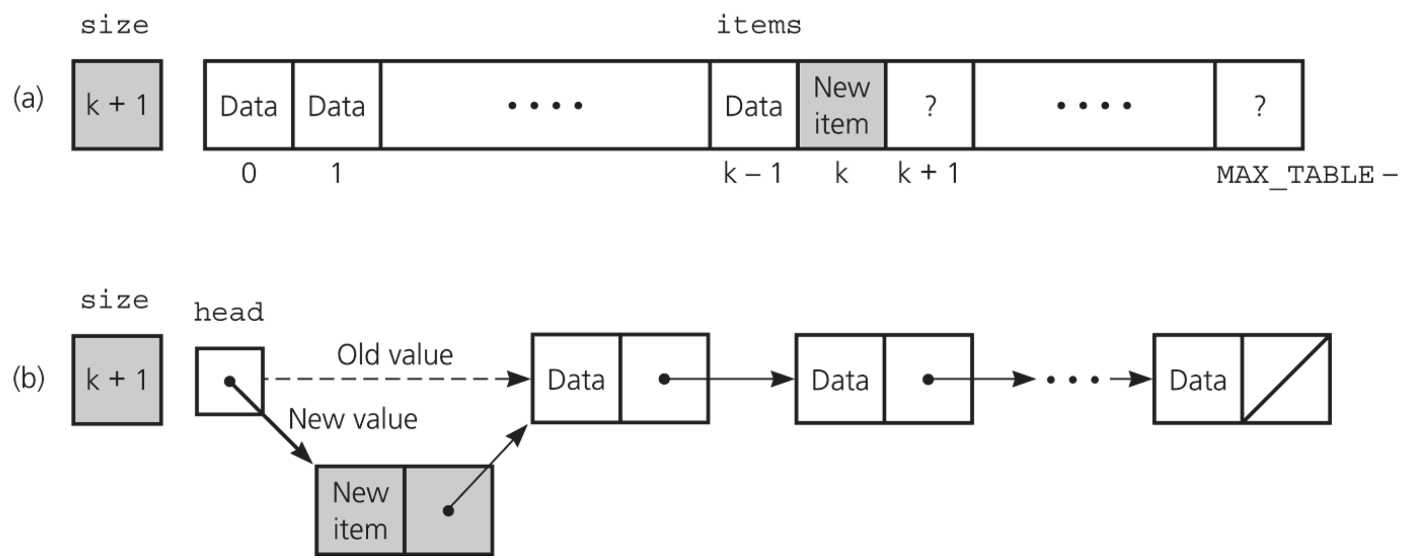


Figure 11-5a Insertion for unsorted linear implementations: array based

Comparing Linear Implementations

- Sorted array-based implementation
  - Both insertions and deletions require shifting data
  - Retrieval can use an efficient binary search

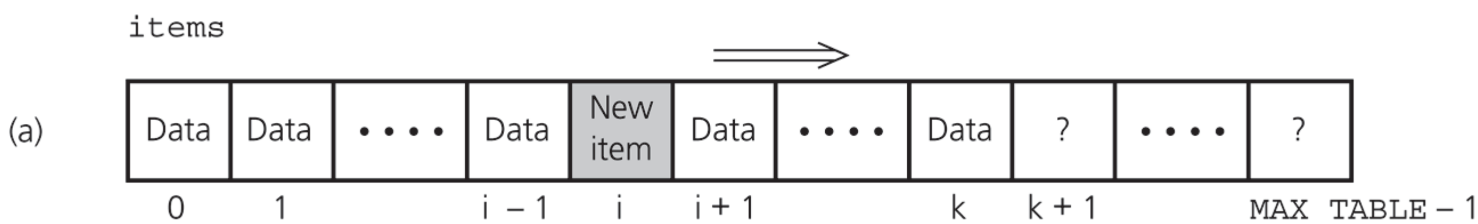


Figure 11-6a Insertion for sorted linear implementations: array based

Comparing Linear Implementations

- Unsorted pointer-based implementation
  - No data shifts
  - Insertion is made efficiently at the beginning of a linked list
  - Deletion requires a sequential search
  - Retrieval requires a sequential search

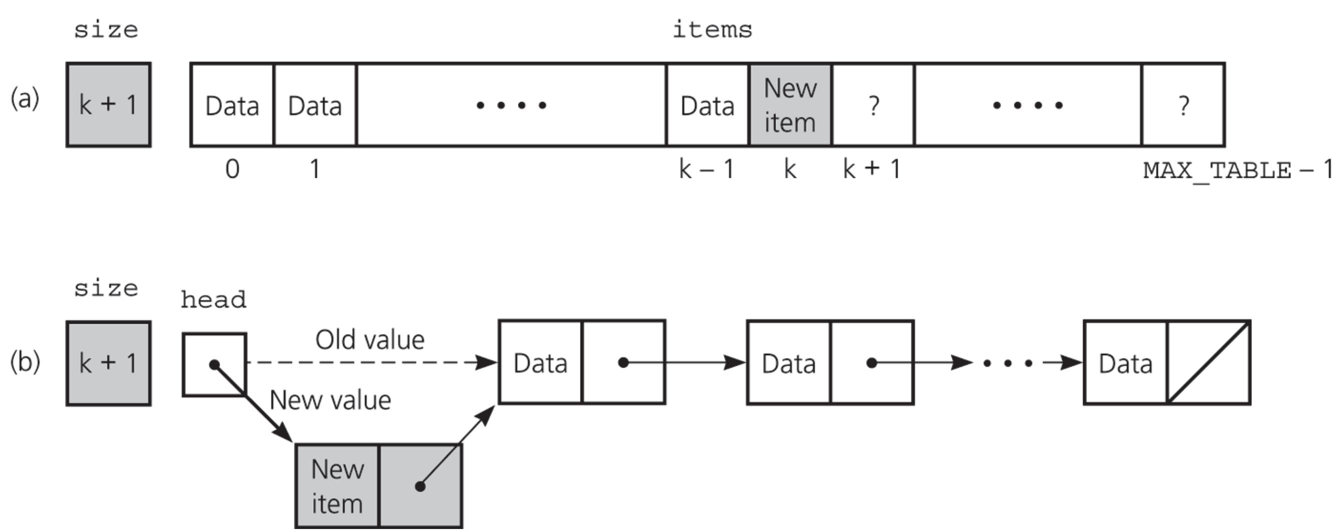


Figure 11-5b Insertion for unsorted linear implementations: pointer based

Comparing Linear Implementations

- Sorted pointer-based implementation
  - No data shifts
  - Insertions, deletions, and retrievals each require a sequential search

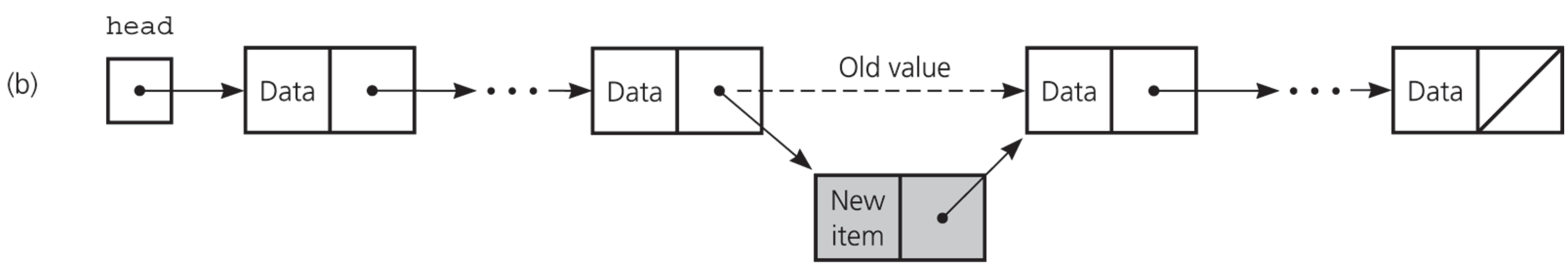


Figure 11-6b Insertion for sorted linear implementations: pointer based

Selecting an Implementation

- Linear
  - Easy to understand conceptually
  - May be appropriate for small tables or unsorted tables with few deletions
- Nonlinear
  - Is usually a better choice than a linear implementation
  - A balanced binary search tree
    - Increases the efficiency of the table operations

Selecting an Implementation

	Insertion	Deletion	Retrieval	Traversal
Unsorted array based	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Unsorted pointer based	$O(1)$	$O(n)$	$O(n)$	$O(n)$
Sorted array based	$O(n)$	$O(n)$	$O(\log n)$	$O(n)$
Sorted pointer based	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Binary search tree	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$

Figure 11-7 The average-case order of the ADT table operations for various implementations

Selecting an Implementation for a Particular Application

- Frequent insertions and infrequent traversals in no particular order
  - Unsorted linear implementation
- Frequent retrievals
  - Sorted array-based implementation
    - Binary search
  - Balanced binary search tree
- Frequent retrievals, insertions, deletions, traversals
  - Binary search tree (preferably balanced)

Generalized Data Set Management

- Problem of managing a set of data items occurs many times in many contexts
  - arbitrary set of data represented by an arbitrary key value within the set
- Strict separation of the set of data from the key helps with abstraction and generalization
- Data Set
  - class or structure defined in application terms
- Container class
  - STL terminology
  - holds key and data set items

Keyed Base Class

- Create base class for associating *key* with an arbitrary item
- Maintains key outside the item fields
- Rows of Table are derived classes of this class
- Inserting item in Table creates instance of derived class and stores it under key

```
#include <string>
using namespace std;
typedef string KeyType;

class KeyedItem
{
public:
   KeyedItem() {}
   KeyedItem(const KeyType&
keyValue)
    : searchKey(keyValue){}
   KeyType getKey() const {
      return searchKey;
   }
private:
   KeyType searchKey;
};
```

Table Item Class

- Create table of cities indexed by city name
- Might create *struct* for each city
  - name, popu., country
- Or, might derive this class from KeyedItem
- Delegates chosen key to base class storage

```
class City : public KeyedItem
{
public:
  City() : KeyedItem() {}
  City(const string& name,
         const string& ctry,
         const int& num)
   : KeyedItem(name),
     country(ctry), pop(num) {}

  string cityName() const;
  int getPopulation() const;
  void setPopulation(int newPop);
private:
  // city's name is search-key value
  string country;
  int    pop;
};
```

A Sorted Array-Based Implementation of the ADT Table

- Default constructor and virtual destructor
- Copy constructor supplied by the compiler
- Has a typedef declaration for a “visit” function
- Public methods are virtual
- Protected methods: setSize, setItem, and position

A Binary Search Tree Implementation of the ADT Table

- Reuses `BinarySearchTree`
  - An instance is a private data member
- Default constructor and virtual destructor
- Copy constructor supplied by the compiler
- Public methods are virtual
- Protected method: `setSize`

Priority Queue

- Binary Search Tree is an excellent data structure, but not always
  - simple in concept and implementation
  - BST supports many useful operations well
    - insert, delete, deleteMax, deleteMin, search, searchMax, searchMin, sort
  - efficient average case behavior T(n) = O(log n)
- However, BST is not good in all respects for all purposes
  - brittle with respect to balance
  - worst case T(n) = O(n)
- Balanced Trees are possible but more complex

Priority Queue

- Priority Queue semantics are useful when items are added to the set in arbitrary order, but are removed in either ascending or descending priority order
  - priority can have a flexible definition
  - any property of the set elements imposing a total order on the set members
  - If only a partial order is imposed (multiple items with equal priority) a secondary tiebreaking rule can be used to create a total order

Priority Queue

- The deletion operation for a priority queue is different from the one for a table
  - general ‘delete’ operation is not supported
  - item removed is the one having the highest priority value
- Priority queues do not have retrieval and traversal operations

ADT Priority Queue

PriorityQueue
items
createPriorityQueue() *destroyPriorityQueue()* *pqIsEmpty()* *pqInsert()* *pqDelete()*

Figure 11-8 UML diagram for the class *PriorityQueue*

The ADT Priority Queue: Possible Implementations

- Sorted linear implementations
  - Appropriate if the number of items in the priority queue is small
  - Array-based implementation
    - Maintains the items sorted in ascending order of priority value
    - items[size - 1] has the highest priority

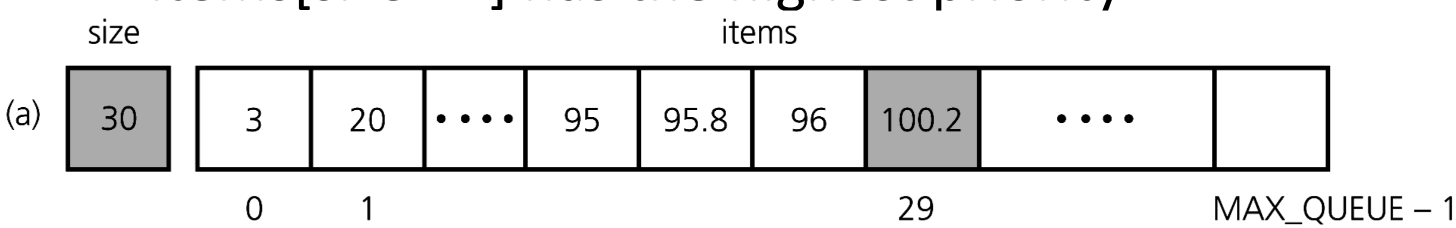


Figure 11-9a An array-based implementation of the ADT priority queue

The ADT Priority Queue: Possible Implementations

- Sorted linear implementations (continued)
  - Pointer-based implementation
    - Maintains the items sorted in descending order of priority value
    - Item having the highest priority is at beginning of linked list

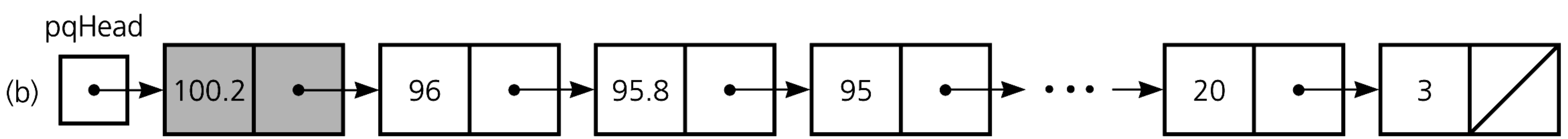


Figure 11-9b A pointer-based implementation of the ADT priority queue

The ADT Priority Queue: Possible Implementations

- Binary search tree implementation
  - Appropriate for any priority queue
  - Largest item is rightmost and has at most one child

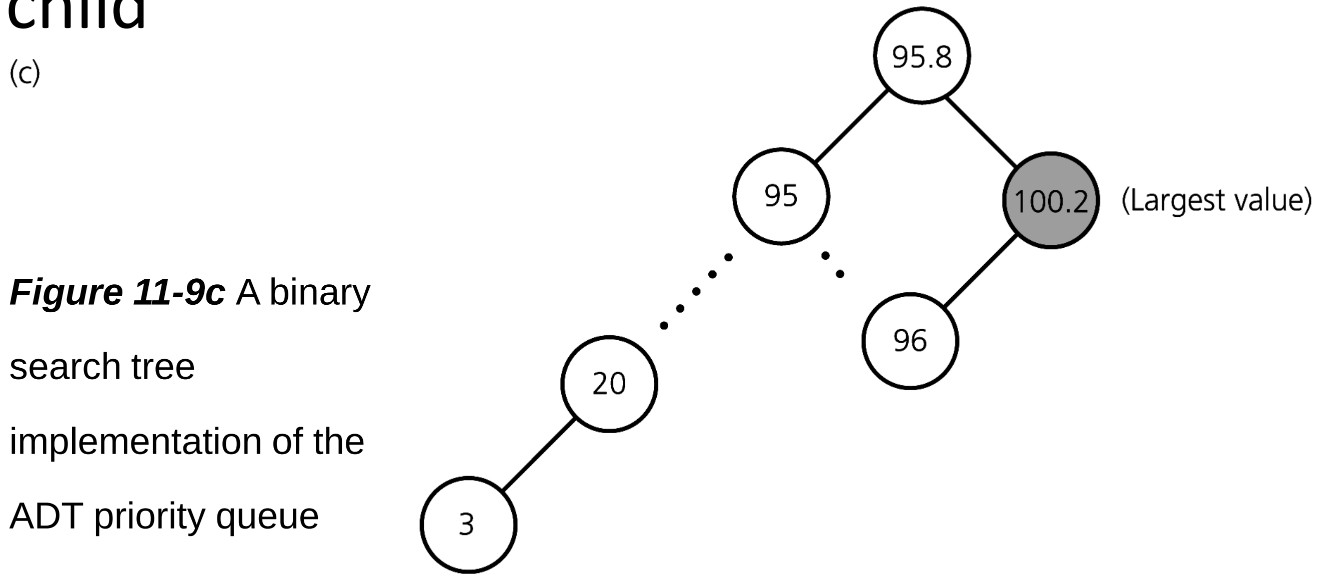


Figure 11-9c A binary search tree implementation of the ADT priority queue

The ADT Priority Queue: Heap Implementation

- A heap is a complete binary tree
  - that is empty, OR
  - whose root contains a search key >= the search key in each of its children, and whose root has heaps as its subtrees
- Heap is the best approach because it is the most efficient for the specific PQ semantics
- Heap provides a partially ordered tree
  - avoids brittleness of BST and has lower overhead than balanced search trees

Heaps

- Note:
  - The search key in each heap node is >= the search keys in each of the node's children
  - The search keys of a node's children have no required relationship

Heaps

- A maximum, binary, heap H is a complete binary tree satisfying the *heap-ordered* tree property:
  - *Complete*: Every level complete, except possibly the last, and all leaves are as far left as possible
  - *Heap Ordered*: Priority of any node is >= priority of all its descendants
  - maximum element of set is thus at root
- A minimum heap ensures that all nodes have priority values <= all its descendants
  - minimum element at root

Heap – ADT

Heap
items
createHeap() *destroyHeap()* *heapIsEmpty()* *heapInsert()* *heapDelete()*

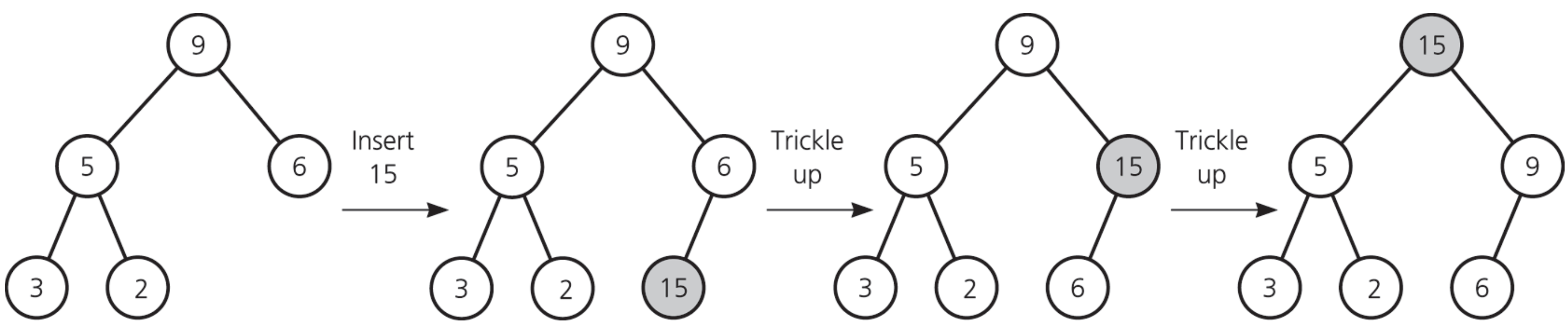
Figure 11-10 UML diagram for the class *Heap*

Heap – Implementation

- Considering typical heap operations, for example, insert into heap
- Result must be a complete tree satisfying the heap property that all nodes are >= descendants
- Two step insert process works well
  - insert the new item in the next “open” slot for keeping H a complete binary tree
  - restructure H to make it satisfy the heap-ordered property
- Two step remove
  - client code save root value for use
  - Replace root with “last” node in level-order
  - Restructure H to migrate/percolate new root to the correct tree location

Heap – Implementation

- Traversal of the inserted node to its proper place requires at most O(log n) operations
  - since the height of a complete binary tree is O(log n)



Heap – Implementation

- **Deletion** is similar
  - always deletes the root of the tree, left with two disjoint subtrees
  - place item in last node in the root
  - out of place item in root node should percolate down to its proper position
  - O(log n)

Heap – Implementation

- Data structure suitable for heap implementation must
  - support efficient determination of where next and last slots in a complete tree are located for insert and delete, respectively
  - support efficient percolation of misplaced nodes
- Percolation down is simple using standard child references and comparison of parent to child values
- Percolation up is almost as simple, but requires a parent reference at each node
- Knowing the last occupied and next open slots under different data structures is more subtle under some data structures than others

Heap – Implementation

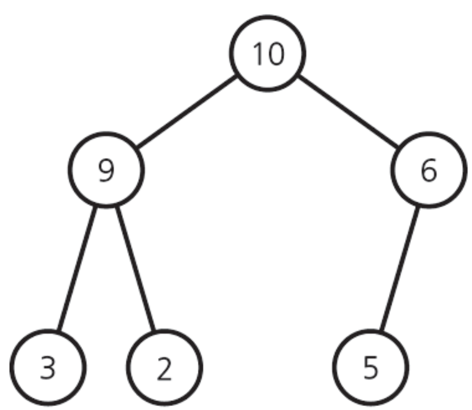
- Pointer based heaps require two child and one parent pointer at each node
  - can use additional state information to track location of next and last complete tree slots
- Array based heap implementation simplifies parent and child references by making them calculated
  - lowers space overhead
  - not clear execution time would be lower
    - array index calculation vs. pointer access
- Similarly, location of the next and last slots for the complete tree can be calculated from the number of nodes in the tree, which is simple to track

Heap – Array Implementation

- In an array representation of a binary tree T
  - Root of T is at A[0]
  - left and right children of A[i] are at A[2i+1] and A[2i+2]
  - parent of a node A[i] is at A[(i-1)/2]
  - for n>1, A[i] is a leaf iff 2i>n
  - in a heap with n elements the last element of the complete binary tree is at A[n-1] and the next element (element n+1) will be added at A[n]

Heap – Array Implementation

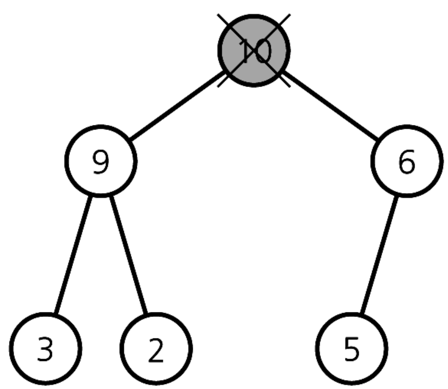
- An array-based representation is attractive
  - need to know the heap's maximum size
- Constant MAX_HEAP
- Data members
  - items: an array of heap items
  - size: an integer equal to the current number of items in the heap



0	10
1	9
2	6
3	3
4	2
5	5

Heap – Array Implementation

- heapDelete operation with arrays
- Step 1: Return the item in the root
  - rootItem = items[0]



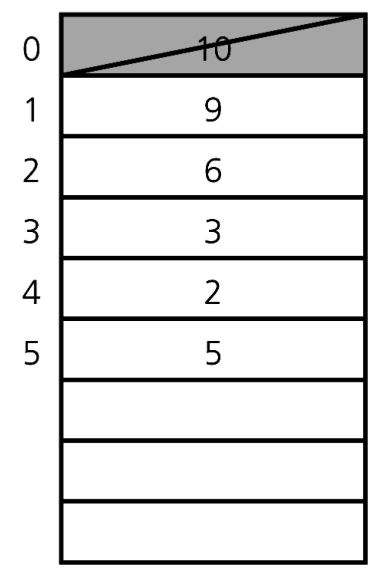


Figure 11-12a Disjoint heaps
(a)

Heap – Array Implementation

- Step 2: Copy the item from the last node into the root: items[0]= items[size-1]
- Step 3: Remove the last node: --size
  - Results in a semiheap

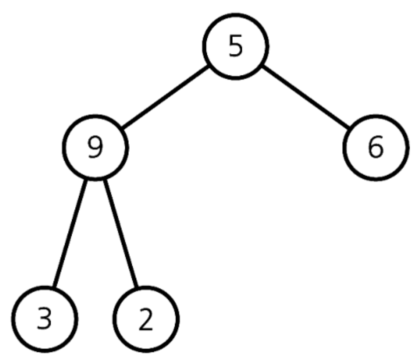
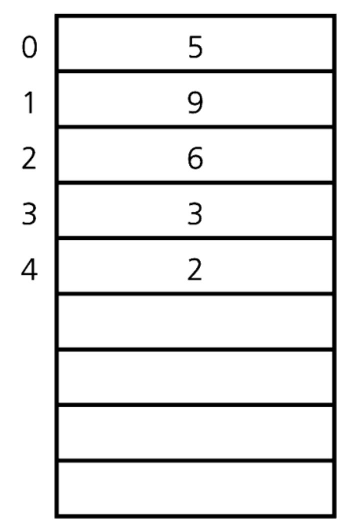


Figure 11-12b A semiheap
(b)



Heap – Array Implementation

- Step 3: Transform the semi-heap back into a heap
  - use the recursive algorithm heapRebuild
  - the root value trickles down the tree until it is not out of place
    - if the root has a smaller search key than the larger of the search keys of its children, swap the item in the root with that of the larger child

A Heap Implementation of the ADT Priority Queue

- Priority-queue operations and heap operations are analogous
  - the priority value in a priority-queue corresponds to a heap item's search key
- One implementation
  - has an instance of the Heap class as a private data member
  - methods call analogous heap operations

A Heap Implementation of the ADT Priority Queue

- disadvantage
  - requires the knowledge of the priority queue's maximum size
- advantage
  - a heap is always balanced

- Another implementation
  - a heap of queues
  - useful when a finite number of distinct priority values are used, which can result in many items having the same priority value

Heapsort

- Strategy
  - transform the array into a heap
  - remove the heap's root (the largest element) by exchanging it with the heap’s last element
  - transforms the resulting semiheap back into a heap

Heapsort

	Array anArray					
Original anArray	6	3	5	9	2	10
After heapRebuild(anArray, 2, 6)	6	3	10	9	2	5
After heapRebuild(anArray, 1, 6)	6	9	10	3	2	5
After heapRebuild(anArray, 0, 6)	10	9	6	3	2	5

Tree representation of anArray

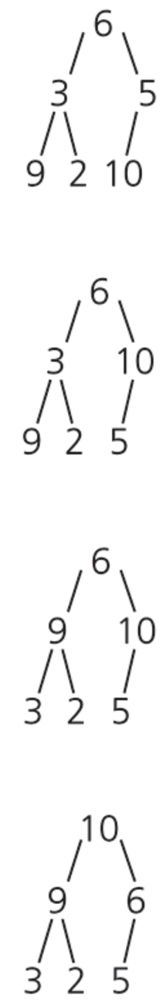


Figure 11-17 Transforming the array anArray into a heap

Heapsort

- Compared to mergesort
  - both heapsort and mergesort are O(n * log n) in both the worst and average cases
  - however, heapsort does not require second array
- Compared to quicksort
  - quicksort is O(n * log n) in the average case
  - it is generally the preferred sorting method, even though it has poor worst-case efficiency : $O(n^2)$

Summary

- The ADT table supports value-oriented operations
- The linear implementations (array based and pointer based) of a table are adequate only in limited situations
  - when the table is small
  - for certain operations
- A nonlinear pointer based (binary search tree) implementation of the ADT table provides the best aspects of the two linear implementations
  - dynamic growth
  - insertions/deletions without extensive data movement
  - efficient searches

Summary

- A priority queue is a variation of the ADT table
  – its operations allow you to retrieve and remove the item with the largest priority value
- A heap that uses an array-based representation of a complete binary tree is a good implementation of a priority queue when you know the maximum number of items that will be stored at any one time

Summary

- Heapsort, like mergesort, has good worst-case and average-case behaviors, but neither sort is as good as quicksort in the average case
- Heapsort has an advantage over mergesort in that it does not require a second array