Principles & Applications of Random FM Radar Waveform Design

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I. INTRODUCTION

As its name implies, noise radar involves the generation of noise to serve as the illuminating signal with which to elicit scattering that is collected and processed by a suitable radar receiver. Consequently, this noise radar signal is modulated in both phase and amplitude, thereby providing intrinsic low probability of intercept (LPI) characteristics, though the presence of amplitude modulation does impose a linearity requirement on transmitter amplification that may preclude noise radar from applications where high power is necessary.

However, there is a special case of noise radar in which the random nature is restricted to frequency modulation (FM). While the above LPI attribute may no longer apply, FM noise waveforms possess a constant amplitude, continuous structure that is well suited for high power applications. The purpose of this paper is to familiarize the reader with this particular type of waveform, introduce some of the ways that FM noise signals can be generated, discuss the inherent trade-offs that arise, and survey some of the emerging applications being facilitated by random FM.

The earliest determinable efforts involving the use of random FM for radar can be traced to a U.S. Navy patent application filed in 1956 by Whiteley and Adrian [1], though the patent was not actually issued until 1980. Then beginning in 1984, Liu Guosui led a group at Nanjing University

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of Science & Technology to investigate the implementation and analysis of radar signals in which frequency is modulated by white noise [2]. Further theoretical analysis subsequently followed, first by Axelsson [3], and more recently by Pralon, et al [4], that established closed form expressions for peak-to-sidelobe level (PSL) ratio, pulse compression gain, range resolution, etc. under the assumption that the modulating random process is Gaussian and wide sense stationary.

In [5], Jakabosky, et al, took the position that an arbitrary random FM initialization for a given pulsed waveform (or continuous-wave (CW) segment) could be optimized according to a desired power spectrum while retaining both the FM structure and the inherent randomness. Due to the non-convex nature of this optimization problem and the dependence of each locally optimal solution on the particular random initialization, a class of "designed" FM noise waveforms has arisen that provides significant practical utility and flexibility, with further design approaches continuing to emerge [6-12]. For the sake of brevity, and because the generated signal does not repeat (i.e. is unique from pulse to pulse), we shall refer to all of these waveforms, whether optimized or not, simply as FM noise or random FM.

Figure 1 provides a conceptual illustration of an FM noise signal (CW in this case) by comparing it to a standard FMCW chirp in terms of instantaneous frequency as a function of time. Where the FMCW chirp repeats in a structured manner, the FM noise signal meanders randomly, thereby avoiding repetition. Consequently, the time-bandwidth product (*BT*) for the FMCW signal is the product of the frequency interval traversed during one sweep and the temporal extent of that sweep. The number of sweeps in a coherent processing interval (CPI) for subsequent slow-time processing – Doppler processing for moving target indication (MTI) or cross-range processing for synthetic aperture radar (SAR) – does provide additional integration gain and discrimination capability in the Doppler/cross-range domain, but does not provide further waveform-domain dimensionality (e.g. for separability purposes).

In contrast, due to the nonrepeating nature of random FM, it has a corresponding "aggregate" *BT* defined by some measure of the spectral content, such as 3 dB bandwidth or maximum swept bandwidth, and the temporal extent of the entire coherent processing interval (CPI). The result is a potential for extremely high signal dimensionality that facilitates rather useful properties such as enhanced receive separability and natural sidelobe suppression.

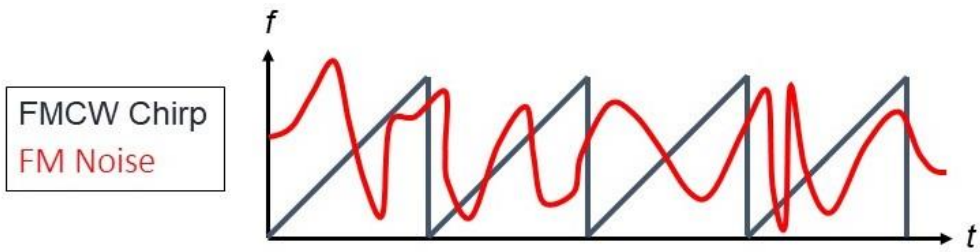

Fig. 1: Conceptual representation of FM noise, here depicted as CW but could also be pulsed

The nonrepeating structure of FM noise can introduce a trade-off, however, where clutter is concerned. Specifically, because the matched filter response of each waveform (or CW segment) has a unique autocorrelation sidelobe structure, the subsequent combining of these pulse compressed responses when performing slow-time processing realizes a *range sidelobe modulation* (RSM) effect upon the clutter [13, 14]. Unless the dimensionality, in terms of aggregate *BT*, is high enough to drive it below the post-processing noise level, the RSM establishes a floor on sensitivity because it is a nonstationary component that cannot be adequately suppressed through standard clutter cancellation. That said, the next section briefly discusses some methods that have been developed to combat RSM.

II. Design & Operational Principles

Consider the complex baseband representation of an arbitrary FM waveform of pulsewidth T denoted as

$$s_k(t) = \exp\left(j\theta_k(t) \right), \tag{1}$$

where $\theta_k(t)$ is the continuous, instantaneous phase and the subscript designates the kth pulse, for $k = 1, 2, \cdots, K,$ that are transmitted during the CPI. Extension of (1) to CW segments is easily achieved by phase rotating the kth waveform segment so that its starting phase is aligned with the ending phase from the preceding segment (to maintain phase continuity). The instantaneous frequency of the signal in (1) is obtained via the scaled derivative

$$\frac{1}{2\pi}\frac{d\theta_k(t)}{dt} = f_k(t)\ , \tag{2}$$

the existence of which necessitates that $\theta_k(t)$ be continuous over the pulsewidth (or segment). Thus (1) may alternatively be expressed in the explicit FM form

$$s_k(t) = \exp\left(j2\pi\int_0^t f_k(\tau)\, d\tau \right). \tag{3}$$

In [4] it was analytically shown that if $f_k(t)$ is a wideband, Gaussian-distributed random process, then the autocorrelation of $s_k(t)$ approaches a Gaussian shape, which subsequently means that the power spectral density likewise approaches a Gaussian shape. This result is important because low autocorrelation sidelobes are obtained when a waveform's spectrum tapers towards the band edges (see [15] and references therein). That said, to achieve arbitrary power spectral shapes, or if the intended signal is not wideband, some manner of spectral shaping optimization is necessary.

As discussed in [16, 17], the optimization of physical waveforms that are frequency modulated requires operating on a discretized representation of the signal that is sufficiently "over-sampled" (e.g. with respect to 3-dB bandwidth) such that an adequate portion of the spectral roll-off is captured to keep unavoidable aliasing to an acceptable minimum (recall that a time-limited pulse has theoretically infinite bandwidth). Further, to retain the diversity afforded by random waveforms it is important that the optimization procedure push towards relatively good solutions while still preserving the freedom provided by the initializing random process. This condition can be accomplished by designing for a desired power spectrum (e.g. like the Gaussian shape above) while allowing the spectral phase response to remain arbitrary.

A variety of methods [5-12] have thus far been developed based on this general notion of designing the power spectrum (with the arbitrary phase providing the randomness). Specifically, [5, 8, 9] employed a form of alternating time/frequency projections, [6, 7, 10, 12] defined metrics that could be optimized via gradient descent (where in [7] a Gaussian-like power spectrum results as a by-product), and [11] formulated the optimized design of a *shaping filter* that can convert a stream of discrete values independently drawn from a white Gaussian process into a continuous FM signal possessing a desired power spectrum. What all of these methods have in common is that they produce physically-realizable FM waveforms that can be experimentally tested in hardware with very little distortion, and are consequently amenable for generation by high-power radar transmitters.

For example, Figs. 2 and 3 illustrate free-space MTI measurements collected using the pulsed form of the pseudo-random optimized (PRO-FM) method of [5] followed by normalized matched filtering and Doppler processing on receive. Multiple cars and trucks can be observed traversing a traffic intersection near the University of Kansas campus. Here the stationary platform (roof of a campus building) permits simple projection-based cancellation of the clutter at zero-Doppler. Figure 2 depicts the use of 10^4 unique PRO-FM pulsed waveforms, while Fig. 3 shows what occurs after down-selecting every 100^{th} pulse, resulting in a total of 100 random FM waveforms. The former has a pulse repetition frequency (PRF) of 100 kHz, and thus the latter has an effective PRF of 1 kHz.

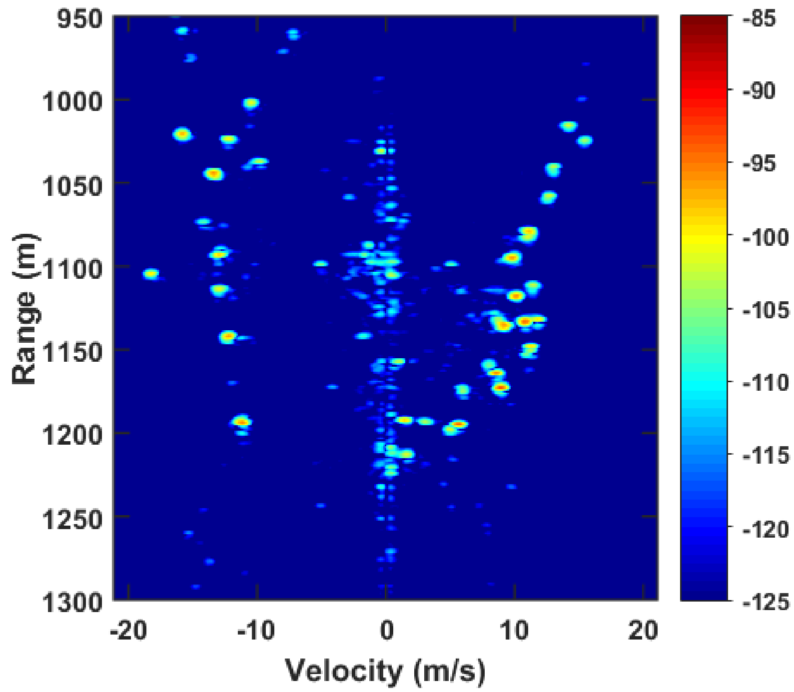

Fig. 2: Range-Doppler response (dB scale) after cancellation of zero-Doppler clutter using 10^4 PRO-FM pulsed waveforms and normalized matched filtering [5] (dB scale)

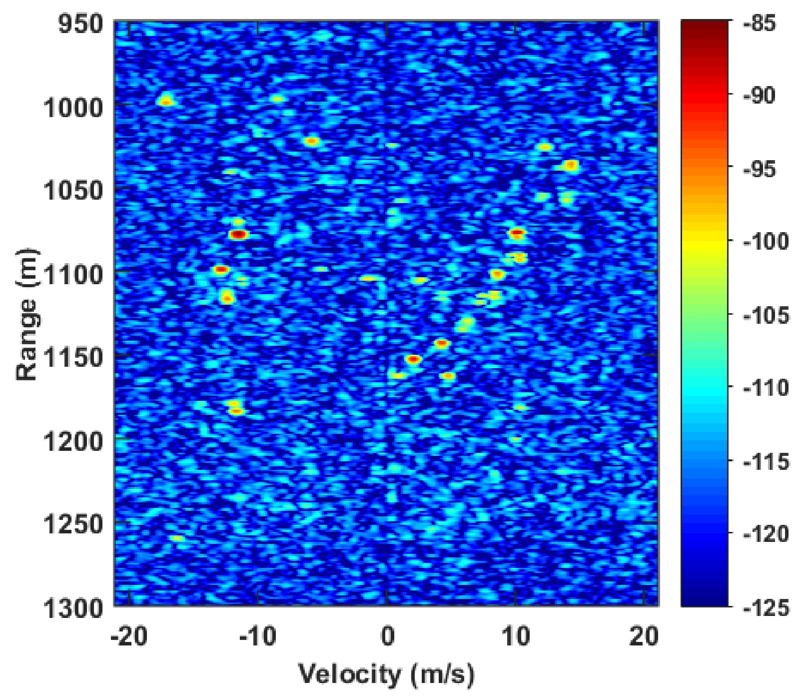

Fig. 3: Range-Doppler response (dB scale) after cancellation of zero-Doppler clutter using 100 PRO-FM pulsed waveforms and normalized matched filtering [5] (dB scale)

What quickly becomes evident when comparing these two range-Doppler responses is that, as one would expect, the one employing 100× more pulses (Fig. 2) benefits from a 20 dB higher signal-to-noise ratio (SNR), since the moving cars/trucks are far more visible. A more subtle distinction, however, is the increase in what appears to be background noise when a lower number of pulses is used (see Fig. 3). In fact, this response is the result of range sidelobe modulation (RSM) of clutter, which smears across Doppler and thus goes largely uncanceled using standard methods for clutter suppression.

An early approach to compensate for RSM was denoted as joint least squares (JLS) [13] and involves the joint design of mismatched filters (MMFs) that, when applied to a set of diverse waveforms, produce "homogenized" range sidelobe responses. A computationally efficient version of JLS where the MMFs are computed in the frequency domain was also recently developed [18]. For the special case of only two unique waveforms that are used repeatedly in some randomly alternating fashion, it was shown in [13] that each waveform is the matched filter of the other, and thus RSM could be completely mitigated if fast-time Doppler is negligible. Of course, this two-waveform case is rather limiting, and the use of JLS MMFs only allows expansion to a few additional unique waveforms before the trade-off (lower RSM at the cost of higher overall range sidelobes) becomes less useful in general.

As an alternative, [5] sought to avoid this RSM vs. sidelobe level trade-off by simply constructing an optimized MMF for each individual pulsed waveform without consideration of the sidelobe structure similarity to other waveform/filter pairs. The justification for this approach is that, if the range sidelobes can be sufficiently suppressed using individual MMFs, the benefits garnered from their averaging during subsequent slow-time processing may be enough to drive them below the noise.

For example, Fig. 4 illustrates the lower PRF range-Doppler response from Fig. 3, albeit when using the optimized MMFs from [5]. While the moving targets still have a lower SNR compared to the higher dimensionality of Fig. 2, the RSM has been reduced by about 17 dB relative to Fig. 3.

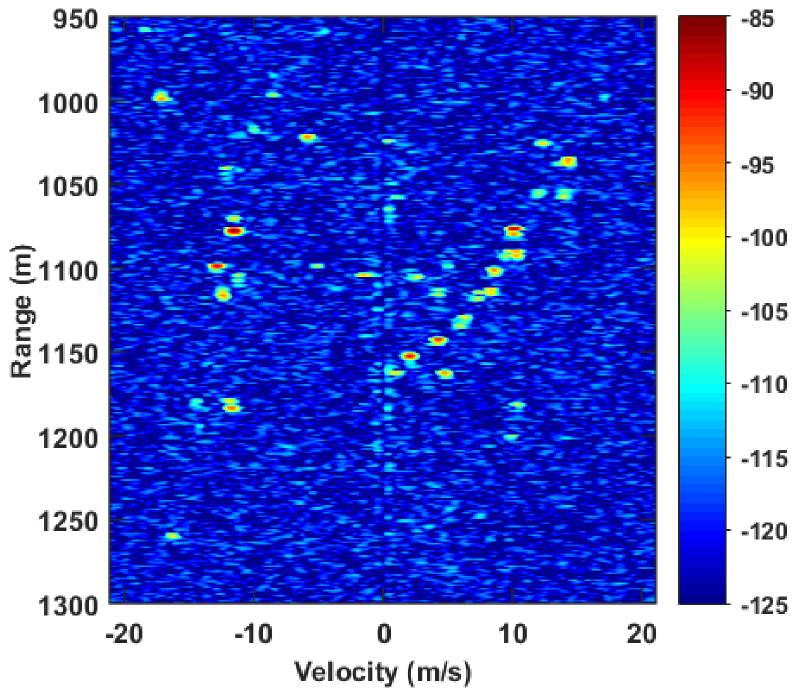

Fig. 4: Range-Doppler response (dB scale) after cancellation of zero-Doppler clutter using 100 PRO-FM pulsed waveforms and individually optimized MMFs [5] (dB scale)

This notion of mitigating RSM by simply suppressing the sidelobes that cause it can be taken a step further by performing pre-summing on the pulse compression responses, under the condition that they possess a complementary relationship. While the idea behind complementary coding and combining has been known since 1961, it is generally not considered to be all that practical given the high sensitivity to Doppler and transmitter distortion, which is particularly deleterious to codes (see [16]). However, when subsets within a larger CPI of random FM waveforms can be designed to have a complementary property, their intrinsic diversity and enhanced robustness to transmitter distortion and Doppler has been experimentally demonstrated [7] to provide significant practical sidelobe suppression. For instance, Fig. 5 depicts the loopback measured sidelobe suppression capability of these “Comp-FM” waveforms. Moreover, it has even recently been experimentally shown using free-space measurements [19] that subsets of complementary-enabling MMFs can be generated for arbitrary nonrepeating FM waveforms (i.e. not designed to be complementary) that achieve this same benefit.

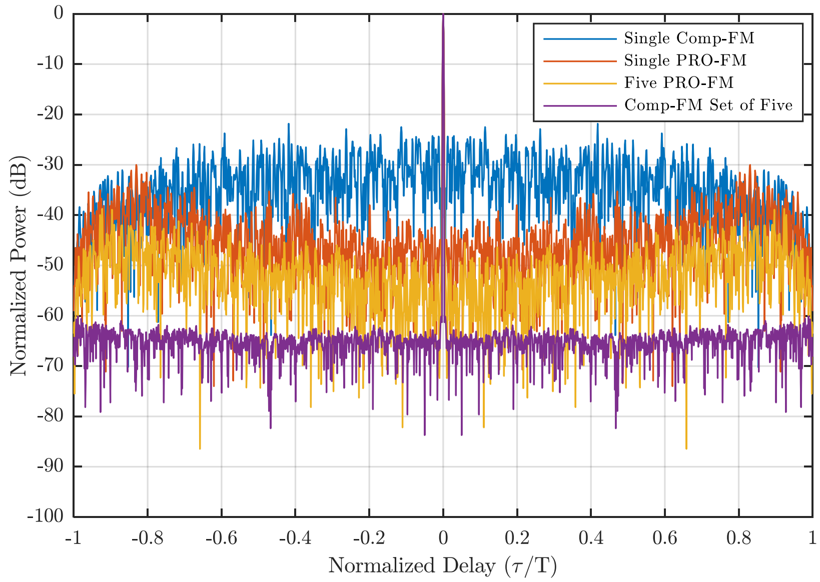

Fig. 5: Autocorrelation response of individual PRO-FM and Comp-FM waveforms and combinations of subsets of 5 [7]

Fundamentally, the root cause of RSM is that we are attempting to perform standard radar receive processing, which involves range-domain pulse compression followed by processing in slow-time across the pulses in a CPI (or sweeps/segments if FMCW). Such an approach makes sense when the same waveform is repeated, but is actually not consistent with the emission structure when employing nonrepeating waveforms. In the nonrepeating context it would be more appropriate to perform joint fast-time/slow-time receive processing. Different approaches to this manner of combined processing were considered in [20-22], with a subsequent reduced complexity method presented in [23] that addresses the attendant high computational cost that accompanies any form of joint domain processing (e.g. like space-time adaptive processing (STAP) for clutter cancellation from a moving platform).

Finally, at higher transmit powers, for which random FM waveforms are well suited, one can also expect to realize extended (if not completely eliminated) range ambiguity, to the degree that the waveforms are sufficiently separable on receive. Of course, this prospective benefit also brings with it a more complicated arrangement for multiple time-around clutter, which may necessitate more sophisticated joint-domain methods for cancellation such as those noted above.

III. Applications of Random FM Waveforms

A. MTI and SAR

Some of the applications of FM noise waveforms are the same as those that have been examined for standard noise radar, such as MTI and SAR. However, the FM property permits operation at greater ranges through the use of efficient power amplifiers (class C or higher) operated in saturation. Figures 2-4 (and [5, 7, 9-11, 24]) experimentally illustrate the efficacy and trade-offs involved with applying random FM to a stationary MTI mode. Similar behavior is expected for airborne/space-based MTI, though the coupling between slow-time (Doppler) and fast-time (range) discussed in the previous section may lead to further interactions since a moving platform likewise induces coupling between Doppler and the spatial domain (hence the use of STAP).

For SAR, where both waveform BT and the number of pulses in the CPI are generally much higher than for MTI, even greater freedom may be possible because sensitivity-limiting RSM may instead become a benefit. Specifically, well-designed waveforms can achieve a peak sidelobe level on the order of $-20 \log_{10} (BT)$ dB [15] and slow-time processing of M random waveforms elicits an additional $10 \log_{10} (M)$ dB in sidelobe reduction because sidelobes add incoherently [8]. When these values are large, high dynamic range (if sidelobe limited) can be achieved without the need for well-known tapering approaches that impose degradation in both resolution and SNR. A preliminary simulation example of SAR using FM noise waveforms was presented in [7], with experimental effects still to be explored. Given the high transmit power afforded by the FM structure and the separability enabled by individual uniqueness, waveforms of this type could also prove useful for differentiating different range ambiguities that otherwise become superimposed on receive, particularly for space-based applications.

Finally, polarimetric scattering is useful for many remote sensing applications of SAR and could likewise aid in target discrimination for MTI. Like standard noise radar, the separability of random FM waveforms permits simultaneous dual-polarized operation (i.e. transmit on both orthogonally polarized channels concurrently), thereby avoiding the timeline trade-off required to alternate polarizations. In [24] a simultaneous dual-polarized MTI mode was experimentally demonstrated that provides coherent versions of the HH, HV, VH, and VV channels, which may be beneficial for subsequent polarimetric processing. Of course, the fundamental limitation in this context is the achievable isolation (or purity) between the dual-polarized channels due to their close proximity (roughly –10 dB to –30 dB is typical, though further enhancement is possible).

B. Cognitive Radar Spectrum Sharing

Waveform design to facilitate transmit spectral notching, an ongoing topic of research for some time, has become increasingly important within the context of cognitive sensing as one of the means to address demands for radar to share spectrum resources. However, the introduction of spectral notches into the radar's passband, which involves significant changes to the original waveform, likewise tends to produce a rather substantial increase in range sidelobes. In contrast, the incoherent averaging effect that provides a $10 \log_{10} (M)$ dB sidelobe reduction for random FM waveforms still largely holds true, with one caveat.

Figure 6 [9] shows the aggregate autocorrelation (summed over the CPI) that is obtained when four different types of 2500 unique FM noise waveforms are coherently combined after matched filtering (i.e. the zero-Doppler response to a point scatterer). The most prominent distinction is observed between the case involving no spectral notch (blue trace) and when the set of unique waveforms contains a sharp notch for 10% of the 3-dB bandwidth (red trace), with the latter exhibiting a $\sin(x)/x$ sidelobe roll-off in range sidelobes that is associated with a rectangular spectral shape (due to the sharp notch "corners"). While not eliminated, this effect is noticeably diminished when tapering of the notch edges is employed (here using a Tukey window), as observed for the yellow and purple traces.

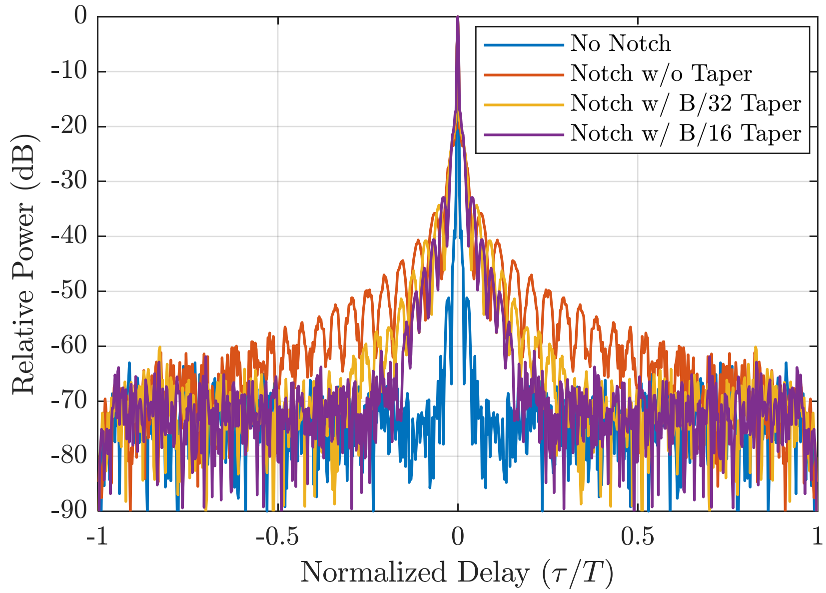

Fig. 6: Normalized aggregate autocorrelation (coherent combining of 2500 random FM waveforms) with different spectral notch characteristics [9]

It has also been observed experimentally [9, 25] that if spectral notch locations move during the CPI – e.g. to keep pace with dynamically changing radio frequency interference (RFI) that resides in the radar passband – then in addition to RSM the clutter response is also modulated by changes to the pulse compression mainlobe of each unique waveform. This phenomena produces streaking artifacts in the range/Doppler response after standard clutter cancellation [9], even when RSM-reducing approaches such as MMFs are employed. This mainlobe modulation effect arises directly from changes in the number and location(s) of the passband spectral notch(es), and necessitates its own form of compensation (e.g. see [25]).

C. Dual-function Radar/Communications

Communication signals inherently possess randomness because they convey information, and thus the incorporation of an information-bearing payload into a radar emission likewise introduces some degree of variability. If the resulting signal has an FM structure, which as previously discussed is naturally amenable to high-power operation, then this signal constitutes some form of random FM. Of course, the precise nature of this randomness depends on how the signal is constructed (e.g. digital symbol constellations would impose some structure to the random signal).

For example, it was shown in [26] that a baseline radar waveform such a linear FM (LFM) chirp could be combined, at the level of instantaneous phase, with a digital FM communication signal based on continuous phase modulation (CPM). This combination, denoted as phase-attached radar/communication (PARC), is mathematically represented as [26]

$$s_k(t) = \exp\left(j\left[\theta(t) + h\psi_k(t) \right] \right), \tag{4}$$

where $\theta(t)$ is the instantaneous phase for the radar component (e.g. quadratic function for an LFM chirp), $\psi_k(t)$ is the CPM phase component that is unique to the kth pulse, and h is the "modulation index" that scales the degree of phase deviation from the baseline radar waveform, which subsequently introduces a performance trade-off between the radar and communication functions. An FMCW version of (4) has recently been experimentally demonstrated using free-space measurements [27] that leverages other recent work by some of the same authors enabling stretch processing of arbitrary chirp-like waveforms. A similar approach to [26, 27] was also recently presented in [28]. It was alternatively shown in [29] that the CPM component $\psi_k(t)$ could be replaced by a form similar to constant-envelop (CE) orthogonal frequency division multiplexing (OFDM). All of these approaches do maintain some structure due to the presence of the baseline waveform so it would perhaps be more appropriate to consider them as quasi-random FM, with clutter cancellation limitations due to the RSM dependence on h [14] (i.e. is eliminated if h = 0).

A completely different dual-function mode was proposed in [8] and subsequently experimentally demonstrated whereby the transmit spectral notches discussed earlier are filled with appropriately shaped OFDM subcarriers. Denoted as tandem-hopped radar and communications (THoRaCs), because the notch and subcarrier locations can move around the passband in tandem during the CPI of pulses, an overall FM structure is imposed upon the combined signal by enforcing

$$s_k(t) = r_k(t) + e_k(t) = \exp\left(j\theta_k(t) \right), \tag{5}$$

through the use of an alternating-projections optimization procedure and a random FM initialization. Here $r_k(t)$ is the OFDM combination of symbol-weighted subcarriers, with the inclusion of additional amplitude weighting to preserve the desired power spectrum, and $e_k(t)$ is the “excess signal” needed for the combination $r_k(t) + e_k(t)$ to adhere to the FM structure in (5). This manner of waveform construction has less structure than the PARC and related dual-function approaches, and thus is closer to truly random FM.

D. Nonlinear Radar

An application that necessitates transmission at high power is nonlinear radar because sufficient power density is required to cause a device under test (DUT) to produce a nonlinear response. The standard approach relies on the measurement of harmonics generated by the DUT, which likewise necessitates extreme spectral purity on the part of the radar transmitter to avoid contamination by transmitter harmonics that would otherwise hinder detection of DUT harmonics.

In contrast, the intermodulation approach, generally involving the simultaneous illumination of the DUT by two signals in order to produce a third intermodulated response, has received far less attention. One reason is because the intermodulation response resides in the same spectral region as the second-order transmitter harmonics, thus resulting in similar contamination effects. In other words, this approach needs an additional transmitter without providing a clear advantage over standard harmonic radar.

However, it was recently shown [30] that the high dimensionality provided by FM noise waveforms enables the separability needed to discriminate the intermodulation response from the harmonics. Denoted as shared-spectrum pseudo-random intermodulation (SSPRInt), the approach involves illuminating the DUT by a sequence of unique pulse pairs ($s_{k,1}(t)$ and $s_{k,2}(t)$ based on (1)), such that each intermodulation response

$$s_{k,12}(t) = \exp\left(j\left[\theta_{k,1}(t) + \theta_{k,2}(t) \right] \right) \tag{6}$$

is likewise unique from the corresponding harmonics

$$\begin{aligned} s_{k,11}(t) &= \exp\left(j\left[2\theta_{k,1}(t) \right] \right) \\ s_{k,22}(t) &= \exp\left(j\left[2\theta_{k,2}(t) \right] \right), \end{aligned} \tag{7}$$

all three of which reside at twice the center frequency of the illuminating waveforms. Subsequent receiver matched filtering via (6) permits coherent integration of the intermodulation response relative to noise and the uncorrelated harmonic responses. Figure 7 shows loopback measurements that demonstrate how a large number of nonrepeating waveform pairs produces an intermodulation response that can be easily discriminated from the harmonics.

E. Practical MIMO Operation

Finally, the very low cross-correlation provided by high-dimensional, nonrepeating waveforms (as demonstrated in Fig. 7 for nonlinear radar), provides the means with which to achieve the degree of separability promised by so-called “orthogonal” waveforms for multiple-input multiple-output (MIMO) radar. Consequently, these waveforms represent an enabling technology that may subsequently facilitate experimental evaluation for the large body of theoretical work in the MIMO radar literature.

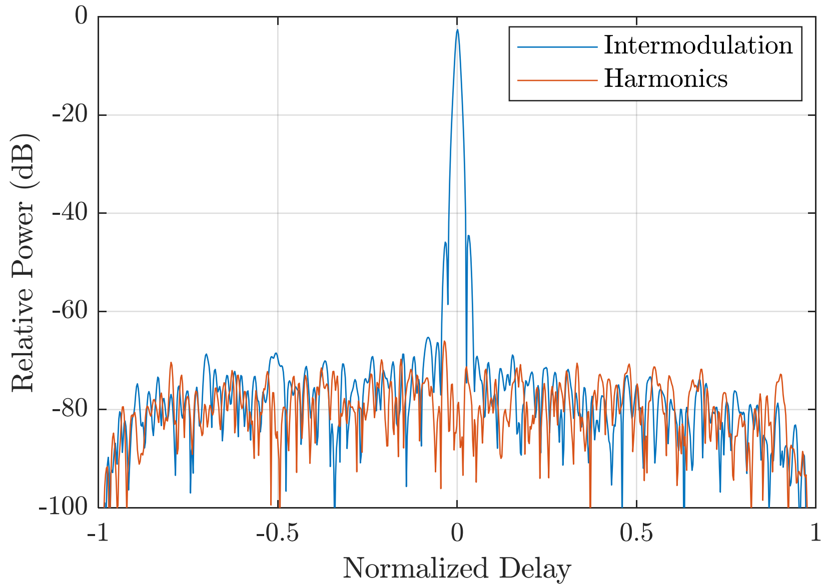

Fig. 7: Response from loopback measurement of nonlinear radar after matched filtering to the intermodulation signal and coherent integration of 10^5 unique pairs of PRO-FM pulsed waveforms [30]

Moreover, high-dimensional separability also supports the practical realization of various multistatic modes in which spatially separated radar emitters occupy the same spectrum. Of course, care must still be taken to avoid receiver saturation (or even damage) that could arise for geometries in which a transmit mainbeam (or possibly even a sidelobe) directly illuminates a sensitive receiver.

A very specific form of MIMO radar has also arisen from these nonrepeating FM waveforms by introducing the additional random FM signals $b_{\mathrm{x}}(t)$ and $b_{\mathrm{z}}(t)$ in the waveform domain across the elements in a planar MIMO antenna array in the x-z plane. Denoted as spatial modulation [31], random FM waveform $s_k(t)$ for the kth pulse is further modulated according to the particular antenna element, indexed by m_{x} and m_{z}, as

$$s_{k,m_{\mathrm{x}},m_{\mathrm{z}}}(t) = \exp\big(j\theta_k(t)\big)\exp\big(-jm_{\mathrm{x}}\phi_k(t)\big)\exp\big(-jm_{\mathrm{z}}\varphi_k(t)\big), \qquad (8)$$

where $\phi_k(t)$ and $\varphi_k(t)$ are the continuous, instantaneous phase functions for the x and z directions, respectively, that are independent and randomly generated for each pulse. Inspired by the random perturbations involved with *fixational eye movement* for human vision, in this context a coherent transmit beam is randomly moved in fast-time about the center look direction. Like in vision, the benefit is enhanced discrimination as illustrated in simulation for a collection of four point scatterers in Figs. 8-10. Figures 8 and 9 depict the response induced by a static beam, where the latter has been phase-dithered to broaden the beam somewhat. By comparison, the spatially modulated beam result in Fig. 10 demonstrates significant enhancement with regard to identifying and discriminating the four point scatterers. Experimental evaluation of this approach will take place in the very near future.

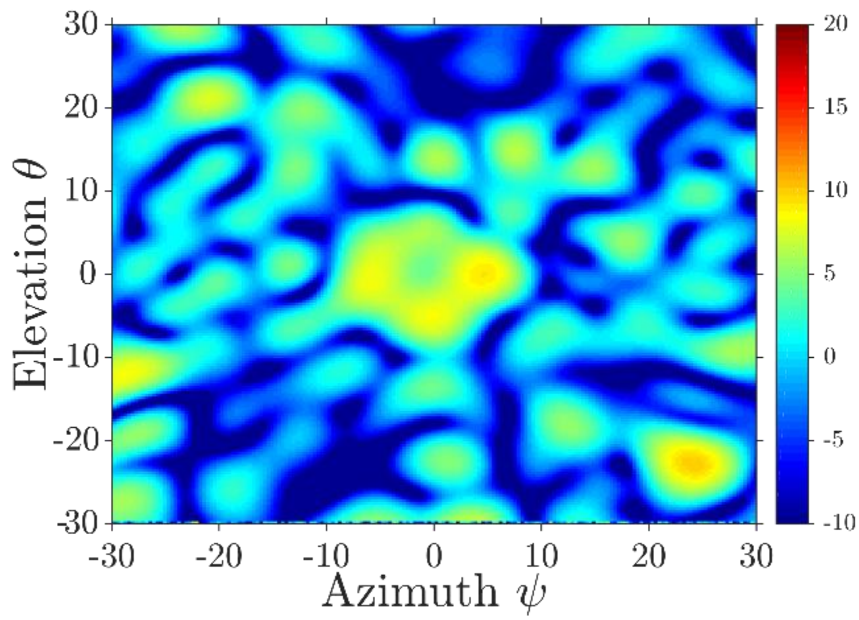

Fig. 8: Azimuth-elevation cut for a staring beam with null-to-null beamwidth of ±5.7° [31] (dB scale)

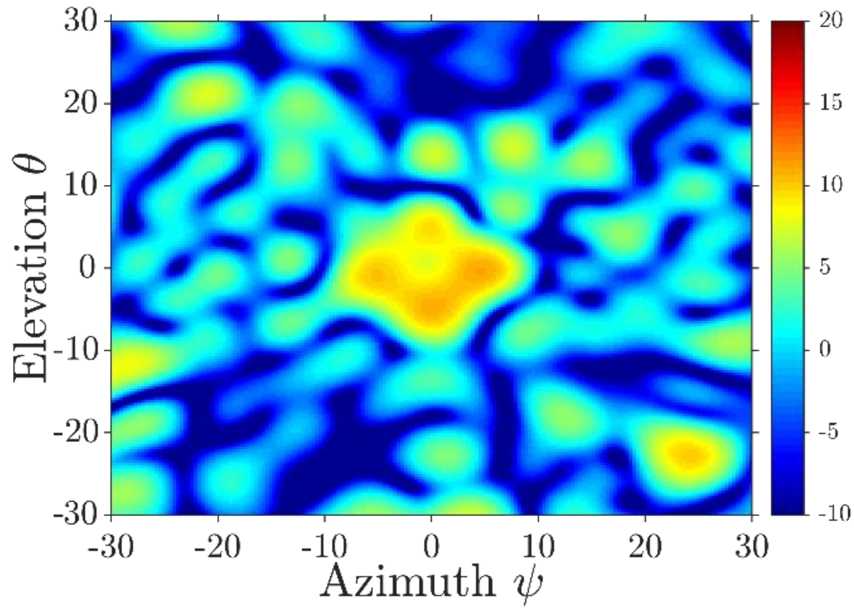

Fig. 9: Azimuth-elevation cut for a static, spatially-dithered beam [31] (dB scale)

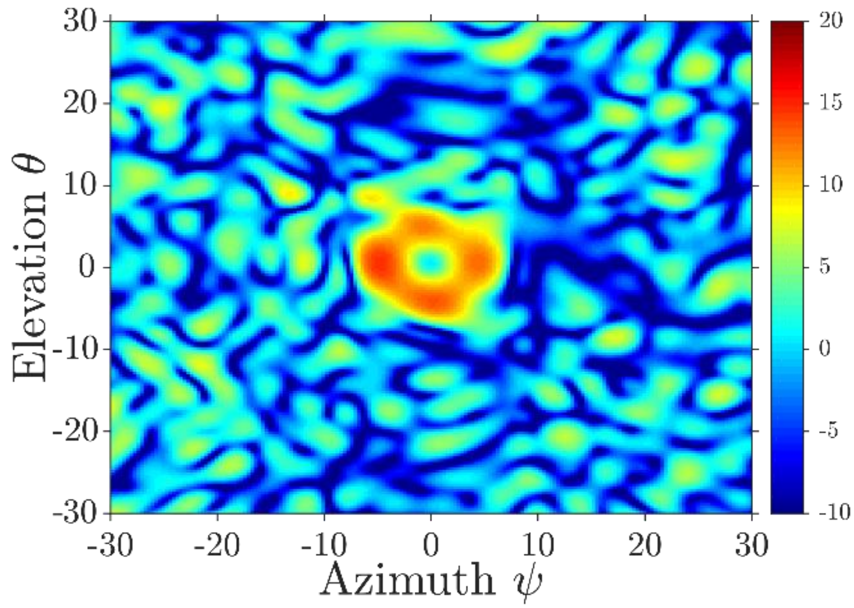

Fig. 10: Azimuth-elevation cut for a spatially-modulated beam [31] (dB scale)

IV. CONCLUSIONS

Random FM waveforms are a special case of the more general class of noise radar waveforms that, while no longer truly LPI, do possess attributes that make them amenable to high-power operation. Moreover, the high dimensionality that can be achieved makes this type of waveform useful for a variety of emerging sensing modes that continue to be explored, including

MIMO, nonlinear radar, dual-function radar/communications, cognitive sensing, and more. Some versions can also be parameterized with underlying codes that can be easily implemented as physically realizable signals due to their FM structure. Consequently, random FM waveforms represent a form of waveform diversity with tremendous design freedom that can be readily evaluated using experimental measurements.

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