JOINT ADAPTIVE PULSE COMPRESSION TO ENABLE MULTISTATIC RADAR

Shannon D. Blunt, Karl Gerlach

Radar Division
US Naval Research Laboratory
Washington DC

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Abstract

The fundamental limitation on the concurrent use of the same spectrum by two or more radars is the cross-correlation between the waveforms employed by the respective radars. The waveform transmitted by each individual radar is reflected by an illuminated range profile and is then received by all of the radars (accounting for aspect angle diversity). The received signal at a given radar is therefore the superposition of the return signals from all radars. Hence the use of standard matched filtering can result in masking that arises from the inherent waveform cross-correlations (in addition to the masking that results from autocorrelation range sidelobes). This paper presents a method for adaptively pulse compressing two or more concurrently received radar return signals that occupy the same portion of the frequency spectrum. The proposed method extends the recently developed concept of Reiterative Minimum Mean-Square Error (RMMSE) estimation to estimate adaptively the appropriate pulse compression filter for each particular range cell of each received radar return signal. The resulting algorithm, denoted as Multistatic Adaptive Pulse Compression (MAPC), effectively mitigates the interference caused by autocorrelation range sidelobes and waveform cross-correlations.

1 Introduction

The increased desire for ever greater sensor coverage will inevitably result in proximate radars overlapping (at least partially) their respective operating frequency bands and as such will become sources of mutual interference. This problem is further exacerbated by the fact that the US Federal Communications Commission continues to sell portions of the frequency spectrum, previously allocated for radar, to the wireless communications industry [1]. It therefore becomes necessary to begin exploring ways to enable concurrent, shared-spectrum radar operation in order to mitigate mutual interference, as well as to exploit the potential benefits that such an arrangement would provide such as aspect angle diversity, greater area coverage with shorter revisit times, and anti-stealth sensing capability.

It is well known that two or more radars operating in relatively close proximity, at the same time, and in the same spectrum will interfere with one another – often to the point of achieving complete RF fratricide. This is because it is impossible to generate waveforms that are orthogonal to one another at all possible respective delays and Doppler frequency shifts. The result is that a relatively large target return associated with one of the received signals can mask target returns from the other received signals.

A significant amount of work has been done to design sets of waveforms/matched filter pairs [2] that possess suitable ambiguity and cross-ambiguity characteristics (for example [3,4]). These waveforms are designed such that the overall ambiguity (*i.e.* range sidelobe levels and cross-correlations) is minimized on average over all delay/Doppler shifts and cross-correlations. However, as long as the radar receivers rely on standard matched filtering (pulse compression), there remains the distinct possibility that a large target can mask small targets that may exist in nearby range cells (within the same range profile) or from small targets in another range profile from which the reflected signal arrives nearly coincident in time at the receiver. The combination of range sidelobes and waveform cross-correlation can collectively be considered as self-interference.

Conceptually, in order to mitigate the masking problem, a receive filter for a particular waveform at a particular range cell must be closely matched to the given transmitted waveform while also cancelling the interference from targets in nearby range cells (range sidelobes) as well as from target returns from other received signals (waveform cross-correlations). Hence, the receive filters must be adaptive to the actual received signals since the appropriate receive filter will be unique for each individual range cell associated with each received signal.

Recently, an approach known as Reiterative Minimum Mean-Square Error (RMMSE) estimation was developed [5]-[7] which, for the monostatic radar case, is capable of accurately estimating the range profile illuminated by a radar by suppressing range sidelobes to the level of the noise floor. This is accomplished by adaptively estimating the appropriate receiver pulse compression filter to use for each individual range cell. Furthermore, the RMMSE algorithm, which has also been denoted as Adaptive Pulse Compression (APC) when applied to the radar matched filtering problem, has been shown to be robust to rather severe Doppler mismatch [6,7]. In this paper we extend the concept of monostatic APC to encompass multistatic radar. The resulting algorithm, which we denote as Multistatic Adaptive Pulse Compression (MAPC), adaptively estimates a unique pulse compression filter for each individual range cell of each range profile. The extension of APC to the multistatic scenario enables the joint estimation of multiple independent range profiles. MAPC may also be used to estimate the profile of a single spatial region from multiple aspect angles or different portions of a single extended range profile made possible by transmitting a series of pulses, each modulated with a different waveform.

2 Multistatic Adaptive Pulse Compression

Consider K radars (designated individually as radar k, for $k = 1,2,\ldots,K$) that operate simultaneously in the same spectrum each with a distinct transmitted waveform. We denote the discrete-time version of the k^{th} radar's waveform as the column vector $\mathbf{s}_k$ having length N, and $\mathbf{r}_k = [1 \;\, e^{j\theta_k} \;\, \cdots \;\, e^{j(M-1)\theta_k}]^T$ as the spatial steering vector corresponding to the angle-of-arrival (AOA) of the k^{th} radar return signal received at radar 1. Note that without loss of generality the same processing is to be performed at each of the radars; thus for this development we consider only the 1st radar. It is assumed either that the range profiles of interest lie in the collective far field of the group of radars (for instance a cluster of ground-looking space-based radars) or that sufficient pulse chasing (sweeping the mainbeam to follow a changing AOA) is enabled so that the appropriate spatial steering vector as a function of delay (range) is employed. Let radar 1 have an M-length linear array (in general MAPC can be applied to any array geometry). Then the ℓ^{th} time sample of the K received radar return signals on the m^{th} antenna element is defined as

$$y_m(\ell) = \sum_{k=1}^{K} \mathbf{x}_k^T(\ell)\, \mathbf{s}_k \; e^{j(m-1)\theta_k} + v_m(\ell) \qquad (1)$$

for $\ell = 0,\cdots,L+N-2$ the indices of the received signal samples of interest (used to estimate the L-length processing windows of the respective range profiles) where $\mathbf{x}_k(\ell) = [x_k(\ell) \;\; x_k(\ell-1) \;\; \cdots \;\; x_k(\ell-N+1)]^T$ is the N-length vector of discrete range profile samples at delay ℓ with which the discrete transmitted waveform $\mathbf{s}_k$ convolves, $v(\ell)$ is additive noise, and $(\bullet)^T$ is the transpose operation. The M received radar return signals for the ℓ^{th} time sample are collected into the vector $\mathbf{y}(\ell) = [y_0(\ell) \;\, y_1(\ell) \;\, \cdots \;\, y_{M-1}(\ell)]^T$.

Let each antenna array element possess its own receive channel (frequency down-conversion, A/D converter, etc…) thus enabling digital beamforming. A separate beamformer is applied for each of the K received signals across the M outputs of the antenna array. Note that it is assumed that each radar possesses knowledge of the respective AOAs of the received radar return signals (which could be a function of the delay time if pulse chasing is necessary). The ℓ^{th} time sample of the k^{th} received radar return signal after beamforming (and normalization), denoted as $z_k(\ell)$, is

$$z_k(\ell) = M^{-1}\mathbf{r}_k^H \mathbf{y}(\ell) = \sum_{i=1}^{K} \eta_{ki}\, \mathbf{x}_i^T(\ell)\, \mathbf{s}_i \; + u_k(\ell), \qquad (2)$$

where $u_k(\ell) = M^{-1}\mathbf{r}_k^H [v_0(\ell) \;\, v_1(\ell) \;\, \cdots \;\, v_{M-1}(\ell)]^T$ is additive noise after beamforming, $\eta_{ki} = M^{-1}\mathbf{r}_k^H \mathbf{r}_i$ is the normalized correlation between the k^{th} and i^{th} spatial steering vectors, and $(\bullet)^H$ is the complex conjugate transpose, or Hermitian, operation. By collecting N samples of the received radar return signal after beamforming, the resulting signal model can be expressed as

$$\mathbf{z}_k(\ell) = \sum_{i=1}^{K} \eta_{ki}\, \mathbf{X}_i^T(\ell)\, \mathbf{s}_i \; + \mathbf{u}_k(\ell) \qquad (3)$$

where $\mathbf{z}_k(\ell) = [z_k(\ell) \;\; z_k(\ell+1) \;\; \cdots \;\; z_k(\ell+N-1)]^T$ is an N-tuple of contiguous temporal samples of the received signal after beamforming, $\mathbf{u}_k(\ell) = [u_k(\ell) \;\; u_k(\ell+1) \;\; \cdots \;\; u_k(\ell+N-1)]^T$ is a vector of additive noise after beamforming, and $\mathbf{X}_k(\ell) = [\mathbf{x}_k(\ell) \;\; \mathbf{x}_k(\ell+1) \;\; \cdots \;\; \mathbf{x}_k(\ell+N-1)]$ is an $N \times N$ matrix comprised of N-length sample-shifted snapshots (in the columns) of the k^{th} range profile.

After beamforming, the standard matched filtering operation [3] dictates convolving the received radar return signal with the time-reversed complex conjugates of the transmitted waveforms in order to obtain the K respective range profiles. The outputs of the matched filtering operation can be expressed in the digital domain as

$$\hat{x}_{MF,k}(\ell) = \mathbf{s}_k^H \mathbf{z}_k(\ell) \qquad (4)$$

for $k = 1,2,\cdots,K$ and $\ell = 0,1,\cdots,L-1$. However, since ideal matched filtering assumes only a single received signal in noise, it is expected that the matched filter will perform poorly in the multistatic scenario, as the received signals will effectively jam one another.

To accommodate for multiple, simultaneously received signals in the same spectrum, the Multistatic Adaptive Pulse Compression (MAPC) algorithm replaces the matched filter $\mathbf{s}_k$ in (4) with the RMMSE-based filter [5]-[7] which, for the k^{th} radar's waveform and ℓ^{th} range gate, minimizes the MMSE cost function [8]

$$J_k(\ell) = E\left[\left| x_k(\ell) - \mathbf{w}_k^H(\ell)\, \mathbf{z}_k(\ell) \right|^2\right] \qquad (5)$$

for each k where $E[\bullet]$ denotes expectation. The solution to (5) takes the form

$$\mathbf{w}_k(\ell) = \hat{\rho}_k(\ell) \left(\sum_{i=1}^{K} \eta_{ki} \, \mathbf{C}_i(\ell) + \mathbf{R}_k \right)^{-1} \mathbf{s}_k \qquad (6)$$

for each k, where $\hat{\rho}_k(\ell) = |\hat{x}_k(\ell)|^2$ is the estimated power of $x_k(\ell)$ and $\mathbf{R}_k = E[\mathbf{u}_k(\ell) \; \mathbf{u}_k^H(\ell)]$ is the temporal (range) noise covariance matrix after beamforming in the direction of the k^{th} AOA. The matrix $\mathbf{C}_i(\ell)$ is defined as

$$\mathbf{C}_i(\ell) = \sum_{n=-N+1}^{N-1} \hat{\rho}_i(\ell+n) \; \mathbf{s}_{i,n} \mathbf{s}_{i,n}^H \;, \qquad (7)$$

where $\mathbf{s}_{i,n}$ contains the elements of the waveform $\mathbf{s}_i$ shifted by n samples and the remainder is zero-filled. For example, $\mathbf{s}_{i,2} = [0 \;\; 0 \;\; s_i(0) \;\; \cdots \;\; s_i(N-3)]^T$ for $n = 2$ and $\mathbf{s}_{i,-2} = [s_i(2) \;\; \cdots \;\; s_i(N-1) \;\; 0 \;\; 0]^T$ for $n = -2$.

To employ (6) and (7) requires initial estimates of the K range profiles as well as knowledge of the noise covariance matrices $\mathbf{R}_k$, $k = 1,2,\cdots,K$. Assuming the noise covariance is white Gaussian, $\mathbf{R}_k$ simplifies to $\sigma_v^2 \mathbf{I}$, where $\mathbf{I}$ is the $N \times N$ identity matrix and σ_v^2 is the noise power which can be assumed known since internal thermal noise is known to dominate the external noise at microwave frequencies (where most radars operate) [9]. The initial estimates of the K range profiles can be obtained either by using standard matched filtering or by initializing the power estimates of all of the KL range cells to be equal and assuming the noise is negligible initially. In the latter case, (6) reduces to

$$\tilde{\mathbf{w}}_k = \left(\sum_{i=1}^{K} \eta_{ki} \, \tilde{\mathbf{C}}_i \right)^{-1} \mathbf{s}_k \qquad (8)$$

for $k = 1,2,\cdots,K$, where the matrix $\tilde{\mathbf{C}}_i$ is defined as

$$\tilde{\mathbf{C}}_i = \sum_{n=-N+1}^{N-1} \mathbf{s}_{i,n} \mathbf{s}_{i,n}^H \;. \qquad (9)$$

The initialization MMSE filters from (8) are range invariant and can therefore be pre-computed. After (8) is applied, as in (4) with $\mathbf{s}_k$ replaced by $\tilde{\mathbf{w}}_k$, and the initial KL range cell power estimates have been obtained, (6) is subsequently used to estimate the refined receive filters which are then applied to the beamformed received signals. The refined receive filters are better able to mitigate the masking effects caused by waveform cross-correlation and range sidelobes due to the fact that they are estimated based upon some *a priori* knowledge regarding the larger targets, which was obtained by a previous stage. The re-estimation of the individual receive filters and range cells is repeated for a pre-determined number of stages. Hence, as long as sufficient adaptive degrees of freedom are available, the MAPC filters at each successive stage will further refine the estimate of the range profiles until reaching the noise floor.

Note that for (6) and (8) the MAPC algorithm performance degrades gracefully when the steering vectors for individual received signals become more closely aligned which effectively reduces the adaptive degrees of freedom. The MAPC algorithm still surpasses the performance of the standard matched filters, however, as they represent the non-adaptive solution. Finally, the structure of MAPC enables fast implementation via the matrix inversion lemma through a straightforward extension of the method described in [7].

3 Simulation Results

We first consider the simultaneous reception of two random-phase waveforms of length N = 30 received at angles of -20º and +10º off boresight of an 11-element uniform linear array. The autocorrelations of the waveforms and their cross-correlation (neglecting spatial beamforming suppression) are depicted in Figs. 1-3.

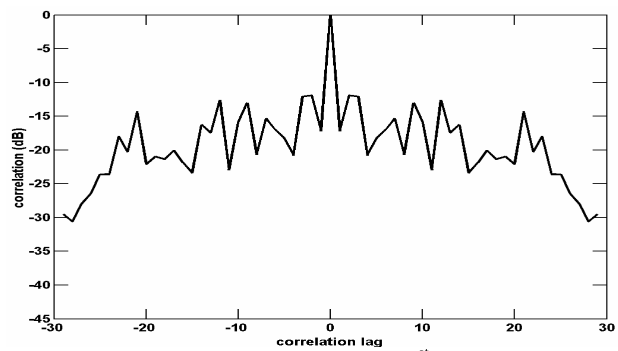

Fig. 1. Autocorrelation of the 1st waveform

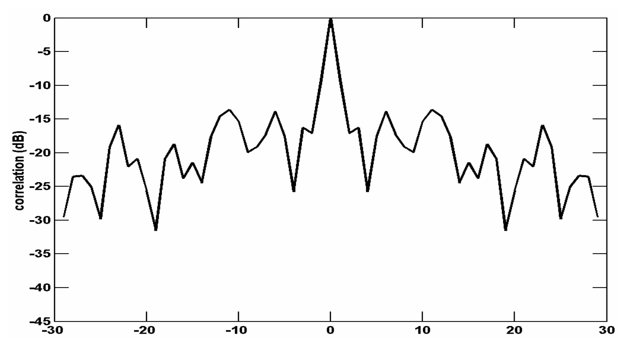

Fig. 2. Autocorrelation of the 2nd waveform

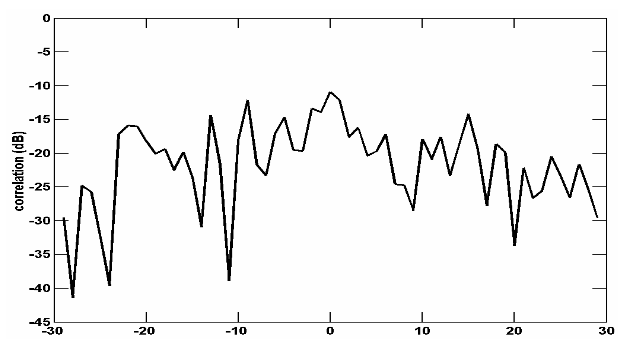

Fig. 3. Cross-correlation between waveforms

The waveforms have normalized peak sidelobe levels of -12 dB and -13 dB, respectively, and their cross-correlation peaks at -11 dB. In this case the difference in AOA enables an additional 21 dB of mutual interference suppression by using spatial beamforming. As is presented in Fig. 4, the ground truth of the respective range profiles (represented by the solid lines) is comprised of many closely spaced targets with highly disparate power levels and -60 dB noise (with respect to the largest target power). As expected, even after beamforming the matched filters perform poorly due to the combined effects of range sidelobes and waveform cross-correlation. The MAPC algorithm is employed with four stages and, for the given scenario, suppresses both the range sidelobes and the cross-correlation interference to the level of the noise floor as is shown in Fig. 5 in which the MAPC range profile estimates closely overlap ground truth. In terms of overall mean-square error (MSE), the (normalized) matched filters yield an MSE value of -14 dB while the MAPC algorithm achieves an MSE of -63 dB, an improvement of nearly 50 dB.

The second case we consider is when two waveforms are received at the same AOA - in this case directly along boresight. This situation could correspond to two radars that are illuminating the same region to obtain aspect angle diversity or it could occur whenever a monostatic radar employs a different waveform for every other pulse in order to double the maximum unambiguous range of the radar without reducing the pulse repetition frequency (PRF). For this scenario, Figs. 6 and 7 present the results for the standard matched filters and the MAPC algorithm, respectively. Whereas matched filtering again performs very poorly, the MAPC algorithm experiences only a slight degradation relative to the previous case. Furthermore, it can be seen that the MAPC algorithm yields better results without a beamforming gain (Fig. 7) than the matched filters do with a beamforming gain (Fig. 4). In terms of overall MSE performance, matched filtering attains an MSE of -13 dB while the MAPC algorithm achieves -43 dB, a 30 dB improvement.

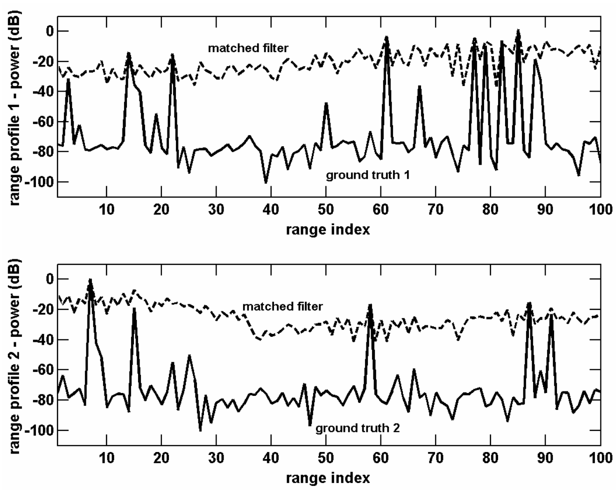

Fig. 4. Matched filter results for multistatic radar reception with spatial beamforming separation

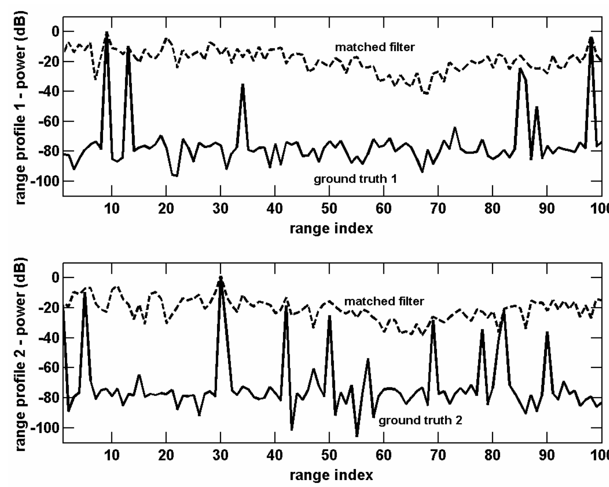

Fig. 6. Matched filter results for multistatic radar reception without spatial beamforming separation

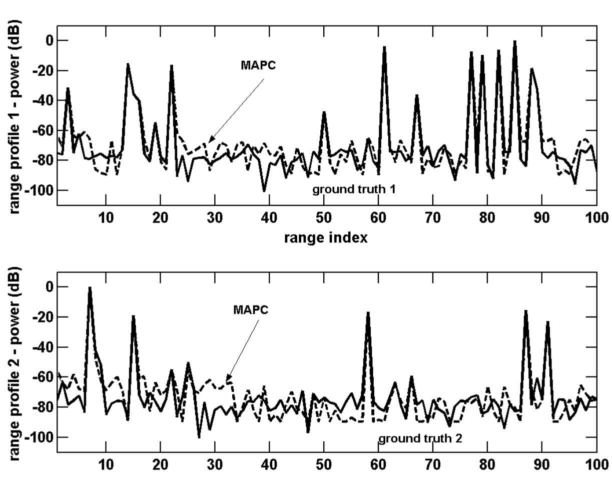

Fig. 5. MAPC results for multistatic radar reception with spatial beamforming separation

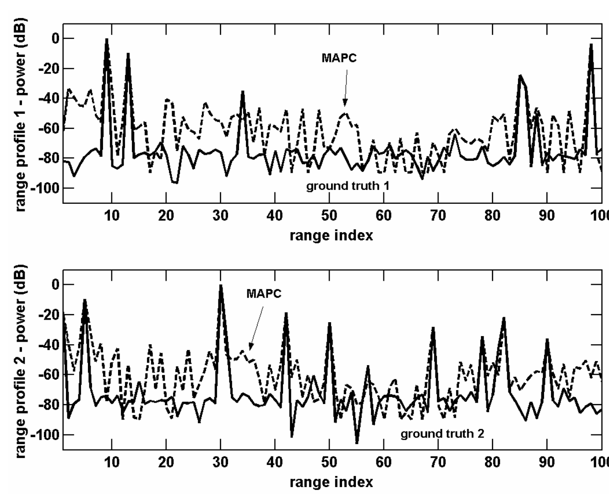

Fig. 7. MAPC results for multistatic radar reception without spatial beamforming separation

4 Conclusions

Standard matched filtering (pulse compression) is well known and is employed ubiquitously in radar applications. However, matched filtering for pulse compression assumes the reception of a single radar return signal and noise. In addition, the single radar return signal is assumed to possess only point targets that have significant range separation so that they do not interfere with one another. Hence, the matched filter suffers from range sidelobes that mask small targets when larger targets are nearby. Furthermore, the presence of multiple received signals, such as occurs in the shared-spectrum multistatic scenario, is known to cause deleterious effects to matched filter performance due to the cross-correlations between waveforms.

To mitigate the effects of range sidelobes and waveform cross-correlations, both of which are essentially forms of self-interference, this paper has introduced the Multistatic Adaptive Pulse Compression (MAPC) algorithm. The MAPC algorithm extends the concept of Reiterative Minimum Mean-Square Error (RMMSE) estimation to estimate jointly the receive filters needed for each individual range cell of each received signal in order to suppress the self-interference resulting from range sidelobes and waveform cross-correlation. The MAPC algorithm has been shown to suppress the self-interference to the level of the noise floor when coupled with beamforming and thereby achieving a 50 dB improvement over matched filtering in terms of the mean-square error (MSE) of range profile estimation. Without the benefit of beamforming, the MAPC algorithm is found to degrade slightly but was still found to demonstrate 30 dB MSE improvement over matched filtering.

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