ADAPTIVE REPAIR OF PULSE-COMPRESSED RADAR WAVEFORMS: SEEING THE FOREST DESPITE THE TREES

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Abstract

The standard matched filter used in radar pulse compression is known to generate range sidelobes in the vicinity of large targets which can result in the masking of smaller nearby targets. Significant research has been done to obtain waveform/filter pairs that possess low sidelobes with minimal mismatch loss. However, there exists no single waveform/filter pair that can completely eliminate masking for all possible scenarios as demonstrated by the well-known ambiguity function. This paper presents an adaptive approach to radar pulse compression in which the received signal is employed to adaptively determine the appropriate receive filter to apply for each individual range cell. Furthermore, in contrast to previous work on adaptive pulse compression, the work herein addresses the repair (*i.e.* unmasking) of range cells masked by sidelobes from a large target after standard pulse compression has occurred. Therefore, the proposed approach, denoted as Pulse Compression Repair (PCR), is applicable to legacy radar systems in which replacing the current pulse compression system is not feasible.

1 Introduction

Pulse compression allows a radar to obtain the range resolution of a short pulse without the need for very high peak transmit power [1]. This is accomplished by transmitting a long pulse that is phase or frequency modulated. The modulated pulse (or waveform) is reflected back to the radar by scatterers that fall within the beam of the radar. The received return signal at the radar can therefore be viewed as the convolution of the waveform with an impulse response that is representative of the range profile illuminated by the radar. The purpose of pulse compression is then to extract an estimate of the radar impulse response from the noisy received return signal based upon the known transmitted waveform.

The classical approach to pulse compression is known as matched filtering [1] which has been shown to maximize the received signal-to-noise ratio (SNR) for a solitary point target in the presence of white noise and is accomplished by filtering the received radar return signal with the time-reversed complex-conjugate of the transmitted waveform. In the digital domain matched filtering can be represented as

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H \mathbf{y}(\ell)\,, \tag{1}$$

where $\hat{x}_{MF}(\ell)$, for $\ell = 0, \cdots, L-1$, is the estimate of the ℓ^{th} range cell within the processing window of interest, $\mathbf{s} = [s_1 \;\; s_2 \;\; \cdots \;\; s_N]^T$ is the length-N sampled version of the waveform transmitted by the radar, $\mathbf{y}(\ell) = [y(\ell) \;\; y(\ell+1) \;\; \cdots \;\; y(\ell+N-1)]^T$ is a vector of N contiguous received radar return samples, and $(\bullet)^T$ and $(\bullet)^H$ are the transpose and conjugate transpose (or Hermitian) operations, respectively. Each individual radar return sample can be expressed as

$$y(\ell) = \mathbf{x}^T(\ell)\, \mathbf{s} + v(\ell)\,, \tag{2}$$

where $\mathbf{x}(\ell) = [x(\ell) \;\; x(\ell-1) \;\; \cdots \;\; x(\ell-N+1)]^T$ consists of samples of the true radar impulse response and $v(\ell)$ is additive noise. The matched filter output can therefore be written as

$$\hat{x}_{MF}(\ell) = \mathbf{s}^H \mathbf{A}^T(\ell)\, \mathbf{s} + \mathbf{s}^H \mathbf{v}(\ell)\,, \tag{3}$$

where $\mathbf{v}(\ell) = [v(\ell) \;\; v(\ell+1) \;\; \cdots \;\; v(\ell+N-1)]^T$ and

$$\mathbf{A}(\ell) = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+N-1) \\ x(\ell-1) & x(\ell) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(\ell+1) \\ x(\ell-N+1) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix} \tag{4}$$

is a collection of sample-shifted snapshots of the radar impulse response.

From (4), it is apparent that whenever any one of the off-diagonal elements of $\mathbf{A}(\ell)$ are relatively large, the estimation of $x(\ell)$ may be masked by range sidelobes. Previously, Reiterative Minimum Mean-Square Error (RMMSE) estimation was proposed [2]-[4] which adaptively suppresses range sidelobes to the level of the noise floor. The algorithm, which when applied is known as Adaptive Pulse Compression (APC), iteratively estimates the *N*-length receive filter for each individual range cell based upon the actual received radar return signal. For a given range cell, the resulting APC receive filter possesses nulls at the locations pertaining to other nearby targets. Hence, the effects of interfering scatterers in neighboring range cells are effectively suppressed such that the radar range profile can essentially be estimated to within the accuracy of the noise floor.

This paper presents an alternative method to APC whereby range sidelobes are mitigated after standard pulse compression. The proposed algorithm, denoted as Pulse Compression Repair (PCR), treats the waveform autocorrelation which results from matched filtering as an "effective received waveform" and then adaptively suppresses the range sidelobes surrounding a large target. Pulse Compression Repair is applicable to legacy radar systems in which replacing the existing pulse compression system is not feasible. Furthermore, the computational cost of PCR is offset by the fact that it needs to be applied only when a sufficiently large target is detected.

2 Pulse Compression Repair

In some legacy radar systems it is not feasible to replace the existing pulse compression apparatus to enable robust range sidelobe suppression. However, range sidelobe suppression can still be achieved by post-processing the matched filter output. This is possible because the operations of convolution of the transmitted waveform with the radar impulse response in (2) and the convolution of the received return signal with the time-reversed, complex conjugated waveform in (1) can be combined such that (3) is re-expressed as

$$\hat{x}_{MF}(\ell) = \tilde{\mathbf{x}}^T(\ell)\,\tilde{\mathbf{r}} + u(\ell) \tag{5}$$

where $\tilde{\mathbf{x}}(\ell) = [x(\ell+N-1)\ \cdots\ x(\ell+1)\ x(\ell)\ x(\ell-1)\ \cdots\ x(\ell-N+1)]^T$, $u(\ell)$ is additive noise correlated by the matched filter, and $\tilde{\mathbf{r}}$ is the length $2N-1$ autocorrelation of $\mathbf{s}$. We treat the matched filter output as the received return signal (as in (2)) by collecting $2N-1$ contiguous samples of the matched filter output $\hat{x}_{MF}(\ell)$ from (5) into $\tilde{\mathbf{y}}(\ell) = [\hat{x}_{MF}(\ell-N+1)\ \cdots\ \hat{x}_{MF}(\ell)\ \cdots\ \hat{x}_{MF}(\ell+N-1)]^T$ which can be expressed as

$$\tilde{\mathbf{y}}(\ell) = \mathbf{B}^T(\ell)\,\tilde{\mathbf{r}} + \tilde{\mathbf{u}}(\ell), \tag{6}$$

where $\tilde{\mathbf{u}}(\ell) = [u(\ell-N+1)\ \cdots\ u(\ell)\ \cdots\ u(\ell+N-1)]^T$ and

$$\mathbf{B}(\ell) = \begin{bmatrix} x(\ell) & x(\ell+1) & \cdots & x(\ell+2N-2) \\ x(\ell-1) & x(\ell) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(\ell+1) \\ x(\ell-2N+2) & \cdots & x(\ell-1) & x(\ell) \end{bmatrix} \tag{7}$$

is a $2N-1 \times 2N-1$ matrix of sample-shifted snapshots of the radar impulse response.

The matched filter output $\tilde{\mathbf{y}}(\ell)$ is adaptively pulse compressed with the respective MMSE receive filter obtained by minimizing the MMSE cost function [5]

$$J(\ell) = E\left[\left|x(\ell) - \tilde{\mathbf{w}}^H(\ell)\ \tilde{\mathbf{y}}(\ell)\right|^2\right] \tag{8}$$

where $\tilde{\mathbf{w}}(\ell)$ is the length $2N-1$ MMSE receive filter specific to the estimation of range cell $x(\ell)$.

Solving (8) for $\tilde{\mathbf{w}}(\ell)$ yields PCR filter for each individual range cell as

$$\tilde{\mathbf{w}}(\ell) = \hat{\rho}(\ell)\ (\mathbf{C}(\ell) + \mathbf{R})^{-1}\,\tilde{\mathbf{r}} \tag{9}$$

where

$$\hat{\rho}(\ell) = |\hat{x}(\ell)|^2 \tag{10}$$

is the estimated power of $x(\ell)$, $\mathbf{R} = E[\mathbf{u}(\ell)\ \mathbf{u}^H(\ell)]$ is the noise covariance matrix correlated by the matched filter, and the $(i,j)^{th}$ element of the matrix $\mathbf{C}(\ell)$ is defined as

$$c_{i,j}(\ell) = \sum_{n=\kappa_L}^{\kappa_U} \hat{\rho}(\ell-n+i-1)\ r(n)\ r^*(n-i+j) \tag{11}$$

where $\kappa_L = \max\{0, i-j\}$, $\kappa_U = \min\{2N-2, 2N-2+i-j\}$, and $(\bullet)^*$ is the complex conjugate operator. The full matrix $\mathbf{C}(\ell)$ can be written as

$$\mathbf{C}(\ell) = \sum_{n=-2N+2}^{2N-2} \hat{\rho}(\ell-n)\ \tilde{\mathbf{r}}_n\ \tilde{\mathbf{r}}_n^H \tag{12}$$

where $\tilde{\mathbf{r}}_n$ contains the elements of the length $2N-1$ waveform autocorrelation $\tilde{\mathbf{r}}$ shifted by *n* samples and the remainder zero filled. For example, $\tilde{\mathbf{r}}_2 = [0\ 0\ r_0\ \cdots\ r_{2N-4}]^T$ for $n = 2$ and $\tilde{\mathbf{r}}_{-2} = [r_2\ \cdots\ r_{2N-2}\ 0\ 0]^T$ for $n = -2$. Also, assuming that the noise power is small compared to the power of the radar returns and that the waveform has relatively good autocorrelation properties (sufficiently low sidelobe levels), the noise covariance matrix $\mathbf{R}$ can be approximated as $\sigma_v^2\mathbf{I}$, where σ_v^2 is the noise power. The noise power can be assumed known since internal thermal noise is known to dominate the external noise at microwave frequencies (where most radars operate) [6].

As (9) and (12) illustrate, the PCR receive filter for a particular range cell is a function of the current power estimate of the range cell of interest as well as the surrounding range cells. Initial estimates of the range cell powers can be obtained from the normalized output of the matched filter as

$$\hat{\rho}_0(\ell) = \left| \frac{\hat{x}_{MF}(\ell)}{\|\mathbf{s}\|^2} \right|^2 . \qquad (13)$$

The power estimates from (13) are inserted into (9) and (12) to generate the respective PCR receive filters which are then applied onto the corresponding $\hat{\tilde{\mathbf{x}}}_{MF}(\ell)$ to re-estimate the complex range cell amplitudes. This process is repeated, with the range cell power estimates obtained from the previous stage, for a predetermined number of stages. It has been found via simulations that only 1 or 2 stages of PCR is sufficient to mitigate the range sidelobes resulting from very large (and possibly densely spaced) targets.

3 Implementation Issues

The PCR algorithm is applied after standard pulse compression has been performed upon the received signal. Hence, it is only necessary to employ PCR when the matched filter output contains a target that is sufficiently large to mask other detectable targets. Furthermore, PCR can be implemented using the fast update structure described in [4] in which the matrix inversion lemma enables efficient rank-1 updating of $\mathbf{C}(\ell)+\mathbf{R}$ for each subsequent range cell to be estimated, thereby eliminating the need for all but 1 matrix inverse operation per stage. The selective use of PCR combined with the efficient update implementation thereby keeps the resulting increase in computation to a minimum.

In terms of stability, the PCR algorithm is similar to the Adaptive Pulse Compression (APC) algorithm in that very large targets can cause the matrix $\mathbf{C}(\ell)+\mathbf{R}$ to become ill-conditioned. As with APC the ill-conditioning can be alleviated by compressing the dynamic range of the range cell power estimates and the noise power. This is accomplished by replacing the exponent in (10), as well as (13), with a variable parameter as

$$\hat{\rho}(\ell) = |\hat{x}(\ell)|^{\alpha} \qquad (14)$$

and replacing σ_v^2 with σ_v^{α}, where $0 \le \alpha \le 2$. It has been found via simulation that values of $1.5 \le \alpha \le 1.9$ yield the best results for PCR. Furthermore, α should be set high initially (near 1.9) and decrease at subsequent stages as the estimated dynamic range may become greater which could cause ill-conditioning.

4 Simulation Results

The PCR algorithm need be employed only when a sufficiently large target return is present. Hence, we consider two cases: 1) A single large target return that masks a nearby smaller return, and 2) a dense target scenario containing several targets with disparate power levels. In both cases, the length N = 30 Lewis-Kretschmer P3 code [7] is the transmitted waveform. Note that in the following figures the matched filter outputs are normalized by N in order to make a fair comparison.

For the first case, the large target possesses an SNR (prior to matched filtering) of 60 dB and nominal clutter that is 70 dB less than the large target. The smaller nearby target return is 40 dB less than the large target return and hence is masked by the matched filter as illustrated in Fig. 1. However, after 1 stage of the PCR algorithm using $\alpha = 1.9$, the smaller target is unmasked and the range sidelobes are driven into the noise floor. In terms of mean-square error (MSE), the (normalized) matched filter yields -29 dB while the PCR algorithm with a single stage achieves -54 dB, an improvement of 25 dB.

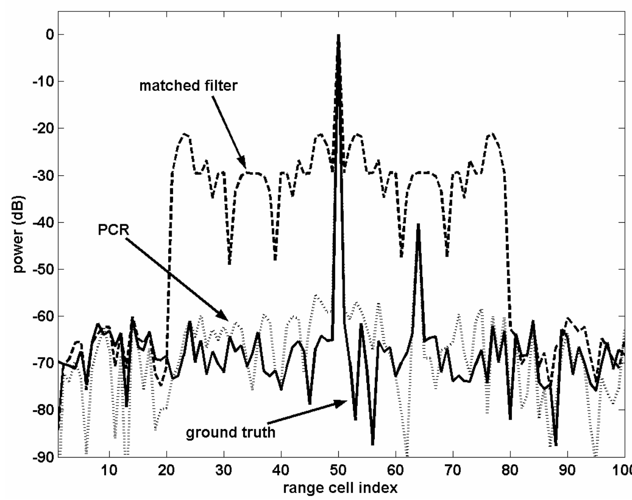

Fig. 1. Performance comparison for masked target scenario

For the second case, the range profile ground truth contains many densely-spaced targets of highly disparate power levels (as much as 50 dB). The noise and clutter are -60 dB and -70 dB with respect to the largest target value, respectively. As expected, the matched filter identifies the largest targets quite well. However, small and even moderately-sized targets are overwhelmed due the presence of a larger nearby target as well as the cumulative effects of dense targets. The PCR algorithm employs 2 stages with $\alpha = 1.9$ and 1.7, respectively. As shown in Fig. 2, PCR estimates the range profile to the accuracy of the noise floor thereby revealing the -50 dB targets. Furthermore, closely space targets such are around range cell index 70 are accurately resolved. In terms of MSE, the matched filter reaches -29 dB while the PCR algorithm after 2 stages achieves -72 dB, an overall improvement of 43 dB.

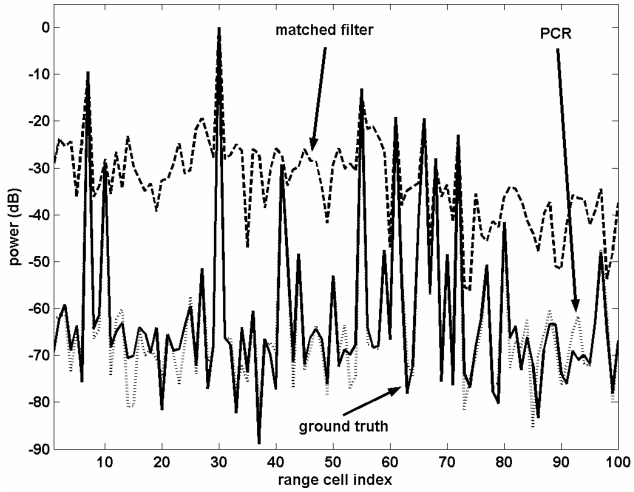

Fig. 2. Performance comparison for dense target scenario

4 Conclusions

Pulse compression enables a radar to obtain the range resolution of a short pulse while maintaining a feasible peak transmit power. For a point target in white noise, the standard matched filter maximizes the received SNR of the target. However, matched filtering suffers from range sidelobes which mask small targets when in the vicinity of large targets. Significant research has been done to obtain waveforms and/or receive filters that preserve adequate SNR while minimizing the range sidelobes. Yet, as demonstrated by the ambiguity function, no single waveform – receive filter pair can completely mitigate range sidelobes for all target scenarios.

In order to completely mitigate range sidelobes Adaptive Pulse Compression has been proposed which employs the concept of Reiterative Minimum Mean-Square Error (RMMSE) estimation. However, for many legacy radar systems it is not feasible to replace the current pulse compression apparatus. This paper has introduced the Pulse Compression Repair (PCR) algorithm which operates on the output of the standard matched filter. PCR treats the waveform autocorrelation as if it were the reflected waveform and adaptively estimates the appropriate receive filter for each particular range cell. The appropriate receive filter is thereby applied to the matched filter output to extract an accurate estimate of the range cell complex amplitude. The PCR algorithm has been shown to suppress the matched filter range sidelobes by 40 dB resulting in significantly greater detection sensitivity.

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