Assessment of Constant Envelope OFDM as a class of Random FM Radar Waveforms

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Motivation

- A variety of new Random FM (RFM) waveform classes possessing advantageous spectral shaping have recently been demonstrated.
- However, all of these methods require some form of optimization to achieve this spectral shaping (to provide low range sidelobes).
- In contrast, the signal structure of CE-OFDM naturally realizes this shaping without any need for optimization.
- Consequently, it represents a form of RFM radar waveform with an inherent dual-function radar/comms (DFRC) capability

RFM Useful Attributes (in General) [1]

- Gaussian-shaped power spectrum
  - Corresponding autocorrelation is therefore also Gaussian … theoretically achieving zero sidelobes
  - Tighter spectral containment can be achieved, but at the cost of higher sidelobes
- Constant amplitude & continuous phase
  - Permits operation of HPA in saturation, while minimizing most of the unavoidable transmitter distortion
- Ability to generate unique FM waveforms having approximately the same spectral content (Gaussian shape realized after averaging)
  - Realize extremely high dimensionality
  - For per-waveform time-bandwidth product BT, resulting aggregate time-bandwidth product is MBT, for M unique waveforms in the coherent processing interval (CPI)

[1] S.D. Blunt, et al., "Principles & applications of random FM radar waveform design," to appear in *IEEE Aerospace & Electronic Systems Magazine*.

Random FM Generation

- Early approaches (dating back to 1956) used FM driven by noise (no spectrum shaping)

- Spectrum shaping optimization realizes much lower sidelobes, with many classes developed and experimentally demonstrated thus far:
  - Via alternating projections: PRO [2], THoRaCs [3], TTE [4]
  - Via gradient descent: FTE [5], Comp-FM [6]
  - Via off-line design of shaping transform: StoWGe [7]

[2] J. Jakabosky, S.D. Blunt, B. Himed, "Spectral-shape optimized FM noise radar for pulse agility," *IEEE Radar Conf*., May 2016.

[3] B. Ravenscroft, P.M. McCormick, S.D. Blunt, E. Perrins, J.G. Metcalf, "A power-efficient formulation of tandem-hopped radar & communications," *IEEE Radar Conf*., Apr. 2018.

[4] C.A. Mohr, S.D. Blunt, "FM noise waveforms optimized according to a temporal template error (TTE) metric," *IEEE Radar Conf*., Apr. 2019.

[5] C.A. Mohr, P.M. McCormick, S.D. Blunt, C. Mott, "Spectrally-efficient FM noise radar waveforms optimized in the logarithmic domain," *IEEE Radar Conf*., Apr. 2018.

[6] C.A. Mohr, P.M. McCormick, S.D. Blunt, "Optimized complementary waveform subsets within an FM noise radar CPI," *IEEE Radar Conf*., Apr. 2018.

[7] C.A. Mohr, S.D. Blunt, "Design and generation of stochastically defined, pulsed FM noise waveforms," *Intl. Radar Conf*., Sept. 2019.

RFM Generation

- Specifically, StoWGe [7] performs optimization offline by designing a transform for a random process that produces unique continuous phase functions with the desired power spectrum in the expectation
    - By avoiding per-waveform optimization (real-time computational cost), the trade-off is somewhat higher sidelobes (but still better than 'no shaping')
- CE-OFDM has been proposed as a power-efficient way to implement OFDM [8], though it has not achieved broad utilization
- On the basis of a single waveform, the sidelobe performance of CE-OFDM is limited due to its thumbtack ambiguity function (vs. that of chirp-like waveforms due to conservation of ambiguity)
- Here we consider CE-OFDM within the nonrepeating RFM context, where the signal structure provides spectrum shaping with no optimization whatsoever

[8] S.C. Thompson, et al, "Constant envelope OFDM," *IEEE Trans. Communications*, vol. 56, no. 8, pp. 1300-1312, Aug. 2008.

OFDM Signal Model

- OFDM is the basis for 4G/5G comms, offers high spectral efficiency, and is easy to demodulate and equalize.
- The baseband signal model can be expressed as

$$u(t) = \sum_{n=1}^{N} \beta_n \exp(j2\pi f_n t)$$

  where the complex exponential at subcarrier frequency f_n is modulated by communication symbol β_n, and subcarriers are spaced in frequency by $1/T$, for T the temporal extent of a symbol.

- Of course, from a radar perspective the non-unity peak-to-average power ratio (PAPR) of OFDM essentially precludes its use for high-power / long-range sensing

CE-OFDM Signal Model

- CE-OFDM was developed to achieve power-efficient comms (via unity PAPR)
- CE-OFDM for a single symbol interval can be expressed as

phase of symbol β_n

$$s(t) = \exp\left(j2\pi h \,\Re\left\{ \sum_{n=1}^{N} \beta_n \exp(j2\pi f_n t) \right\} \right) = \exp\left(j2\pi h \sum_{n=1}^{N} \left|\beta_n\right| \cos(2\pi f_n t + \phi_n) \right)$$

where h is the modulation index and $\Re\{\bullet\}$ extracts the real part of the argument.

- While readily extensible to multiple symbol intervals, here we set the radar pulse width to T as well.

CE-OFDM Signal Model

- It has been shown in [9,10] that the CE-OFDM signal construction can likewise be expressed as

$$s(t) = \prod_{n=1}^{N} \sum_{m=-\infty}^{\infty} d_{n,m} \exp\left(j2\pi m f_n t \right) \mathrm{rect}\left(\frac{t - T/2}{T} \right)$$

in which

$$d_{n,m} = j^m J_m\left(2\pi h \left| \beta_n \right| \right) \exp\left(j m \phi_n \right)$$

mth Bessel function

- Each sum becomes a repeated convolution in frequency, so the central limit theorem implies (average) spectral content tending toward a Gaussian shape without need for spectrum shaping optimization

[9] S.C. Thompson, et al., “Constant envelope OFDM phase modulation: spectral containment, signal space properties and performance,” *IEEE Military Communications Conf.*, Oct./Nov. 2004.

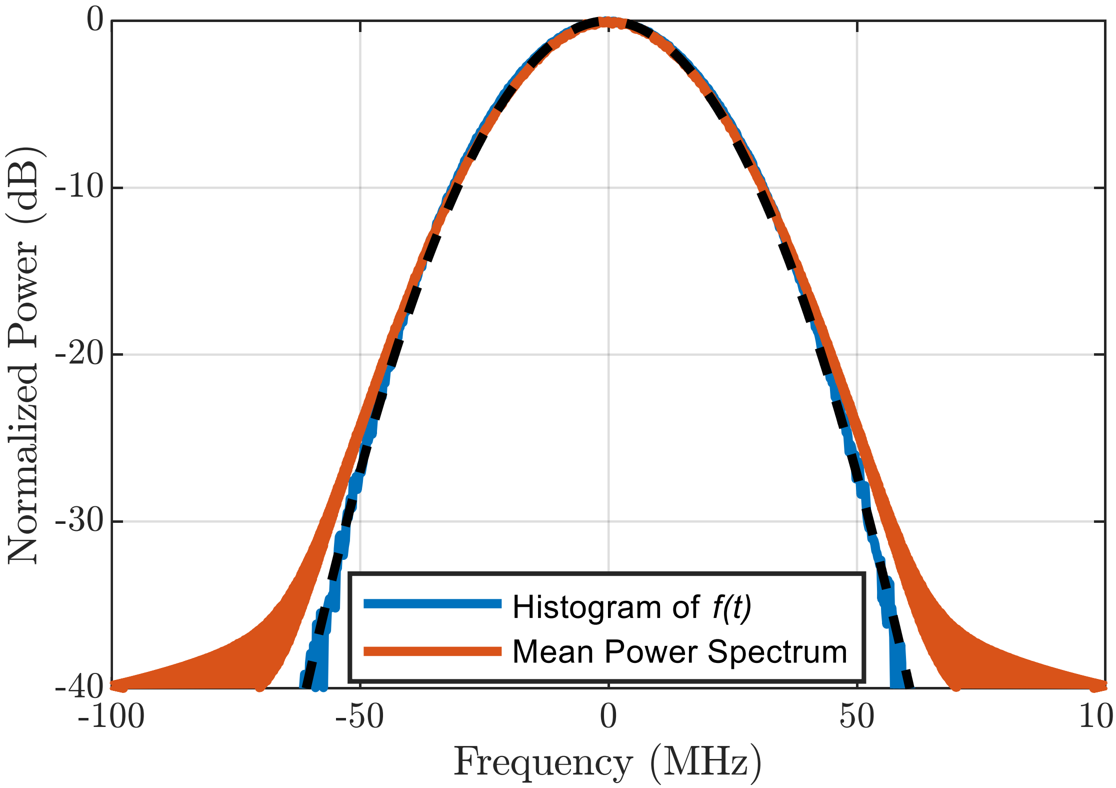
[10] D.A. Hague, P. Kuklinski, “Waveform design using multi-tone feedback frequency modulation,” *IEEE Radar Conf.*, Apr. 2019

Instantaneous Frequency

- Another way to look at the spectral density is to consider instantaneous frequency, where

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \qquad f(t) = -2\pi h \sum_{n=1}^{N} f_n \left|\beta_n\right| \sin\left(2\pi f_n t + \phi_n\right)$$

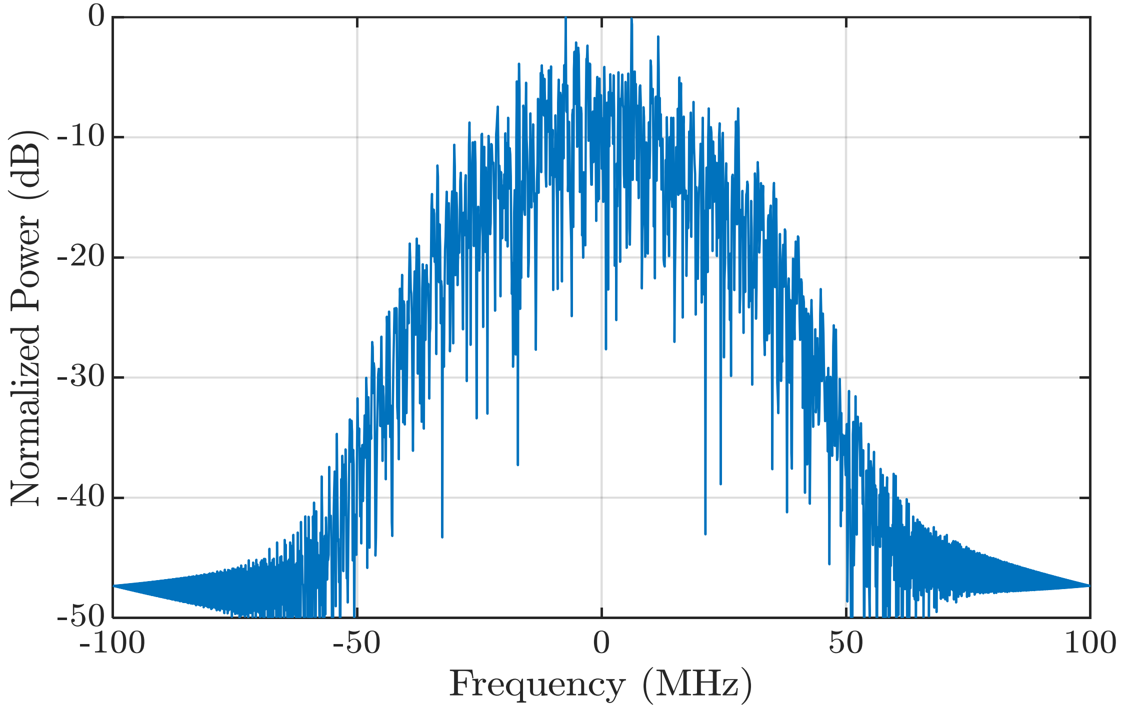
- Generating 10K unique CE-OFDM waveforms and plotting a normalized histogram of instantaneous frequency
- Near perfect match to the normalized mean power spectrum (orange trace) and the expected Gaussian shape (dashed black trace)



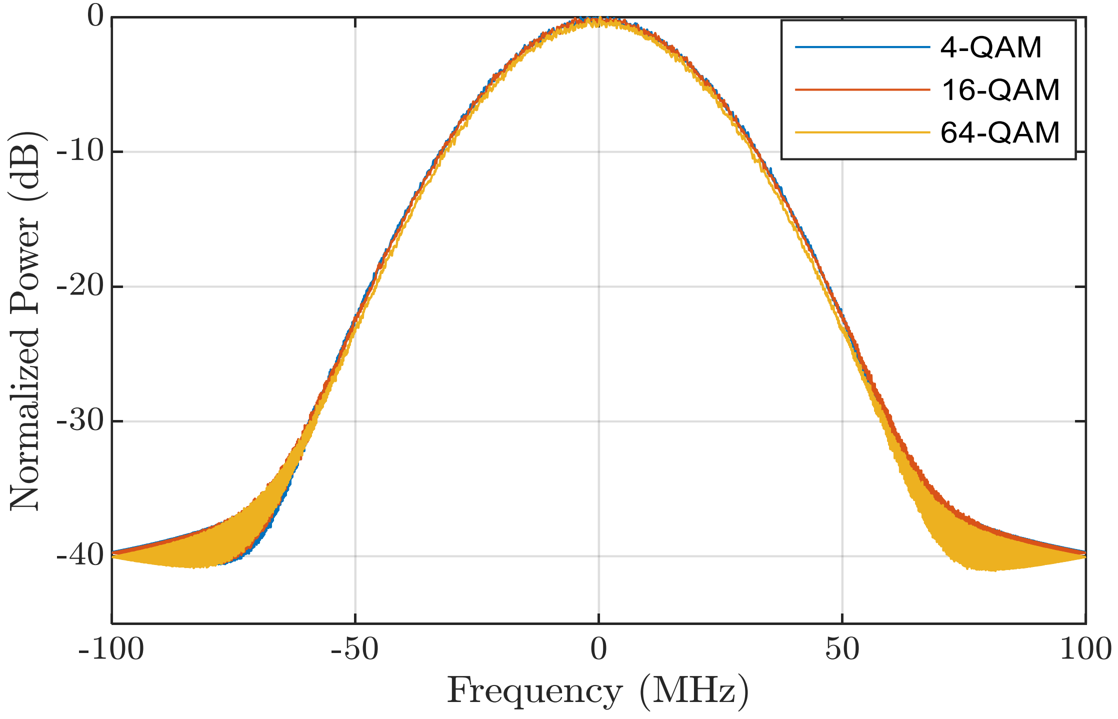
Simulation Results – Power Spectral Density

$BT = 150$ $B_{\text{3-dB}} = 33.3$ MHz $T = 4.5$ μs $N = 35$ subcarriers

Single arbitrary CE-OFDM waveform



Mean of 1K unique CE-OFDM waveforms

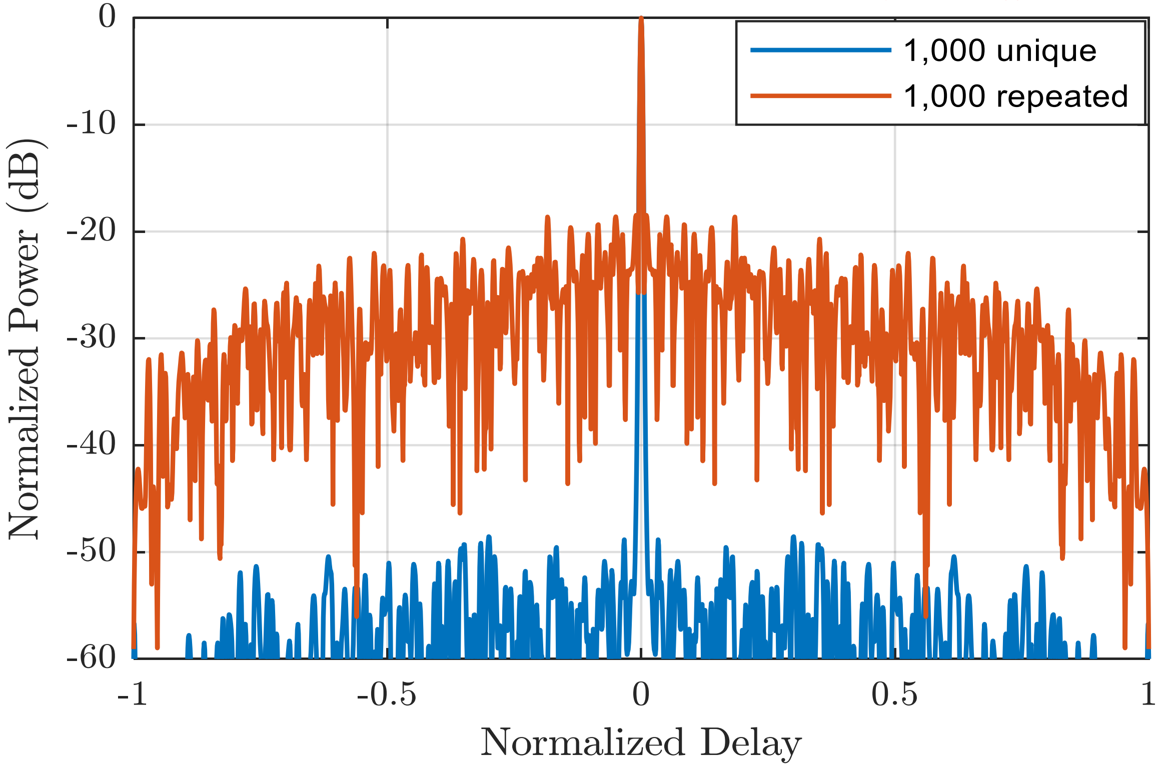


Simulation Results – Autocorrelation Response

$BT = 150$ $B_{3\text{-dB}} = 33.3$ MHz $T = 4.5$ μs $N = 35$ subcarriers

- A single repeated random CE-OFDM waveform achieves a PSL of 19 dB
- Coherent combining (after pulse compression) of 1K unique waveforms achieves a PSL of 49 dB
- The 30 dB = $10 \log_{10}(1{,}000)$ difference is due to the RFM effect of incoherent sidelobe combining, while the mainlobes still combine coherently

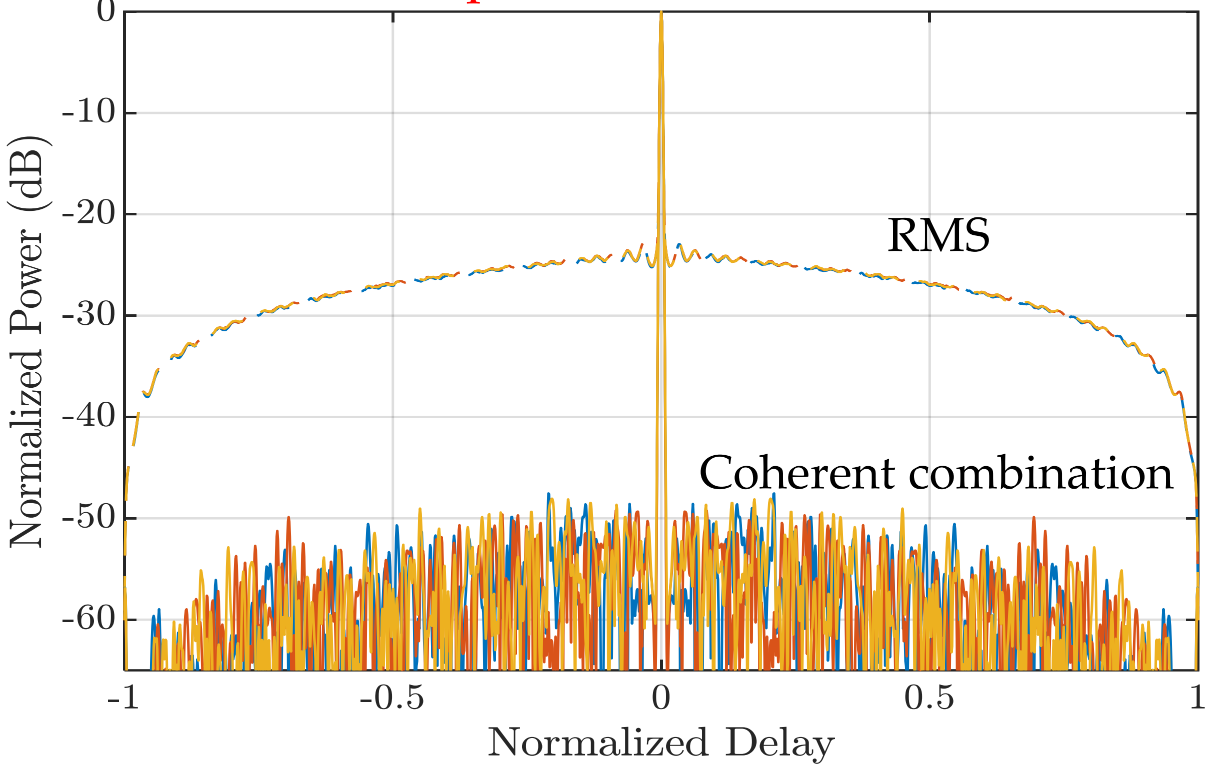
Coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated



Constellation Size Impact

$BT = 150$ $B_{3\text{-dB}} = 33.3$ MHz $T = 4.5$ μs $N = 35$ subcarriers

RMS vs. coherent autocorrelations for 1K unique CE-OFDM waveforms



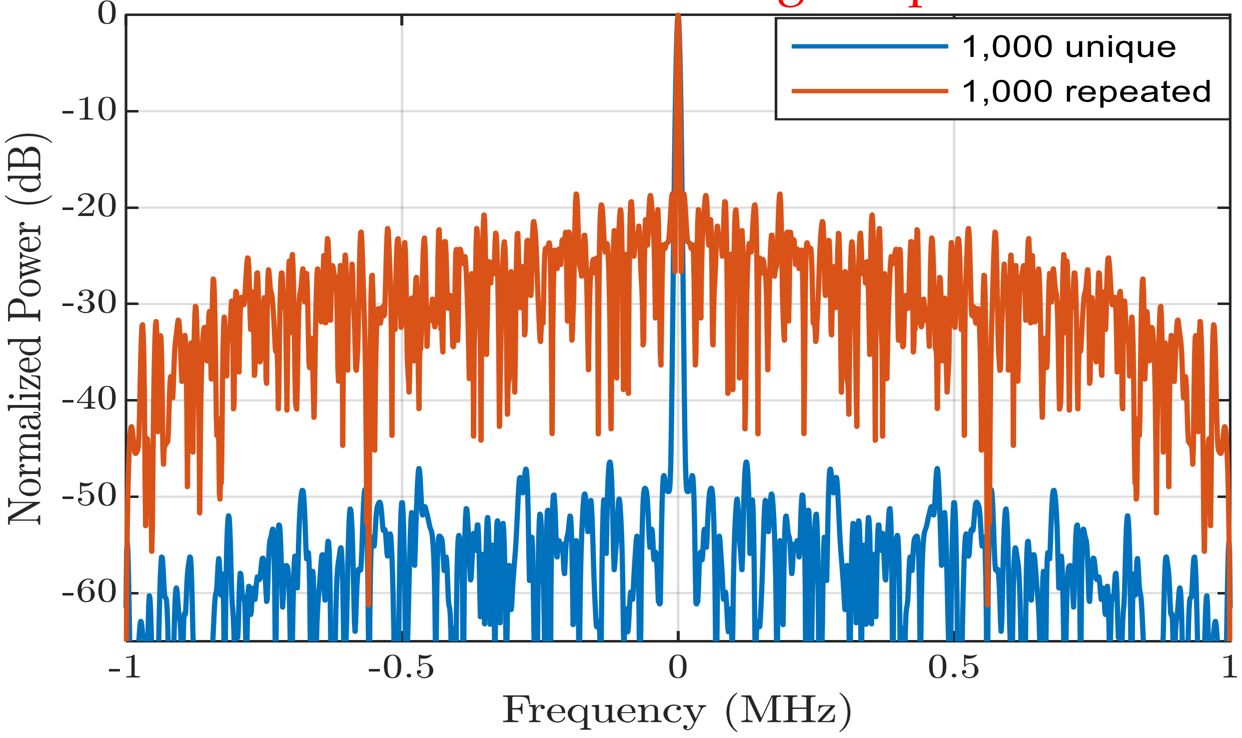
- RMS autocorrelations obtained by averaging (noncoherently) over the 1K unique autocorrelations
- As noted with spectral content, constellation size has no noticeable impact on either RMS or coherently combined autocorrelation responses

Blue = 4-QAM
Orange = 16-QAM
Yellow = 64-QAM

Experimental Loopback Results

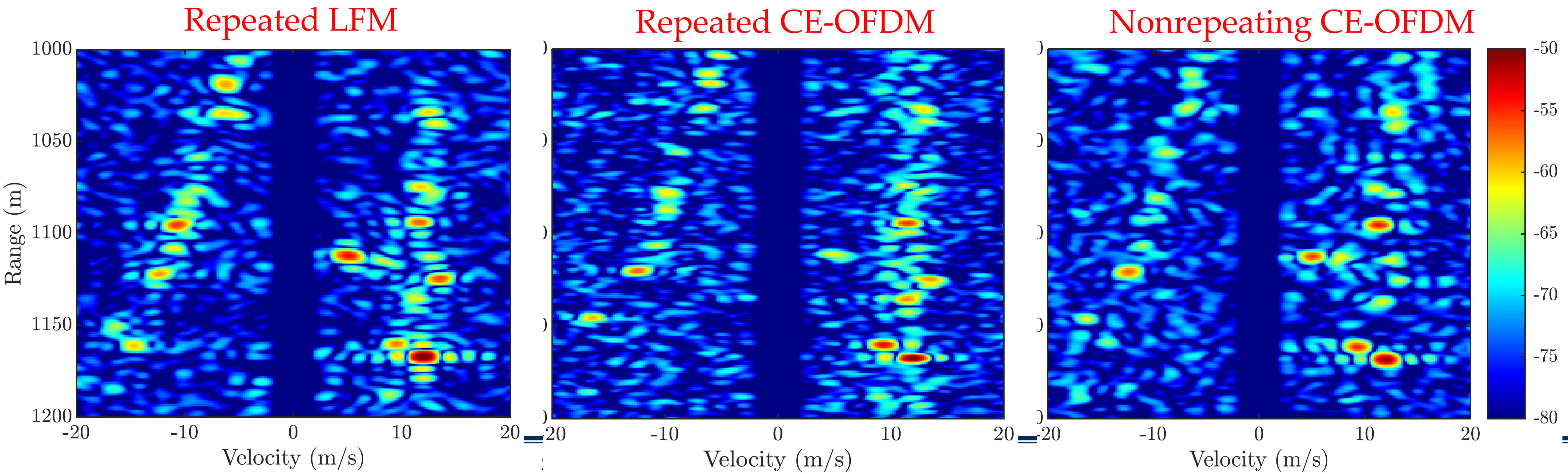
- The 4-QAM case of 1K unique CE-OFDM waveforms was implemented on an arbitrary waveform generator and subsequently captured in loopback
- Loopback test included amplifiers and attenuators to emulate a transmit/receive chain
- Results are indistinguishable from simulation

Loopback version of coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated



Open-Air Experimental Results

- 3 test cases were performed in sequence using different sets of 1K waveforms
  - Repeated LFM, repeated CE-OFDM, and nonrepeating CE-OFDM (random FM version)
- While the scene changes slights during overall collection, it appears that higher sidelobes among the multiple movers at +10 m/s is suppressed in RFM version



Conclusions

- Constant envelope OFDM may have limited utility for comms (alone), but it provides a convenient optimization-free way to generate random FM waveforms with good spectrum shaping and dual-function capabilities
- Like the StoWGe approach, there is a sidelobe suppression performance trade-off by not performing per-waveform optimization
  - Specifically, it has been observed (based on RMS autocorrelation) that CE-OFDM and StoWGe realize PSL of ~$10\log_{10}(BT)$ while per-waveform spectrum shaping optimization methods tend to achieve PSL of ~$20\log_{10}(BT)$
- Of course, with high enough dimensionality in terms of BT and further incoherent sidelobe averaging of $10\log_{10}(M)$ over M unique pulsed waveforms, the above distinction may not matter