

# Assessment of Constant Envelope OFDM as a class of Random FM Radar Waveforms

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- A variety of new Random FM (RFM) waveform classes possessing advantageous spectral shaping have recently been demonstrated.
- However, all of these methods require some form of optimization to achieve this spectral shaping (to provide low range sidelobes).
- In contrast, the signal structure of CE-OFDM naturally realizes this shaping without any need for optimization.
- Consequently, it represents a form of RFM radar waveform with an inherent dual-function radar/comms (DFRC) capability

- Gaussian-shaped power spectrum
  - Corresponding autocorrelation is therefore also Gaussian ... theoretically achieving zero sidelobes
  - Tighter spectral containment can be achieved, but at the cost of higher sidelobes
- Constant amplitude & continuous phase
  - Permits operation of HPA in saturation, while minimizing most of the unavoidable transmitter distortion
- Ability to generate unique FM waveforms having approximately the same spectral content (Gaussian shape realized after averaging)
  - Realize extremely high dimensionality
  - For per-waveform time-bandwidth product  $BT$ , resulting aggregate time-bandwidth product is  $MBT$ , for  $M$  unique waveforms in the coherent processing interval (CPI)

[1] S.D. Blunt, et al., "Principles & applications of random FM radar waveform design," to appear in *IEEE Aerospace & Electronic Systems Magazine*.

- Early approaches (dating back to 1956) used FM driven by noise (no spectrum shaping)
- Spectrum shaping optimization realizes much lower sidelobes, with many classes developed and experimentally demonstrated thus far:
  - Via alternating projections: PRO [2], THoRaCs [3], TTE [4]
  - Via gradient descent: FTE [5], Comp-FM [6]
  - Via off-line design of shaping transform: StoWGe [7]

- [2] J. Jakabosky, S.D. Blunt, B. Himed, “Spectral-shape optimized FM noise radar for pulse agility,” *IEEE Radar Conf.*, May 2016.
- [3] B. Ravenscroft, P.M. McCormick, S.D. Blunt, E. Perrins, J.G. Metcalf, “A power-efficient formulation of tandem-hopped radar & communications,” *IEEE Radar Conf.*, Apr. 2018.
- [4] C.A. Mohr, S.D. Blunt, “FM noise waveforms optimized according to a temporal template error (TTE) metric,” *IEEE Radar Conf.*, Apr. 2019.
- [5] C.A. Mohr, P.M. McCormick, S.D. Blunt, C. Mott, “Spectrally-efficient FM noise radar waveforms optimized in the logarithmic domain,” *IEEE Radar Conf.*, Apr. 2018.
- [6] C.A. Mohr, P.M. McCormick, S.D. Blunt, “Optimized complementary waveform subsets within an FM noise radar CPI,” *IEEE Radar Conf.*, Apr. 2018.
- [7] C.A. Mohr, S.D. Blunt, “Design and generation of stochastically defined, pulsed FM noise waveforms,” *Intl. Radar Conf.*, Sept. 2019.

- Specifically, StoWGe [7] performs optimization offline by designing a transform for a random process that produces unique continuous phase functions with the desired power spectrum in the expectation
  - By avoiding per-waveform optimization (real-time computational cost), the trade-off is somewhat higher sidelobes (but still better than ‘no shaping’)
- CE-OFDM has been proposed as a power-efficient way to implement OFDM [8], though it has not achieved broad utilization
- On the basis of a single waveform, the sidelobe performance of CE-OFDM is limited due to its thumbtack ambiguity function (vs. that of chirp-like waveforms due to conservation of ambiguity)
- Here we consider CE-OFDM within the nonrepeating RFM context, where the signal structure provides spectrum shaping with no optimization whatsoever

[8] S.C. Thompson, et al, “Constant envelope OFDM,” *IEEE Trans. Communications*, vol. 56, no. 8, pp. 1300-1312, Aug. 2008.

- OFDM is the basis for 4G/5G comms, offers high spectral efficiency, and is easy to demodulate and equalize.
- The baseband signal model can be expressed as


$$u(t) = \sum_{n=1}^N \beta_n \exp(j2\pi f_n t)$$

where the complex exponential at subcarrier frequency  $f_n$  is modulated by communication symbol  $\beta_n$ , and subcarriers are spaced in frequency by  $1/T$ , for  $T$  the temporal extent of a symbol.

- Of course, from a radar perspective the non-unity peak-to-average power ratio (PAPR) of OFDM essentially precludes its use for high-power / long-range sensing

- CE-OFDM was developed to achieve power-efficient comms (via unity PAPR)
- CE-OFDM for a single symbol interval can be expressed as

phase of symbol  $\beta_n$

$$s(t) = \exp\left(j2\pi h \Re\left\{\sum_{n=1}^N \beta_n \exp(j2\pi f_n t)\right\}\right) = \exp\left(j2\pi h \sum_{n=1}^N |\beta_n| \cos(2\pi f_n t + \phi_n)\right)$$


where  $h$  is the modulation index and  $\Re\{\bullet\}$  extracts the real part of the argument.

- While readily extensible to multiple symbol intervals, here we set the radar pulse width to  $T$  as well.

- It has been shown in [9,10] that the CE-OFDM signal construction can likewise be expressed as

$$s(t) = \prod_{n=1}^N \sum_{m=-\infty}^{\infty} d_{n,m} \exp(j2\pi m f_n t) \text{rect}\left(\frac{t - T/2}{T}\right)$$

in which

$$d_{n,m} = j^m J_m\left(2\pi h |\beta_n|\right) \exp(jm\phi_n)$$

  $m$ th Bessel function

- Each sum becomes a repeated convolution in frequency, so the central limit theorem implies (average) spectral content tending toward a Gaussian shape without need for spectrum shaping optimization

[9] S.C. Thompson, et al., "Constant envelope OFDM phase modulation: spectral containment, signal space properties and performance," *IEEE Military Communications Conf.*, Oct./Nov. 2004.

[10] D.A. Hague, P. Kuklinski, "Waveform design using multi-tone feedback frequency modulation," *IEEE Radar Conf.*, Apr. 2019

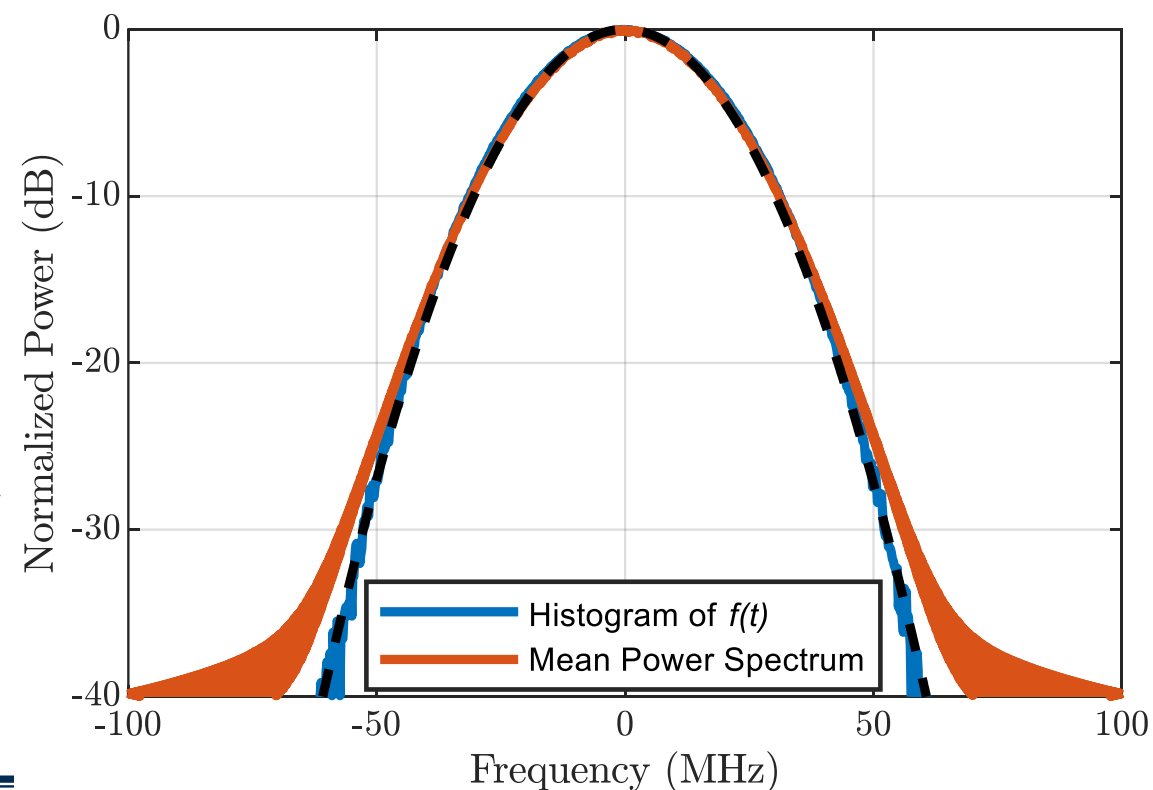


- Another way to look at the spectral density is to consider instantaneous frequency, where

$$f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$f(t) = -2\pi h \sum_{n=1}^N f_n |\beta_n| \sin(2\pi f_n t + \phi_n)$$

- Generating 10K unique CE-OFDM waveforms and plotting a normalized histogram of instantaneous frequency
- Near perfect match to the normalized mean power spectrum (orange trace) and the expected Gaussian shape (dashed black trace)



# Simulation Results – Power Spectral Density

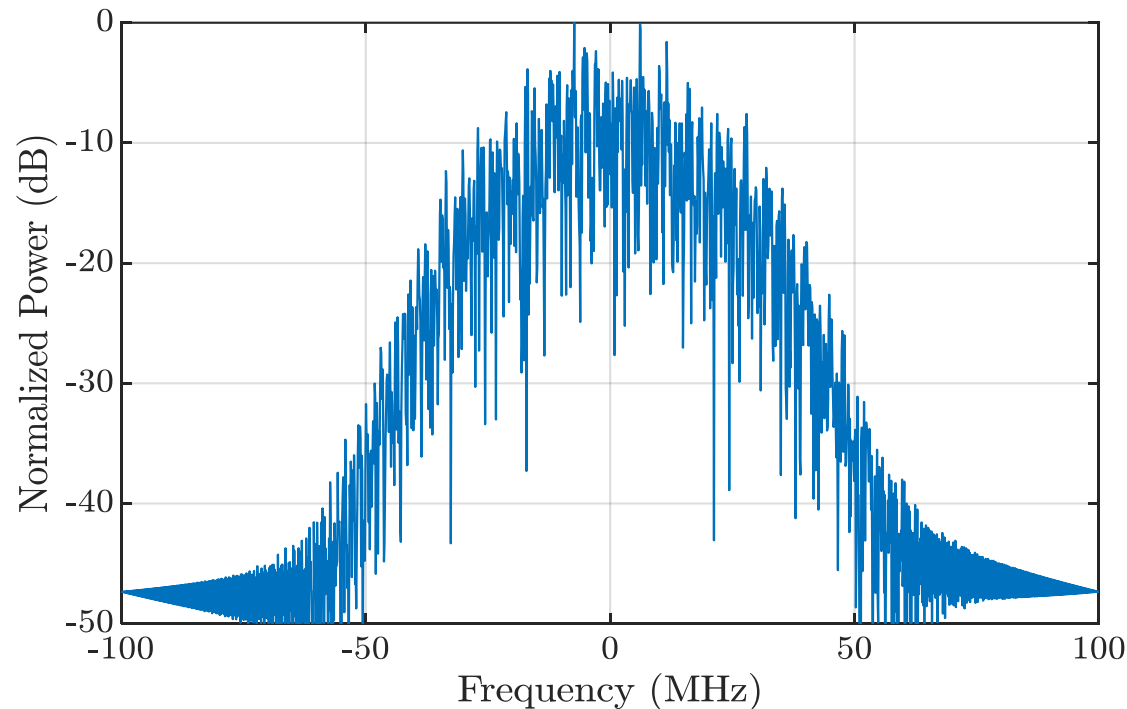
$$BT = 150$$

$$B_{3\text{-dB}} = 33.3 \text{ MHz}$$

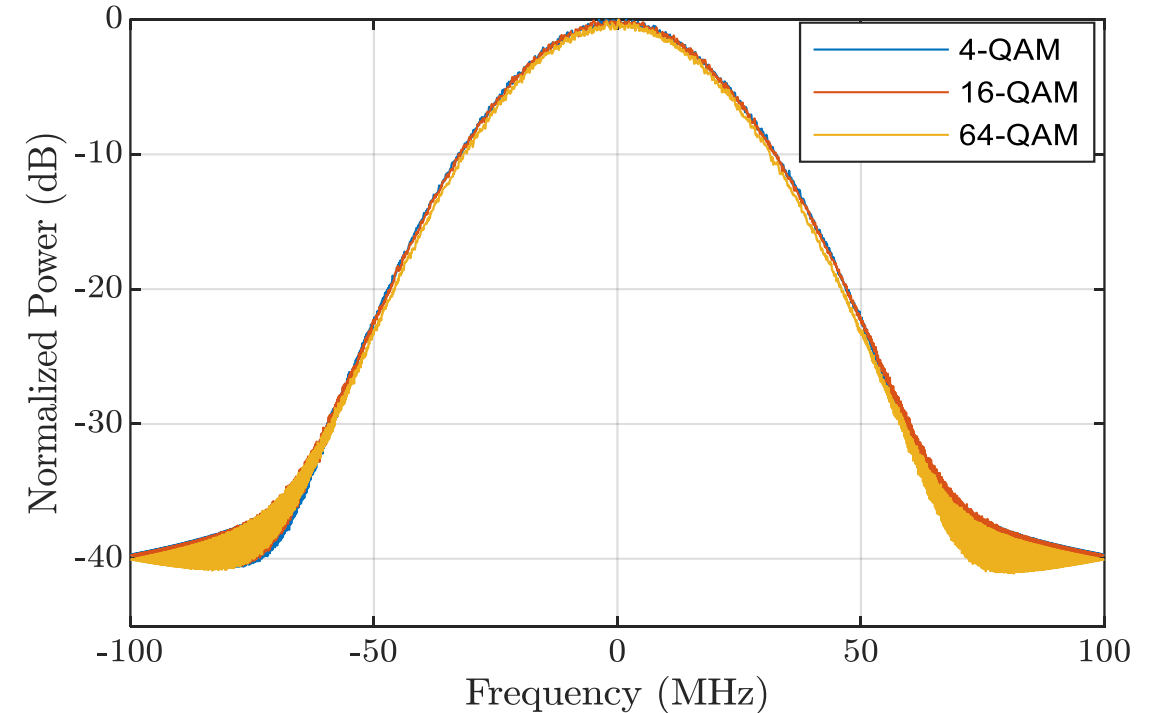
$$T = 4.5 \mu\text{s}$$

$$N = 35 \text{ subcarriers}$$

Single arbitrary CE-OFDM waveform



Mean of 1K unique CE-OFDM waveforms



$$BT = 150$$

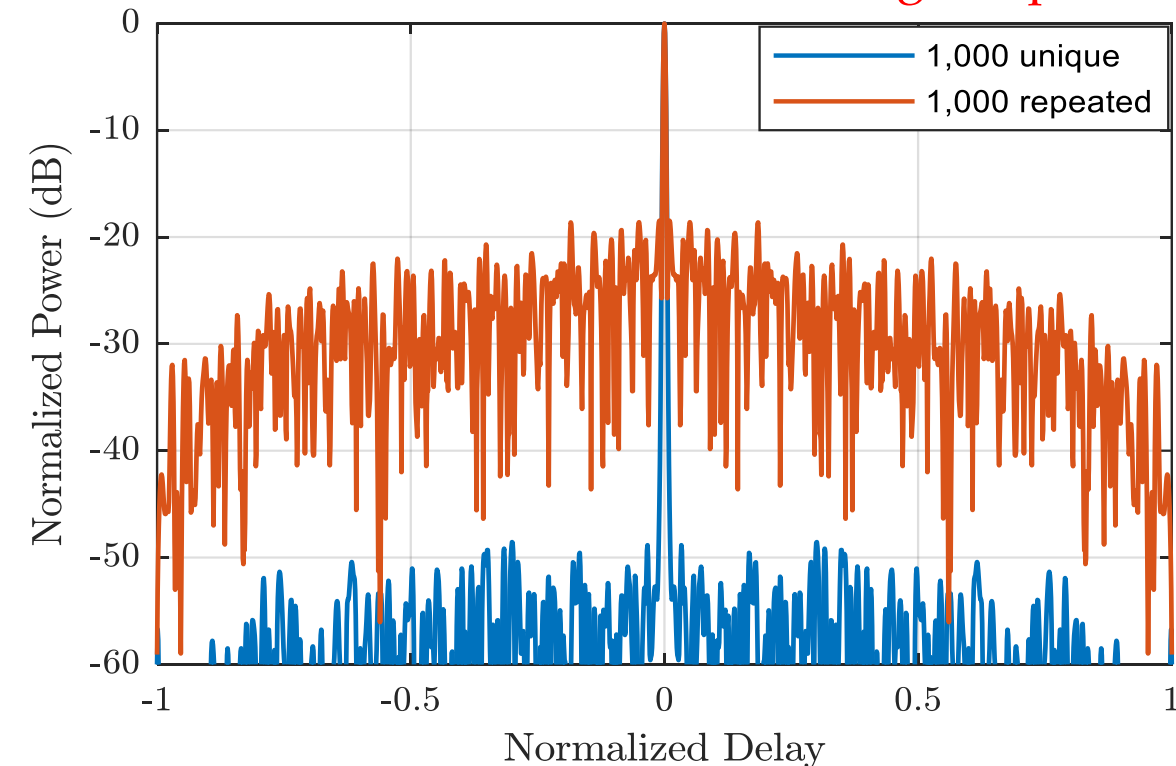
$$B_{3\text{-dB}} = 33.3 \text{ MHz}$$

$$T = 4.5 \text{ } \mu\text{s}$$

$$N = 35 \text{ subcarriers}$$

- A single repeated random CE-OFDM waveform achieves a PSL of 19 dB
- Coherent combining (after pulse compression) of 1K unique waveforms achieves a PSL of 49 dB
- The 30 dB =  $10 \log_{10}(1,000)$  difference is due to the RFM effect of incoherent sidelobe combining, while the mainlobes still combine coherently

Coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated



# Constellation Size Impact

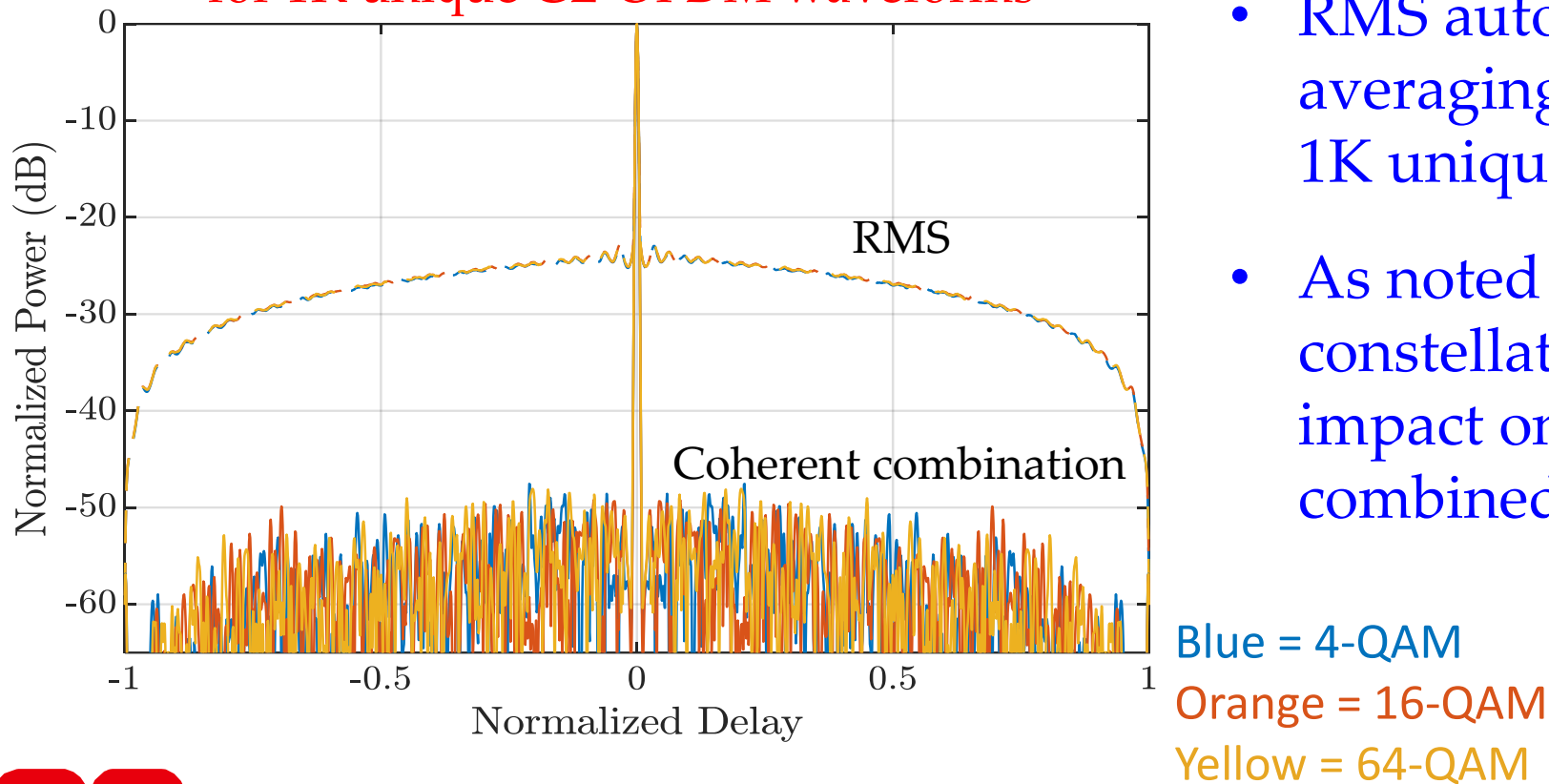
$$BT = 150$$

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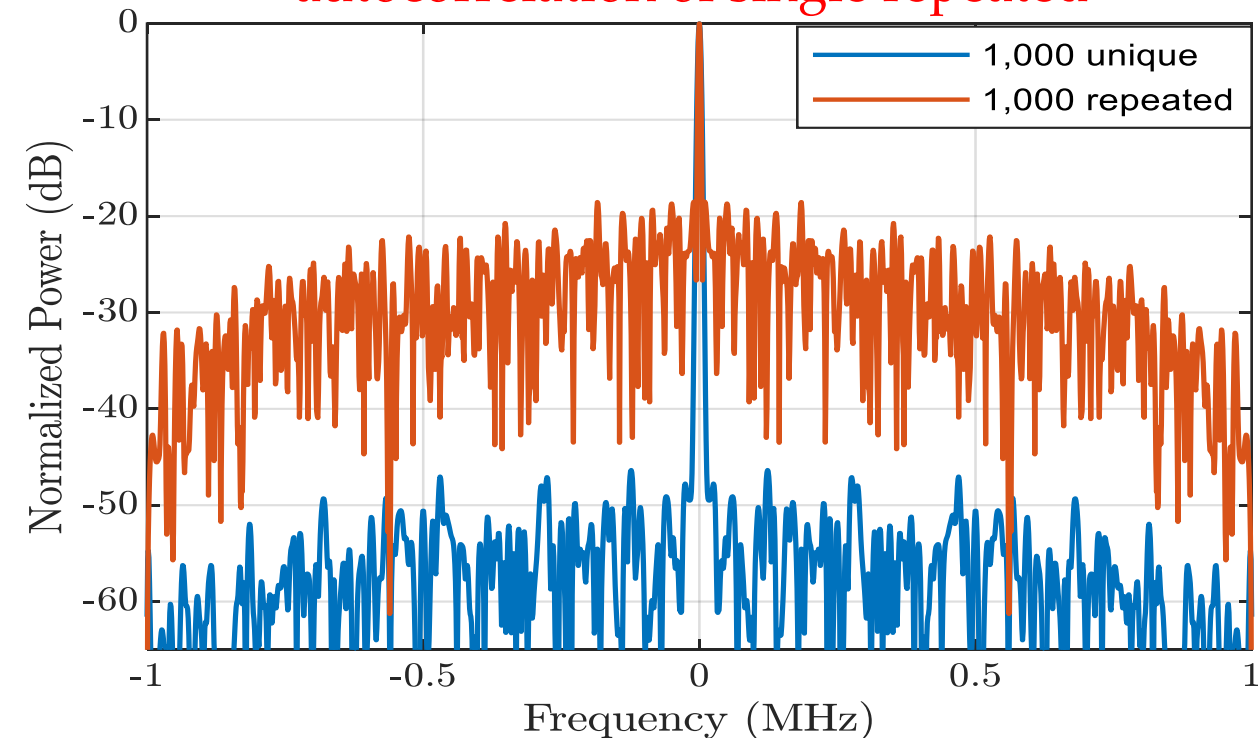
RMS vs. coherent autocorrelations  
for 1K unique CE-OFDM waveforms



- RMS autocorrelations obtained by averaging (noncoherently) over the 1K unique autocorrelations
- As noted with spectral content, constellation size has no noticeable impact on either RMS or coherently combined autocorrelation responses

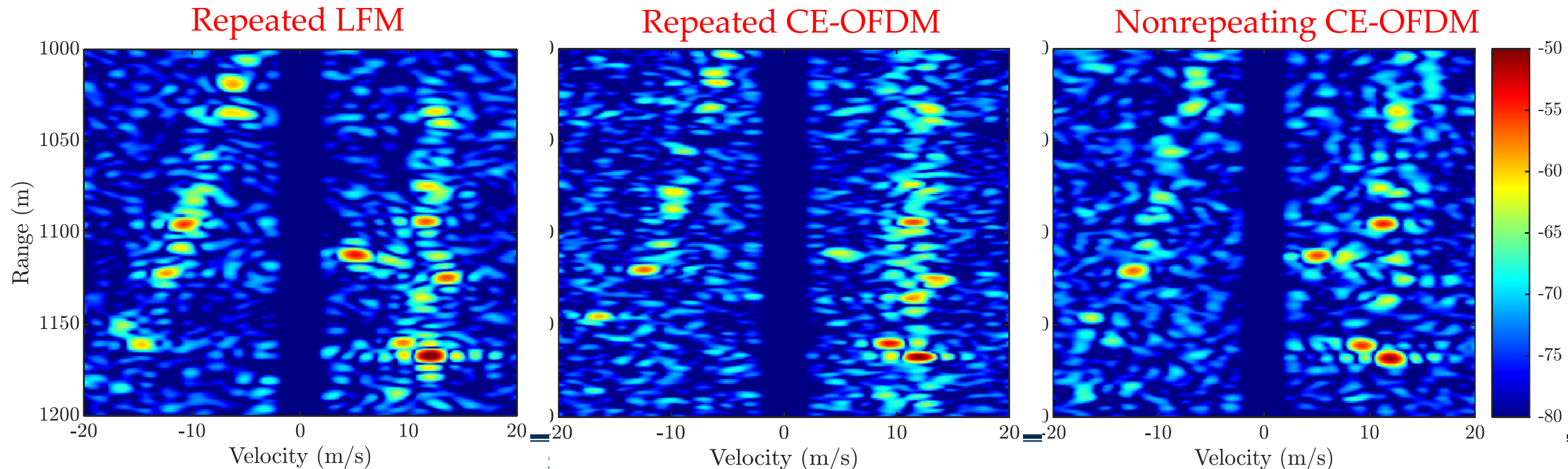
- The 4-QAM case of 1K unique CE-OFDM waveforms was implemented on an arbitrary waveform generator and subsequently captured in loopback
- Loopback test included amplifiers and attenuators to emulate a transmit/receive chain
- Results are indistinguishable from simulation

Loopback version of coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated



# Open-Air Experimental Results

- 3 test cases were performed in sequence using different sets of 1K waveforms
  - Repeated LFM, repeated CE-OFDM, and nonrepeating CE-OFDM (random FM version)
- While the scene changes slightly during overall collection, it appears that higher sidelobes among the multiple movers at +10 m/s is suppressed in RFM version



- Constant envelope OFDM may have limited utility for comms (alone), but it provides a convenient optimization-free way to generate random FM waveforms with good spectrum shaping and dual-function capabilities
- Like the StoWGe approach, there is a sidelobe suppression performance trade-off by not performing per-waveform optimization
  - Specifically, it has been observed (based on RMS autocorrelation) that CE-OFDM and StoWGe realize PSL of  $\sim 10 \log_{10}(BT)$  while per-waveform spectrum shaping optimization methods tend to achieve PSL of  $\sim 20 \log_{10}(BT)$
- Of course, with high enough dimensionality in terms of  $BT$  and further incoherent sidelobe averaging of  $10 \log_{10}(M)$  over  $M$  unique pulsed waveforms, the above distinction may not matter