Assessment of Constant Envelope OFDM as a class of Random FM Radar Waveforms

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Motivation

• A variety of new Random FM (RFM) waveform classes possessing advantageous spectral shaping have recently been demonstrated.

• However, all of these methods require some form of optimization to achieve this spectral shaping (to provide low range sidelobes).

• In contrast, the signal structure of CE-OFDM naturally realizes this shaping without any need for optimization.

• Consequently, it represents a form of RFM radar waveform with an inherent dual-function radar/comms (DFRC) capability
RFM Useful Attributes (in General) \[1\]

- **Gaussian-shaped power spectrum**
  - Corresponding autocorrelation is therefore also Gaussian … theoretically achieving zero sidelobes
  - Tighter spectral containment can be achieved, but at the cost of higher sidelobes

- **Constant amplitude & continuous phase**
  - Permits operation of HPA in saturation, while minimizing most of the unavoidable transmitter distortion

- **Ability to generate unique FM waveforms having approximately the same spectral content (Gaussian shape realized after averaging)**
  - Realize extremely high dimensionality
  - For per-waveform time-bandwidth product $BT$, resulting aggregate time-bandwidth product is $MBT$, for $M$ unique waveforms in the coherent processing interval (CPI)

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Random FM Generation

• Early approaches (dating back to 1956) used FM driven by noise (no spectrum shaping)

• Spectrum shaping optimization realizes much lower sidelobes, with many classes developed and experimentally demonstrated thus far:
  • Via alternating projections: PRO [2], THoRaCs [3], TTE [4]
  • Via gradient descent: FTE [5], Comp-FM [6]
  • Via off-line design of shaping transform: StoWGe [7]

Specifically, StoWGe [7] performs optimization offline by designing a transform for a random process that produces unique continuous phase functions with the desired power spectrum in the expectation

- By avoiding per-waveform optimization (real-time computational cost), the trade-off is somewhat higher sidelobes (but still better than ‘no shaping’)

CE-OFDM has been proposed as a power-efficient way to implement OFDM [8], though it has not achieved broad utilization

On the basis of a single waveform, the sidelobe performance of CE-OFDM is limited due to its thumbtack ambiguity function (vs. that of chirp-like waveforms due to conservation of ambiguity)

Here we consider CE-OFDM within the nonrepeating RFM context, where the signal structure provides spectrum shaping with no optimization whatsoever

• OFDM is the basis for 4G/5G comms, offers high spectral efficiency, and is easy to demodulate and equalize.

• The baseband signal model can be expressed as

\[ u(t) = \sum_{n=1}^{N} \beta_n \exp(j2\pi f_n t) \]

where the complex exponential at subcarrier frequency \( f_n \) is modulated by communication symbol \( \beta_n \), and subcarriers are spaced in frequency by \( 1/T \), for \( T \) the temporal extent of a symbol.

• Of course, from a radar perspective the non-unity peak-to-average power ratio (PAPR) of OFDM essentially precludes its use for high-power / long-range sensing.
CE-OFDM Signal Model

- CE-OFDM was developed to achieve power-efficient comms (via unity PAPR)
- CE-OFDM for a single symbol interval can be expressed as

\[ s(t) = \exp\left(j2\pi h \Re\left\{\sum_{n=1}^{N} \beta_n \exp(j2\pi f_n t)\right\}\right) = \exp\left(j2\pi h \sum_{n=1}^{N}\beta_n \cos(2\pi f_n t + \phi_n)\right) \]

where \( h \) is the modulation index and \( \Re\{\bullet\} \) extracts the real part of the argument.
- While readily extensible to multiple symbol intervals, here we set the radar pulse width to \( T \) as well.
CE-OFDM Signal Model

• It has been shown in [9,10] that the CE-OFDM signal construction can likewise be expressed as

\[ s(t) = \prod_{n=1}^{N} \sum_{m=-\infty}^{\infty} d_{n,m} \exp\left( j2\pi m f_n t \right) \text{rect}\left( \frac{t - T/2}{T} \right) \]

in which

\[ d_{n,m} = j^m J_m(2\pi h|\beta_n|) \exp(jm\phi_n) \]

• Each sum becomes a repeated convolution in frequency, so the central limit theorem implies (average) spectral content tending toward a Gaussian shape without need for spectrum shaping optimization


Another way to look at the spectral density is to consider instantaneous frequency, where

\[
f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}
\]

\[
f(t) = -2\pi h \sum_{n=1}^{N} f_n |\beta_n| \sin(2\pi f_n t + \phi_n)
\]

• Generating 10K unique CE-OFDM waveforms and plotting a normalized histogram of instantaneous frequency

• Near perfect match to the normalized mean power spectrum (orange trace) and the expected Gaussian shape (dashed black trace)
Simulation Results – Power Spectral Density

\( BT = 150 \quad B_{3 \text{-dB}} = 33.3 \text{ MHz} \quad T = 4.5 \mu s \quad N = 35 \text{ subcarriers} \)

Single arbitrary CE-OFDM waveform

Mean of 1K unique CE-OFDM waveforms
Simulation Results – Autocorrelation Response

$$BT = 150 \quad B_{3\text{-}dB} = 33.3 \text{ MHz} \quad T = 4.5 \mu\text{s} \quad N = 35 \text{ subcarriers}$$

- A single repeated random CE-OFDM waveform achieves a PSL of 19 dB
- Coherent combining (after pulse compression) of 1K unique waveforms achieves a PSL of 49 dB
- The 30 dB = 10 log_{10}(1,000) difference is due to the RFM effect of incoherent sidelobe combining, while the mainlobes still combine coherently

Coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated
Constellation Size Impact

- RMS autocorrelations obtained by averaging (noncoherently) over the 1K unique autocorrelations
- As noted with spectral content, constellation size has no noticeable impact on either RMS or coherently combined autocorrelation responses

\[ BT = 150 \quad B_{3\text{-dB}} = 33.3 \text{ MHz} \quad T = 4.5 \ \mu\text{s} \quad N = 35 \text{ subcarriers} \]

RMS vs. coherent autocorrelations for 1K unique CE-OFDM waveforms

- Blue = 4-QAM
- Orange = 16-QAM
- Yellow = 64-QAM
Experimental Loopback Results

- The 4-QAM case of 1K unique CE-OFDM waveforms was implemented on an arbitrary waveform generator and subsequently captured in loopback.
- Loopback test included amplifiers and attenuators to emulate a transmit/receive chain.
- Results are indistinguishable from simulation.

Loopback version of coherent autocorrelation of 1K unique CE-OFDM waveforms vs. autocorrelation of single repeated.
Open-Air Experimental Results

- 3 test cases were performed in sequence using different sets of 1K waveforms
  - Repeated LFM, repeated CE-OFDM, and nonrepeating CE-OFDM (random FM version)
- While the scene changes slightly during overall collection, it appears that higher sidelobes among the multiple movers at +10 m/s is suppressed in RFM version
Conclusions

• Constant envelope OFDM may have limited utility for comms (alone), but it provides a convenient optimization-free way to generate random FM waveforms with good spectrum shaping and dual-function capabilities

• Like the StoWGe approach, there is a sidelobe suppression performance trade-off by not performing per-waveform optimization
  • Specifically, it has been observed (based on RMS autocorrelation) that CE-OFDM and StoWGe realize PSL of $\sim 10 \log_{10}(BT)$ while per-waveform spectrum shaping optimization methods tend to achieve PSL of $\sim 20 \log_{10}(BT)$

• Of course, with high enough dimensionality in terms of $BT$ and further incoherent sidelobe averaging of $10 \log_{10}(M)$ over $M$ unique pulsed waveforms, the above distinction may not matter