Incorporating Hopped Spectral Gaps Into Nonrecurrent Nonlinear FMCW Radar Emissions

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***Abstract* – The time-varying landscape of spectral congestion is driving the investigation into new forms of "spectrally aware" radar emissions based on passive sensing of the environment. The recent Pseudo-Random Optimized FMCW (PRO-FMCW) framework, which can be viewed as an instantiation of FM noise radar, generates a nonlinear FMCW waveform that does not repeat and is designed using spectrally shaped optimization to improve range sidelobe and spectral containment. Here, these concepts are combined to generate time-varying spectral gaps within the PRO-FMCW waveform to avoid in-band interference. The impact on radar range sidelobe performance is considered with regard to both static and time-varying spectral gaps.**

I. Introduction

As the RF spectrum becomes increasingly crowded a new "cognitive" operating paradigm is emerging whereby awareness of other spectrum users and the resulting interference to/from oneself is a chief concern. With 5G commercial communications embracing the notion of "network densification" and hyper connectivity via machine-type communication [1], it is clear that new emission structures and receive processing schemes are imperative for future radar systems to achieve current sensing requirements, much less meet demands for further sensitivity enhancements [2,3]. Generally speaking, what is needed is continued advances in both interference avoidance and interference cancellation, with particular attention given to the potential for a highly non-stationary spectral environment as more systems adopt this cognitive paradigm that may lead to the emergence of cognitive "power struggles" [4] following from a Baldwinian evolutionary perspective [5].

The current radar spectral emission requirements are delineated (at least in the US) in the NTIA "red book" [6] with more or less similar requirements observed worldwide. The main focus of these requirements is on spectral containment, which includes the rate of spectral roll-off from the passband and the out-of-band suppression. With the expectation of a far more dynamic spectral environment in the future combined with the ongoing prospect of emissions from non-compliant devices, the impetus is on the radar to adapt to this environment. In fact, the radar sub-discipline of waveform diversity [7-9] was actually instituted (in 2002) in part for the very purpose of addressing the anticipated problem of spectral congestion. The subsequent notion of cognitive radar, as first articulated by Haykin as a perception-action cycle [10] and later by Guerci in the knowledge-aided context [11], can be viewed in some respects as a proactive extension of waveform diversity [12].

Here we explore the design of radar emissions that require significant bandwidth yet must operate in relatively dense spectral environments (e.g. [13]). One approach to this problem is to pose and solve an optimization problem that is updated in real-time to seek a balance between maximizing bandwidth and acceptable signal to interference plus noise ratio (SINR) based on passive sensing of the available spectrum [14]. We build upon this framework by developing an emission structure that permits incorporation of time-varying spectral gaps to reduce interference to/from the radar and ultimately to enable more advantageous decisions within a bandwidth / SINR trade space.

The notion of spectrally gapped radar waveforms is not new (e.g. [15-17]). However, the important distinctions explored here are 1) the locations of the spectral gaps change with time, which in fact is advantageous from an ambiguity function perspective, and 2) the waveform structure to which the spectral gapping is applied is a physical instantiation of FM noise radar denoted as pseudo-random optimized FMCW (PRO-FMCW) [18]. Along with the ultra-low sidelobe (ULS) emission developed in [19], the PRO-FMCW structure is based on a spectrally shaped optimization procedure. To realize gaps within this spectrally shaped design framework a stage employing the reiterative uniform weighted optimization (RUWO) approach from [20] is also used.

II. Spectrally Shaped Optimization

The FMCW spectrally shaped optimization from [18] involves the generation of a random phase signal segment as initialization that is optimized to match a desired power spectral density (PSD), followed by phase alignment with the previous optimized segment to avoid phase discontinuities. To produce the desired spectral gaps with sufficient null depth, an additional step is added here prior to phase alignment that leverages the RUWO method from [20].

The PRO-FMCW method [18] employs a desired PSD $|G(f)|^2$ having a Gaussian shape due to a well-known property of nonlinear FM waveforms that states that low range sidelobes can be achieved when the signal spectrum decreases towards the band edges [21]. Further, the Gaussian shape in particular is attractive because the Fourier transform of a Gaussian is a Gaussian, thus the autocorrelation is likewise Gaussian-shaped (in theory). In [18] and [19] it was observed via simulation and experimental measurement that, using this Gaussian PSD shaping, the Gaussian roll-off in autocorrelation is achieved until a (quite low) sidelobe floor is reached, which is thought to be a result of finite fidelity. By optimizing according to the PSD, which essentially reflects the average amount of time the FMCW waveform spends in a given frequency, the specific

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time-frequency function is free to vary. As such, the waveform structure needs never repeat.

The spectrally shaped optimization is applied independently to individual segments that collectively comprise the overall FMCW waveform. To incorporate spectral gaps into the formulation from [18], the PSD is modified from its Gaussian shape such that

$$\left|G(f)\right| = 0 \quad \text{for} \quad f \in \Omega\,, \tag{1}$$

where Ω is the set of frequency intervals corresponding to where spectral gaps are desired. For the mth temporal segment of the waveform, the iterative optimization from [18] first applies the alternating projections

$$r_{k+1,m}(t) = \mathbb{F}^{-1}\left\{\left|G(f)\right| \exp\left(j\angle \mathbb{F}\left\{p_{k,m}(t)\right\}\right)\right\} \tag{2}$$

and

$$p_{k+1,m}(t) = u(t)\exp\left(j\angle r_{k+1,m}(t)\right), \tag{3}$$

where $p_{0,m}(t)$ is the initial random phase signal for the mth segment, $u(t)$ is a rectangular window of length T and unit amplitude, $\mathbb{F}$ and $\mathbb{F}^{-1}$ are the Fourier and inverse Fourier transforms, respectively, and $\angle(\bullet)$ extracts the phase of the argument. This alternating process is repeated for K iterations to obtain the mth optimized signal segment denoted $p_{K,m}(t)$.

The process above was shown in [18] to perform well in achieving the desired Gaussian spectral roll-off. However, it has been found to be somewhat limited in the attainment of deep nulled spectral gaps. As such, the optimized segment $p_{K,m}(t)$ is subsequently modified using the RUWO approach from [20] to enhance the null depth of the spectral gaps.

Denote the optimized segment $p_{K,m}(t)$ of temporal extent T in a discretized form as the N-length vector $\mathbf{x}_{0,m}$, which is "over-sampled" relative to 3 dB bandwidth of the segment as discussed in [22] to ensure the representation has sufficient fidelity. The set of frequency intervals in Ω to be nulled are likewise discretized into Q values of f_q having sufficient density to approximate a flat spectral gap adequately. Using these discrete frequency values, an $N \times N$ structured matrix is then constructed as

$$\mathbf{W} = \mathbf{B}\mathbf{B}^H + \delta\mathbf{I}\,, \tag{4}$$

where δ is a diagonal loading term, $\mathbf{I}$ is an $N \times N$ identity matrix, and

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j2\pi f_0} & e^{j2\pi f_1} & \cdots & e^{j2\pi f_{Q-1}} \\ \vdots & \vdots & & \vdots \\ e^{j2\pi f_0 (N-1)} & e^{j2\pi f_1 (N-1)} & \cdots & e^{j2\pi f_{Q-1}(N-1)} \end{bmatrix} \tag{5}$$

is an $N \times Q$ matrix of frequency steering vectors corresponding to the Q discretized frequency values approximating Ω.

The RUWO algorithm [20] is iterated L times according to

$$\mathbf{x}_{l,m} = \exp\left(j\angle\left(\mathbf{W}^{-1}\,\mathbf{x}_{l-1,m}\right)\right), \tag{6}$$

such that the final result $\mathbf{x}_{L,m}$, due to being sufficiently "over-sampled" with respect to 3 dB bandwidth, very accurately translates into the continuous signal $x_{L,m}(t)$ following D/A conversion. This signal is amenable to emission by a high-power transmitter due to being constant modulus and spectrally well-contained as discussed in [22]. Finally, to prevent phase discontinuities, this mth optimized signal is phase rotated to yield the mth waveform segment as

$$s_m(t) = \exp\left(j\phi_{\text{end},m-1}\right) x_{L,m}(t)\,, \tag{7}$$

where $\phi_{\text{end},m-1}$ is the phase at the end of the $(m-1)$th segment.

This optimization procedure can be implemented efficiently using FFTs and a GPU. The structured matrix in (4) only needs to be modified when the spectral environment changes. Receive processing can be performed in the same manner as in [18] by separating the received signal into the constituent segments for matched filter pulse compression and subsequent Doppler processing across the segments.

III. Simulation Results

The ultimate goal for this scheme is to facilitate "on the fly" waveform responsiveness to a dynamic environmental interference. However, the focus here is on the impact these hopping spectral gaps have upon radar performance. As such, the gapped PRO-FMCW scheme is demonstrated using four test cases in simulation.

In the first case a single stationary spectral gap exists. In the second case the effect of two stationary gaps of non-equal width is considered. In the third case, a spectral gap hops sequentially through the radar bandwidth during a single coherent processing interval (CPI) for Doppler processing. Finally, in the fourth case a spectral gap is hopped randomly to represent responsiveness to a changing spectral environment (albeit a simple one with a single interferer). For all cases, the overall waveform has a length of T_{w} = 200 ms with $M = 10^4$ segments. Each segment has length T = 20 μs and a 3 dB bandwidth of B = 80 MHz, yielding an effective time-bandwidth product of 1600 for each segment.

For case 1), the single gap is placed near the edge of the 3 dB bandwidth, with a gap width of $B/5$. The optimization is able to achieve a gap null depth of –70 dB relative to the spectral peak (center frequency) as illustrated in Fig. 1. It is generally known and was demonstrated experimentally in [15] that inserting gaps into the radar passband degrades sensitivity due to increased range sidelobes. Figure 2 depicts the integrated autocorrelation which is obtained via coherent integration across the 10^4 segments after pulse compression matched filtering. Because the segments are unique, since each is initialized with an independent random phase signal, the range sidelobes destructively interfere when the segment matched filter responses are combined. Thus, while this response is degraded relative to the non-gapped result depicted in [18] for the same parameters, the original PRO-FMCW approach achieved such significant range sidelobe suppression that the degraded response in Fig. 2 is still quite good.

Figures 1 and 2 also depict the PSD and integrated autocorrelation for case 2) in which there are two spectral gaps: the previous one with gap width of $B/5$ and an additional gap on the other side of the center frequency (though not symmetric) with a gap spectral width of $B/8$. From Fig. 1 it is observed that the addition of the second gap degrades the gap depth by about 10 dB relative to the single gap case. Further, the integrated autocorrelation in Fig. 2 shows that the two-gap case realizes a range sidelobe degradation of roughly 10 dB relative to the single gap case.

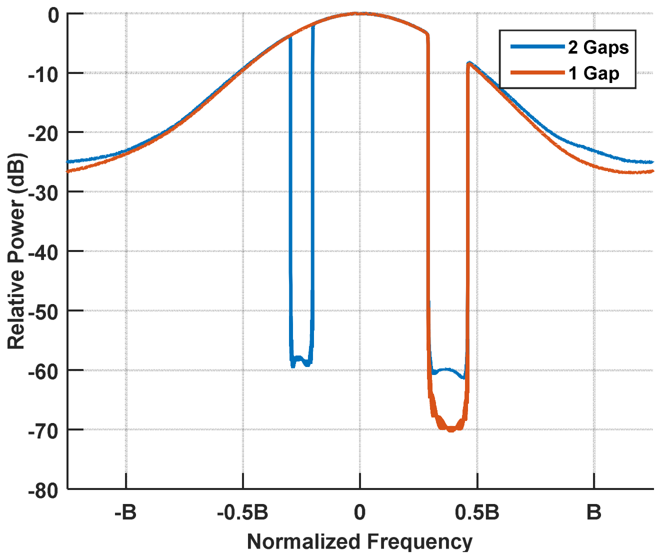

Fig. 1. Mean power spectral density of the 1 and 2 gap cases

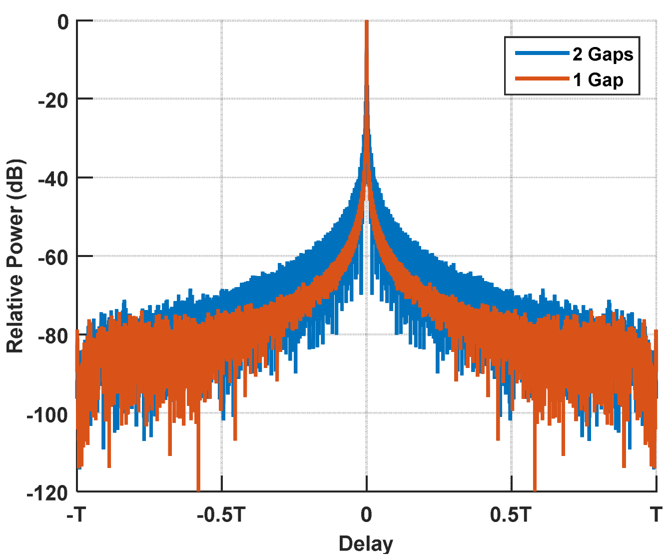

Fig. 2. Integrated autocorrelation of the 1 and 2 gap cases

Now consider the impact of a hopping spectral gap, such as to address non-stationary in-band interference or perhaps to accommodate a frequency hopped communication system that is coordinated with the radar system. Figure 3 shows the PSD for each of 10 different spectral gaps of width $B/8$. Each gap is used in $1/10^{\text{th}}$ (or 10^3) of the total 10^4 waveform segments (so each gap persists for 20 ms). Aside from a specified gap, each segment is otherwise unique due to independent random initialization as previously discussed. Note that the notch depths essentially follow the envelope of the overall PSD.

When the PSD for the entire waveform is considered (all 10^4 segments together) as shown in Fig. 4, a response quite close to the original Gaussian PSD is obtained due to the fact that the different gaps average out. The important of this fact is readily apparent when examining the integrated autocorrelation as depicted in Fig. 5. Whereas the fixed gap cases from Fig. 2 realized an LFM-like range sidelobe roll-off with a peak sidelobe in the neighborhood of –30 dB, the sequential gap case in Fig. 5 realizes a peak sidelobe around –50 dB.

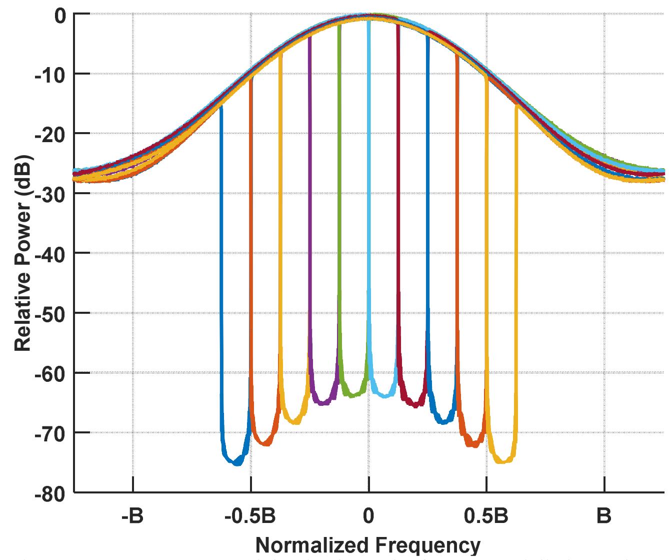

Fig. 3. Mean power spectral density for each of 10 sequentially hopped gaps

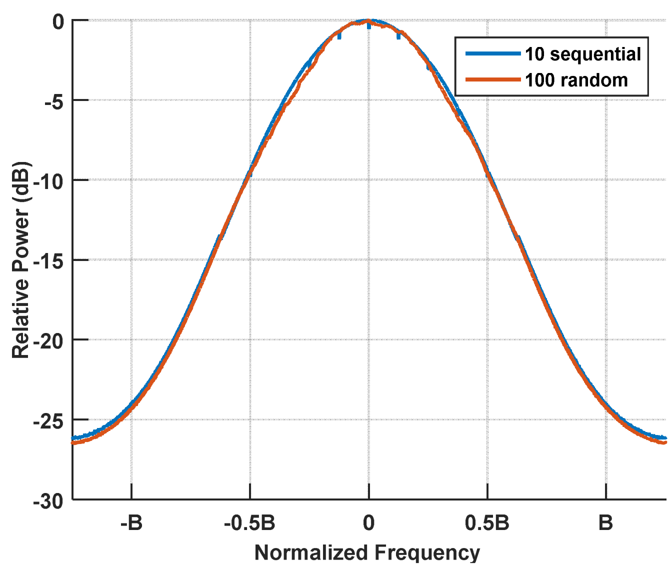

Fig. 4. Mean power spectral density for the sequential and random hopping gap cases

Finally, we consider the effect of a randomly hopped spectral gap of width $B/8$ that can reside over the same frequency interval as depicted in Fig. 3 for the sequential gap case. Here, 100 different gap locations are randomly generated, each pertaining to 10^2 out of the total 10^4 segments (so each gap persists for 2 ms). In Fig. 4 it is observed that the mean PSD over the entire waveform (all 10^4 segments together) is now smoother than that obtained for the sequential case, particularly at the peak of the passband. As a result, the integrated autocorrelation in Fig. 5 reveals a significant overall reduction in range sidelobes, though the peak sidelobe remains roughly the same.

If the rate of spectral gap hopping were even higher, one could expect further sidelobe reduction that could perhaps ultimately converge to the no-gap case from [18]. Simply put, the hopping of spectral gaps serves to smooth out the overall PSD to something closer to that which is desired, thus producing lower integrated autocorrelation sidelobes relative to when a static gap is present.

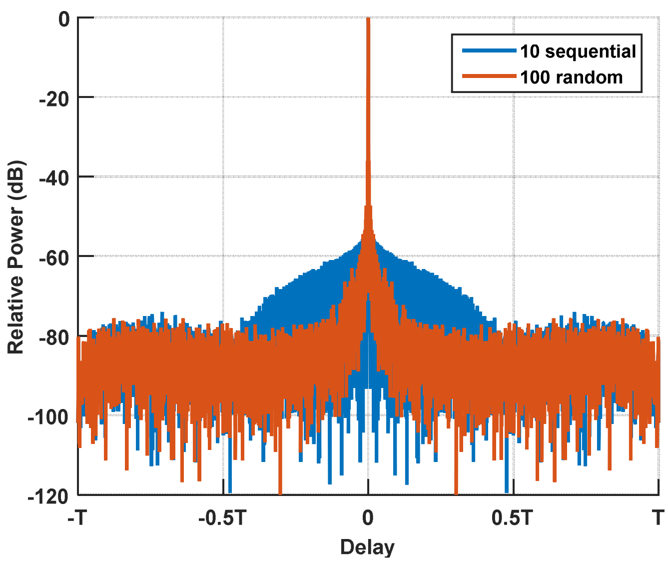

Fig. 5. Integrated autocorrelation of the sequential and random hopping gap case

IV. CONCLUSIONS

The design of a nonrecurrent nonlinear FMCW waveform with hopping spectral gaps was demonstrated. Gap null depths of at least –60 dB were achieved in all cases. As expected, static spectral gaps realize degradation in range sidelobes, though the resulting performance may still be acceptable since the original no-gap performance was quite good to begin with so some degradation margin could be tolerated. When the spectral gaps are allowed to hop the overall PSD of the waveform is smoothed out thereby improving range sidelobes relative to the static gap case. Further, as the rate of gap hopping increases the degree of PSD smoothing is likewise improved. From the standpoint of interference avoidance it is thus preferable for the interference to vary in frequency, as long as the tasks of spectrum monitoring and subsequent segment optimization can be performed in time to respond. In contrast, the notion of a tandem hopped communication signal residing in the radar spectral gaps could be quite well suited to fast hopping. Ongoing work is exploring shaping of gap edges to further reduce range sidelobes, assessment of the impact on Doppler processing due to hopping gaps, and characterization of transmitter distortion on gap depth due to spectral regrowth.

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