COMBINED ADAPTIVE PULSE COMPRESSION AND ADAPTIVE BEAMFORMING FOR MULTISTATIC SHARED-SPECTRUM RADAR

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Abstract

In this paper we present a method to reduce the interference between shared spectrum multistatic radars by performing joint adaptive pulse compression (APC) in conjunction with adaptive beamforming. A recently proposed algorithm based on a Minimum Mean Square Error (MMSE) formulation, Multistatic Adaptive Pulse Compression (MAPC) [1]-[3] has been shown to successfully suppress both range sidelobes and interference from multiple radars operating in the same spectrum. Here, we combine an adaptive beamforming component with the MAPC algorithm to enable further mutual interference suppression and hence better estimation performance such that the number of multistatic radars simultaneously operating in the same spectrum may be increased for the same mean-square estimation error.

1 Introduction

The increasing demand for spectrum usage rights by the communications industry coupled with the requirement for wider instantaneous bandwidths for radar applications is creating an ever growing need for more efficient use of the radio frequency (RF) spectrum. It is well known that two or more radars operating in close proximity, at the same time, and in the same spectrum will interfere with one another – often to the point of achieving complete RF fratricide. It is impossible to generate a set of waveforms that are orthogonal to one another at all possible respective delays, and therefore a large target return from one waveform will appear as a smaller target return from the other waveform(s) as well. However, accurate joint estimation of multiple radar range profiles can be accomplished by iteratively cancelling the mutual interference between the multiple received return signals. This work is an extension of the Multistatic Adaptive Pulse Compression (MAPC) algorithm [1]-[3] wherein multiple known transmitted waveforms are adaptively pulse compressed using Reiterative Minimum Mean Square Error (RMMSE) estimation [4]-[6]. Here, we combine an adaptive beamforming component with the MAPC algorithm to enable better estimation performance and to increase the number of multistatic radars which can simultaneously operate in the same spectrum. Note that this work addresses the interference resulting from mainbeam transmission of other radars which are received in the receiver sidelobe of a particular radar return signal of interest. We currently assume that element tapering or spatial nulling on transmit can sufficiently suppress sidelobe-transmitted signals that would otherwise be present in the receiver mainbeam of the return signal of interest. The more general formulation which includes both of these sources of interference as well as sidelobe-transmit/sidelobe-receive will be addressed in a subsequent paper.

2 Multistatic Adaptive Pulse Compression

For K waveforms transmitted simultaneously in the same spectrum in close proximity, we denote the discrete-time version of the k^{th} waveform as the column vector $\mathbf{s}_k$ having length N, and $\mathbf{r}_k$ as the spatial steering vector corresponding to the angle-of-arrival (AOA) of the k^{th} return signal at the receiver of interest. We assume a uniform linear array with M elements. For this receiver, the l^{th} time sample of the set of received radar return signals on the m^{th} antenna element is defined as

$$y_m(l) = \sum_{k=1}^{K} \mathbf{x}_k^T(l)\,\mathbf{s}_k + v(l) \tag{1}$$

for $l = 0, \ldots, L + N - 2$, where $\mathbf{x}_k(l) = [x_k(l)\ x_k(l-1)\ \ldots\ x_k(l-N+1)]^T$ are N contiguous samples of the range profile impulse response at delay l with which the transmitted waveform $\mathbf{s}_k$ convolves, $v(l)$ is additive noise, $(\cdot)^T$ is the transpose operation, and L is the number of range bins in the processing window. By collecting N samples of the received radar return signal, the system response model (for the m^{th} antenna element) is expressed as

$$\mathbf{y}_m(l) = \sum_{k=1}^{K} \mathbf{X}_k^T(l)\,\mathbf{s}_k + \mathbf{v}(l) \tag{2}$$

where $\mathbf{y}(l) = [y(l)\ y(l+1)\ \ldots\ y(l+N-1)]^T$ is N contiguous samples of the received signal, $\mathbf{X}_k(l) = [\mathbf{x}_k(l)\ \mathbf{x}_k(l+1)\ \ldots\ \mathbf{x}_k(l+N-1)]^T$ is an $N \times N$ matrix, and $\mathbf{v}(l) = [v(l)\ v(l+1)\ \ldots\ v(l+N-1)]^T$. The k^{th} received radar signal after beamforming is given by

$$\mathbf{z}_k(l) = \left(\mathbf{b}_k^H \mathbf{Y}(l)\right)^T \tag{3}$$

where $\mathbf{Y}(l) = [\mathbf{y}_1(l)\ \mathbf{y}_2(l)\ \ldots\ \mathbf{y}_M(l)]^T$ is a matrix containing the received signal vectors from each of the M antenna elements, and $\mathbf{b}_k$ is the beamformer weight vector. By substituting (2) into (3), the k^{th} radar return signal after beamforming can be represented as

$$\mathbf{z}_k(l) = \sum_{i=1}^{K} \eta_{ki} \mathbf{X}_i^T(l)\mathbf{s}_i + \mathbf{u}_k(l)\ , \tag{4}$$

where $\eta_{ki} = \mathbf{b}_k^H\ \mathbf{r}_i$ is the correlation between the k^{th} beamformer weight vector and the spatial steering vector of the i^{th} received signal, and $\mathbf{u}_k(l)$ is the additive noise after beamforming.

Under the standard matched filtering formulation [7] we would assume that the mutual interference is noise. As such, the application of the matched filters involves the convolution of each of the K beamformed return signals with the time-reversed complex conjugate of its respective transmitted waveform. The matched filtering operation can be expressed in the digital domain as

$$\hat{x}_{MF,k}(l) = \mathbf{s}_k^H \mathbf{z}_k(l), \quad k = 1, 2, \cdots, K \tag{5}$$

where $(\cdot)^H$ is the complex conjugate transpose, or Hermitian, operation. However, as ideal matched filtering assumes the presence of only a single received signal in white Gaussian noise (which the mutual interference is most certainly not), the matched filter will perform quite poorly in the multistatic scenario.

To compensate for the presence of the other K-1 return signals, the MAPC formulation [1]-[3] replaces the matched filter with an adaptive filter based on RMMSE estimation, which has been shown to provide excellent performance in the multistatic scenario. In general, the MAPC filter for the k^{th} waveform and l^{th} range bin, can be shown to take the form

$$\mathbf{w}_k(l) = \rho_k(l)\left(\sum_{i=1}^{K} |\eta_{ki}|^2\, \mathbf{C}_i(l) + \mathbf{U}_k\right)^{-1} \mathbf{s}_k \tag{6}$$

where $\mathbf{U}_k = E[\mathbf{u}_k(l)\ \mathbf{u}_k^H(l)]$ is the noise covariance matrix. The $N \times N$ structured signal covariance matrix $\mathbf{C}_k(l)$ is

$$\mathbf{C}_k(l) = \sum_{n=-N+1}^{N-1} \rho_k(l+n)\, \mathbf{s}_{k,n}\mathbf{s}_{k,n}^H \tag{7}$$

where $\rho_k(l) = E[|x_k(l)|^2]$ is the expected power of $x_k(l)$, and $\mathbf{s}_{k,n}$ contains the elements of the k^{th} waveform shifted by n samples and zero-filled in the remaining n samples, e.g. $\mathbf{s}_{k,2} = [0\ 0\ s_k(0)\ \ldots\ s_k(N-3)]^T$ and $\mathbf{s}_{k,-2} = [s_k(2)\ \ldots\ s_k(N-1)\ 0\ 0]^T$.

The beamformer weight vector $\mathbf{b}_k$ can take several forms. The original MAPC algorithm [1]-[3] utilized a non-adaptive beamformer with $\mathbf{b}_k$ equal to the spatial steering vector of the k^{th} signal $\mathbf{r}_k$. While the non-adaptive beamformer does suppress the interfering signals, significantly greater interference suppression can be obtained through the use of an adaptive beamformer which places nulls in the directions of the interfering signals. Here, we use the linearly constrained minimum-variance beamformer [8], with an adaptive weight vector (for the k^{th} received signal) given by

$$\mathbf{b}_k = \frac{\mathbf{R}_k^{-1}\mathbf{r}_k}{\mathbf{r}_k^H \mathbf{R}_k^{-1}\mathbf{r}_k}\,. \tag{8}$$

$\mathbf{R}_k$ is the $M \times M$ spatial covariance matrix with the k^{th} received signal excised from the data. It is critical that the k^{th} received signal be excised from $\mathbf{R}_k$. If it is not, then the adaptive beamformer will attempt to place a null in the direction of the signal that is being estimated. The k^{th} signal component at the l^{th} range bin is removed from the received signal of each of the M spatial channels by projecting the temporal received signal of each of the M spatial channels onto the temporal noise subspace of the $N \times N$ structured signal covariance matrix $\mathbf{C}_k(l)$ of the k^{th} signal component. The temporal noise subspace is associated with the eigenvector associated with the smallest eigenvalue of $\mathbf{C}_k(l)$. Identical temporal processing at a given time instant is implemented on each of the M spatial input channels. With the k^{th} received signal excised from the data, $\mathbf{b}_k$ places spatial nulls in the directions of the remaining $K - 1$ multistatic signals, thereby enabling more of the adaptive degrees-of-freedom to be applied for the adaptive pulse compression of each individual received signal.

Both the MAPC filters (6) and the adaptive beamformer weight vectors (8) are functions of the estimated range profiles $\hat{x}_k(l)$, which in practice are not available at the receiver. Therefore, a reiterative scheme is utilized to estimate the range profile. Assuming the noise covariance is white Gaussian, $\mathbf{U}_k$ simplifies to $\sigma_v^2\mathbf{I}$, where $\mathbf{I}$ is the $N \times N$ identity matrix and σ_v^2 is the noise power, which can be assumed known since internal thermal noise dominates the external noise at microwave frequencies (where most radars operate). The initial estimates of the K range profiles can be obtained either by using standard matched filtering or by initializing the power estimates of all of the range cells to be equal and assuming the noise is negligible initially. In the latter case, (6) reduces to

$$\widetilde{\mathbf{w}}_k = \left(\sum_{i=1}^{K} |\eta_{ki}|^2 \, \widetilde{\mathbf{C}}_i \right)^{-1} \mathbf{s}_k \tag{9}$$

for $k = 1, 2, \cdots, K$, where the matrix $\widetilde{\mathbf{C}}_i$ is defined as

$$\widetilde{\mathbf{C}}_i = \sum_{n=-N+1}^{N-1} \mathbf{s}_{i,n} \mathbf{s}_{i,n}^H \, . \tag{10}$$

The adaptive beamformer is initialized as $\mathbf{b}_k = \mathbf{r}_k$. A predetermined number of stages of estimation are performed where the result of the previous stage is utilized to compute the MAPC filters and the adaptive beamformer weight vectors for the current stage. We denote the adaptive beamforming MAPC algorithm as MAPC-AB. A block diagram of the MAPC-AB algorithm is shown in Fig. 1.

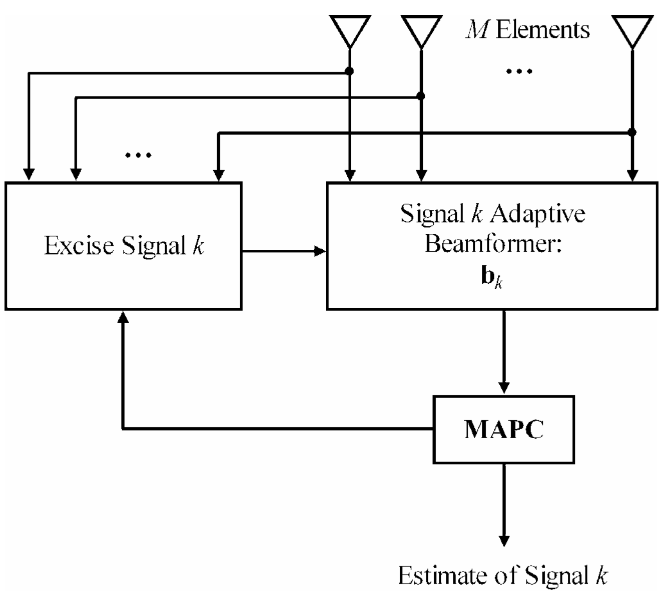

Fig. 1. MAPC-AB block diagram

The operation of the MAPC-AB algorithm progresses as follows. After (9) is applied, as in (5) with $\mathbf{s}_k$ replaced by $\widetilde{\mathbf{w}}_k$, and the initial range cell power estimates have been obtained, (6) is subsequently used to estimate the refined receive filters. The estimate of the signal covariance matrix $\mathbf{C}_k(l)$ is then utilized to excise the k^{th} signal from the M received signals. At this point the adaptive beamformer weight vector is computed using (8) and applied to the data. The refined received filters are then re-applied to the adaptively-beamformed received signals and the range cell complex amplitudes are re-estimated. The refined receive filters are better able to mitigate the masking effects caused by waveform cross-correlation and range sidelobes due to the fact that they are estimated based upon some *a priori* knowledge regarding the relative locations of larger targets, which was obtained in the previous stage. In addition, the power of the interfering signals is significantly decreased by the adaptive beamformer. The re-estimation of the individual receive filters, the adaptive beamformer weight vector, and range cells is repeated for a pre-determined number of stages. It has been determined via simulation that, given the presence of large masking targets, approximately 3 to 6 stages are required for the range profile estimates to reach the noise floor.

3 Simulation Results

To demonstrate the performance of the MAPC-AB algorithm, we compare its performance with that of the MAPC algorithm and that of a bank of matched filters. First, we consider the simultaneous reception of four random polyphase waveforms of length N = 30 received at varying angles off boresight of a 21-element uniform linear array. Two cases are examined, both involving dense stationary target scenarios. For the first case, the angles of arrival of the received signals are separated by 10°, and for the second case the separation is decreased to 5°. In both scenarios, the noise is -60 dB relative to the largest target power before beamforming (with several much smaller targets closer to the noise floor), and four stages of reiteration are utilized. The performance of each algorithm is assessed by computing the mean-square error (MSE) metric associated with estimating the ground truth complex amplitudes of the range profile.

For the first case, the angles of arrival of the received signal are 0°, −10°, −20°, and +10° off boresight. As seen in Fig. 2 the ground truth of the respective range profiles (represented in black) is comprised of many closely spaced targets with highly disparate power levels. As expected, the matched filter (in green) performs poorly due to the accumulated effects of range sidelobes and residual multistatic interference. In contrast, both MAPC (in red) and MAPC-AB (in blue) are able to suppress both the range sidelobes and the multistatic interference, significantly outperforming the matched filter with MAPC-AB markedly better than MAPC. In terms of MSE, the matched filter yields an MSE of −10 dB, the MSE of MAPC is −40 dB, and the MSE of MAPC-AB is −48 dB.

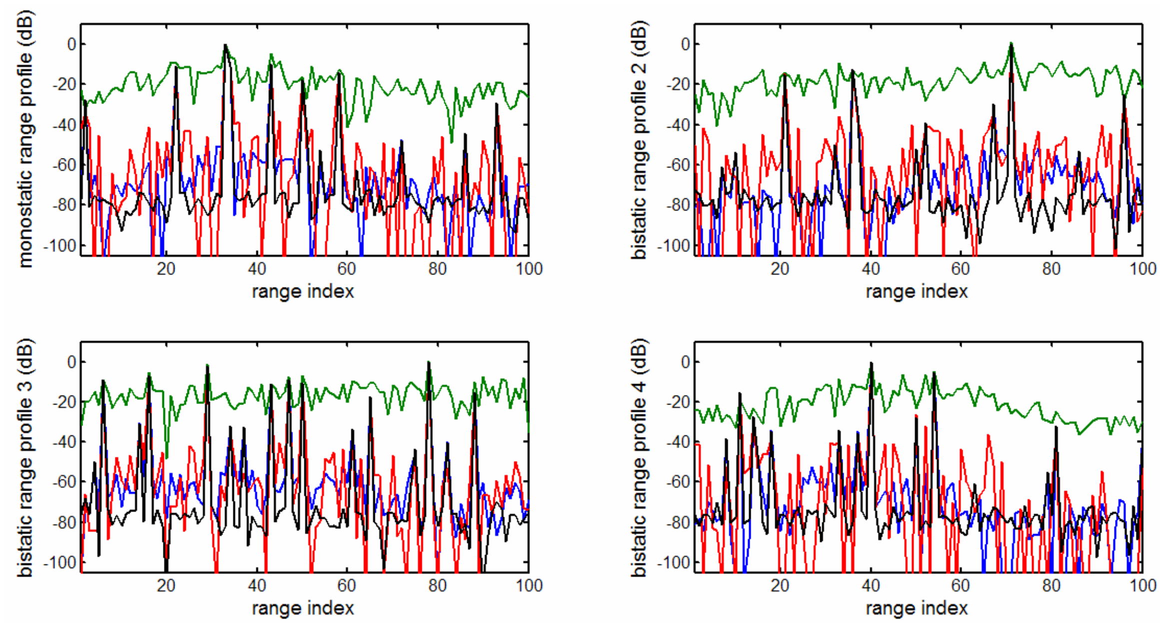

Fig. 2. Range profile estimation for multistatic radar reception (black: ground truth, green: matched filter, red: MAPC, blue: MAP-AB).

When the separation between the angles of arrival of the received waveforms is decreased, the improvement in estimation performance of MAPC when adaptive beamforming is utilized becomes more dramatic. Simulation results for signals received at angles of 0°, −5°, −10°, and +5° off boresight are shown in Fig. 3. For this scenario, the MSE of the matched filter is −7 dB, the MSE of MAPC is −33 dB, and the MSE of MAPC-AB is again −48 dB, an improvement of 15 dB over MAPC and 41 dB over the matched filter.

In order to ascertain the performance of the MAPC-AB algorithm as the number of radars increases, a Monte Carlo simulation was performed with the number of multistatic radars varying from 2 to 30. A total of 100 independent trials were randomly generated for each number of multistatic radars, with the angles of arrival of the received signals randomly distributed between –30° and +30° off boresight, and with randomly generated dense-target range profiles similar to those seen in Figs. 2 and 3. As before, length $N = 30$ random polyphase waveforms are utilized for the simulations. Each estimation approach (bank of matched filters, MAPC, MAPC-AB) was applied to the randomly generated scenarios and the average MSE was computed. These results are shown in Fig. 4. When greater than 12 radars are present, the MSE performance of MAPC-AB is approximately 4 dB better than that of MAPC. The greatest performance increase is obtained when between 7 and 12 multistatic radars are operating, where the MSE of MAPC-AB is as much as 15 dB less than MAPC. It should be noted that the MSE obtained for both MAPC and MAPC-AB when 30 multistatic radars are operating is less than that of the matched filter with only 2 multistatic radars.

4 Summary

The approach described in this paper enables shared-spectrum multistatic radar by performing joint adaptive pulse compression in conjunction with adaptive beamforming. By combining joint RMMSE estimation with adaptive beamforming, the estimation performance of MAPC is increased. Simulation results indicate a decrease in MSE of 8 dB to 15 dB when adaptive beamforming is utilized, depending on the amount of angular separation between the multistatic signals. It was found via Monte Carlo simulation that the greatest increase in performance was obtained when between 7 and 12 multistatic radars are operating. The addition of adaptive beamforming to MAPC allows decreased angular separation without an attendant decrease in estimation performance, enabling a higher density of multistatic radars.

5 Acknowledgement

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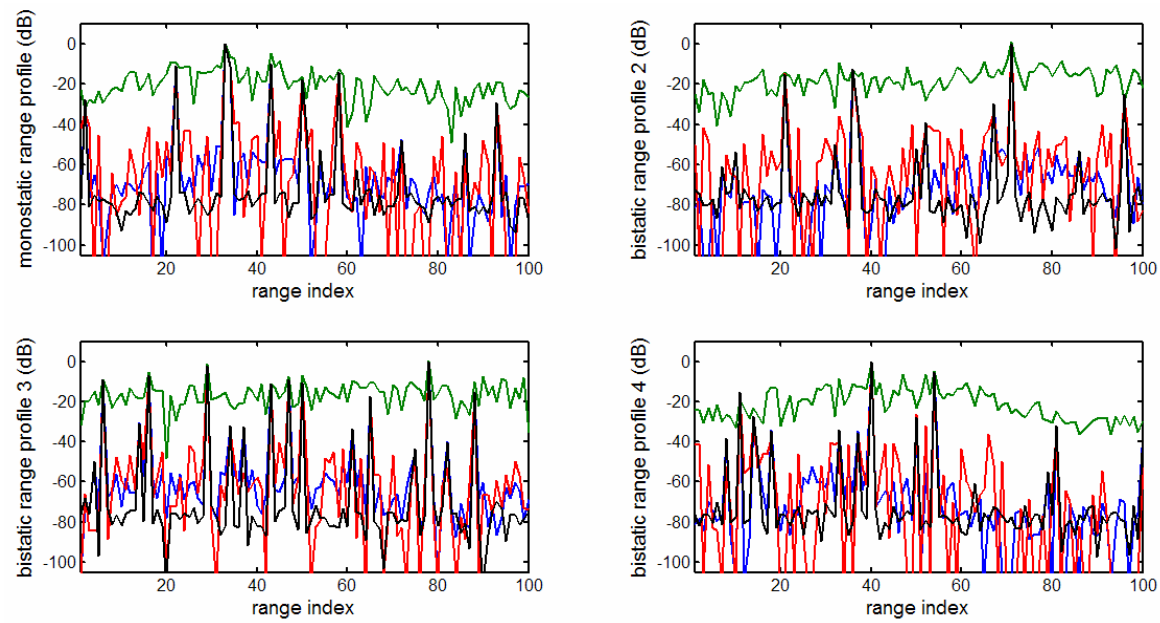

Fig. 3. Range profile estimation for multistatic radar reception with decreased angular separation between received signals (black: ground truth, green: matched filter, red: MAPC, blue: MAPC-AB).

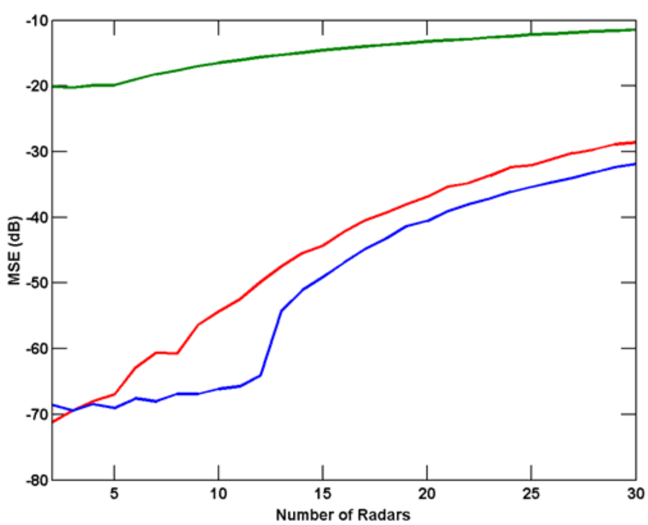

Fig. 4. Monte Carlo simulation for varying numbers of multistatic radars (green: matched filter, red: MAPC, blue: MAPC-AB).

6 References

[1] S. D. Blunt and K. Gerlach, "Joint adaptive pulse compression to enable multistatic radar," *IEE Waveform Diversity and Design Conf.*, Edinburgh, Scotland, (Nov. 2004).

[2] S. D. Blunt and K. Gerlach, "Aspects of multistatic adaptive pulse compression," *IEEE Intl. Conf. on Radar*, Arlington, Virginia, (May 2005).

[3] S. D. Blunt and K. Gerlach, "Multistatic adaptive pulse compression," *IEEE Trans. on Aerospace and Electronic Systems*, submitted.

[4] S. D. Blunt and K. Gerlach, "A novel pulse compression scheme based on minimum mean-square error reiteration," *IEEE Intl. Conf. on Radar*, Adelaide, Australia, pp. 349-353, (Sept. 2003).

[5] S. D. Blunt and K. Gerlach, "Adaptive pulse compression," *IEEE National Radar Conf.*, Philadelphia, Pennsylvania, pp. 271-276, (April 2004).

[6] S. D. Blunt and K. Gerlach, "Adaptive pulse compression via MMSE estimation," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 42, no. 1, Jan. 2006.

[7] M.I. Skolnik, *Introduction to Radar Systems, 3rd ed.*, New York, NY: McGraw-Hill, pp. 339-369, (2001).

[8] S. Haykin, *Adaptive Filter Theory, 4th ed.*, Upper Saddle River, NJ: Prentice Hall, pp. 115-120, (2002).