Structure-Based Adaptive Radar Processing for Joint Clutter Cancellation and Moving Target Estimation

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Abstract—During his many years with the Radar Division of the US Naval Research Laboratory (NRL), Dr. Karl Gerlach made significant contributions to adaptive interference cancellation for radar. For this memorial tribute special session, this paper leverages the reiterative minimum mean square error (RMMSE) estimator, which he also helped to develop, to formulate two techniques whereby interference cancellation is performed jointly with signal estimation as a way to enhance the subsequent range-Doppler response. Experimental results are demonstrated using free-space measurements from pulsed, nonrepeating waveforms at S-band and standard FMCW at W-band.

Keywords—clutter cancellation, signal estimation, adaptive processing

I. INTRODUCTION

Moving target indication (MTI) radars disambiguate targets in the presence of clutter using Doppler as the discriminant. Doing so necessitates estimation and subsequent cancellation of the clutter because it could otherwise mask the presence of moving targets, in some cases to a rather significant degree (see Chap. 17 of [1]).

Dr. Karl Gerlach, for whom this special session is a memorial tribute, made numerous contributions to clutter cancellation (e.g. [2-5] just to name a few). The intent of this paper is to combine clutter cancellation with the structure-based adaptive filtering approach generally denoted as reiterative minimum-mean square error (RMMSE) estimation, for which Karl likewise made a significant impact.

Specifically, two new forms of RMMSE are derived here that incorporate clutter cancellation so that the algorithm can achieve joint cancellation and signal estimation, where the latter property facilitates enhanced discrimination and visibility of moving targets without the SNR loss or resolution degradation otherwise associated with Doppler tapering. The two new approaches are denoted as background supplemental cancellation (BaSC) and background supplemental loading (BaSL), where the former employs “hard” cancellation and the latter represents a form of “soft” cancellation that is performed iteratively. Validation of these methods is demonstrated through application to two sets of experimental free-space measurements, one from an S-band testbed and the other from a W-band testbed.

II. A BRIEF HISTORY OF THE RMMSE CONCEPT

The RMMSE formulation was conceived by Blunt and Gerlach in late 2002 / early 2003. It was initially based on the goal of applying strategies from CDMA multiuser detection [6], where users are separated at the receiver according to their corresponding code, to the application of shared-spectrum multistatic radar. It was then observed that when RMMSE is applied to radar pulse compression, and thus denoted as adaptive pulse compression (APC), it facilitates a beamforming-like capability by nulling the self-interference from range sidelobes [7,8]. The multistatic APC (MAPC) application quickly followed [9,10], along with ways to perform this manner of adaptive processing in legacy systems after analog pulse compression [11], adaptive compensation of pulse eclipsing effects [12], exploitation of fast-time Doppler to perform imaging [13,14], and compensation of Doppler distortion [15].

Subsequent APC-based work included the incorporation of a gain constraint [16], hybridization with the well-known CLEAN technique [17], a reduced-dimension implementation to reduce computational cost [18], and joint range-Doppler [19] and range-angle [20] versions. An RMMSE-based spatial beamformer denoted as reiterative super-resolution (RISR) [21] was then developed for spatial direction-of-arrival (DOA) estimation, though it is likewise applicable to the frequency domain. Gain constrained and “partially constrained” versions of RISR were subsequently developed in [22] to provide enhanced robustness.

These various forms of RMMSE have been used to enhance weather radar [23], synthetic aperture radar (SAR) [24], MEG imaging of brain activity [25], and active sonar [26], where the latter also necessitated incorporation of a covariance matrix taper to address high Doppler sensitivity. More recently, physical attributes of waveforms have been incorporated into the RMMSE paradigm [27,28] that have subsequently permitted experimental demonstrations of enhanced sensitivity and discrimination for simultaneously dual-polarized operation [29], shared-spectrum radar [30], and even stretch processing [31]. Here we build from this litany of previous developments to incorporate a clutter cancellation capability into RMMSE estimation.

III. RISR RECAP FOR SPECTRUM ESTIMATION

In [21] the RMMSE concept was used to obtain an adaptive filter bank to perform DOA estimation using a single snapshot from an arbitrary antenna array, as long as the array manifold is adequately known. Moreover, the resulting RISR formulation includes the means to incorporate array calibration tolerances since the array manifold cannot be known perfectly in practice. By simply considering the spatial steering vectors to instead be frequency steering vectors, RISR can likewise be directly

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applied to spectrum estimation. The following is a brief review of the RISR formulation which will subsequently be modified to incorporate clutter cancellation.

Consider a single vector \( \mathbf{y} \) comprised of \( N \) time samples for which we wish to estimate the spectral content. The importance of this single-snapshot capability will be revealed shortly when the approach is employed in nonstationary applications. The received signal can thus be represented as

\[
\mathbf{y} = \mathbf{Sx} + \mathbf{v},
\]

where \( \mathbf{x} \) is an \( M \times 1 \) vector comprised of \( M \gg N \) frequency-dependent complex amplitudes, \( \mathbf{S} \) is an \( N \times M \) bank of frequency steering vectors, and \( \mathbf{v} \) is additive noise of arbitrary distribution.

In this context, we wish to minimize the objective function

\[
J = \mathbb{E}\left[ \| \mathbf{x} - \mathbf{W}^H \mathbf{y} \|^2 \right],
\]

where \( (\cdot)^H \) is the Hermitian operation, \( \mathbb{E} [\cdot] \) is expectation, and \( \mathbf{W} \) is the resulting \( N \times M \) adaptive filter bank. The general MMSE solution to (2) is

\[
\mathbf{W} = \left( \mathbb{E} \left[ \mathbf{y} \mathbf{y}^H \right] \right)^{-1} \mathbb{E} \left[ \mathbf{y} \mathbf{x}^H \right],
\]

for which the \( m \)th column in the (unconstrained) RMMSE context [21] is the RISR filter

\[
w_{m,i} = p_{m,i} (\mathbf{SP} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m,
\]

for \( \mathbf{s}_m \), the \( m \)th column of \( \mathbf{S} \), and \( \mathbf{R}_{\text{nse}} \) is the \( N \times N \) noise covariance matrix. The \( M \times M \) diagonal matrix

\[
\mathbf{P}_i = \left[ \hat{\mathbf{x}}_i \hat{\mathbf{x}}_i^H \right] \odot \mathbf{I}_{M \times M}
\]

is obtained at the \( i \)th iteration and has the \( m \)th diagonal element \( p_{m,i} \), where \( \mathbf{I}_{M \times M} \) is an identity matrix,

\[
\hat{\mathbf{x}}_i = \mathbf{W}_i^H \mathbf{y}
\]

is the estimate of the complex spectral amplitudes at the \( i \)th iteration, and \( \odot \) is the Hadamard product. This process is initialized by setting

\[
\mathbf{W}_{i=0} = \mathbf{S},
\]

which performs a Fourier transform that is oversampled in the frequency domain since \( \mathbf{S} \) is \( N \times M \).

In [22], the RISR filter bank of (4) was modified to incorporate a gain constraint for each individual filter, with the resulting \( m \)th column of \( \mathbf{W} \) subsequently taking the well-known MVDR form of

\[
w_{m,i,j} = \left( \mathbf{SP} \mathbf{S}^H + \mathbf{R}_{\text{nse}} \right)^{-1} \mathbf{s}_m \left( \mathbf{SP} \mathbf{S}^H + \mathbf{R}_{\text{nse}} \right)^{-1} \mathbf{s}_m.
\]

While this version does not achieve quite the degree of resolution enhancement as (4), it has also been observed to be more robust to mismatch effects and avoids over-suppression of small signal components. Moreover, estimated values of \( \mathbf{x} \) for which there is no signal component present tend to settle around the level of the noise floor when using (8), which is a more realistic response and is useful for subsequent CFAR detection. It is this form that we shall further modify to incorporate clutter cancellation.

IV. INCORPORATING HARD CANCELLATION (BaSC)

The RMMSE-based RISR formulation in (4) or (8) is a recursive approach that seeks to obtain the filter bank that minimizes the mean-square error of the estimate of the underlying signal components according to (2). However, this approach does not discriminate between the desired signal components, moving targets in this case, and the undesired components. We will henceforth apply the term ‘clutter’ for the latter, though use it here to mean the stationary signal components that are persistent. Consequently, this methodology could also be viewed as a form of change detection relative to an estimated background.

Here we wish to supplement the RISR estimation-oriented filter bank with a clutter cancellation component, and shall denote this modified form as background supplementary cancellation (BaSC). First decompose (1) as

\[
\mathbf{y} = \mathbf{Sx} + \mathbf{v} = \mathbf{S}_{\text{clut}} + \mathbf{S}_{\text{rem}} + \mathbf{v},
\]

where we have separated the underlying signal into two parts as \( \mathbf{x} = \mathbf{x}_{\text{clut}} + \mathbf{x}_{\text{rem}} \), in which the first component denotes clutter (stationary background) and the second denotes whatever remains (excluding noise). This form therefore allows for the presence or absence of moving targets since \( \mathbf{x}_{\text{rem}} \) could be a vector of zeroes.

Now define the rank \( k < N \) clutter covariance matrix as

\[
\mathbf{R}_{\text{clut}} = \mathbb{E} [\mathbf{y}_{\text{clut}} \mathbf{y}_{\text{clut}}^H].
\]

For white noise it can thus be readily shown that the inverse of the normalized cancellation transform

\[
\mathbf{R}_{\text{canc}}^{-1} \mathbb{E} [\mathbf{y} \mathbf{y}^H] = \mathbb{E} [\mathbf{y}_{\text{rem}} \mathbf{y}_{\text{rem}}^H] + \mathbb{E} [\mathbf{v} \mathbf{v}^H]
\]

which implies

\[
\mathbf{R}_{\text{canc}}^{-1} \mathbf{v} = \tilde{\mathbf{y}} = \mathbf{y}_{\text{rem}} + \mathbf{v}.
\]

Thus the corresponding clutter-cancelled version of (6) becomes

\[
\hat{\mathbf{x}}_{\text{rem,i}} = \mathbf{W}_i^H \mathbf{R}_{\text{canc}}^{-1} \mathbf{y} = \mathbf{W}_i^H \tilde{\mathbf{y}}
\]

using either (4) or (8) for the RISR filter bank \( \mathbf{W}_i \), which in this case is applied and updated recursively after application of the cancellation transform.
Of course, estimation of the clutter covariance is never perfect and, depending on the particular problem, potentially difficult to obtain precisely (e.g., highly nonstationary environment). Consequently, the result in (13) and (14) would likely still contain at least some residual clutter leakage. Moreover, the hard delineation between the clutter and remaining subspaces may hinder detection of moving targets that are near that cutoff. In [32] joint estimation/cancellation was analytically demonstrated to outperform their sequential application. To that end, a soft cancellation version of the RMMSE framework is likewise examined.

V. INCORPORATING SOFT CANCELLATION (BaSL)
Where (14) employs a sequential cancellation-then-estimation approach, it is worth considering an implementation in which these operations are performed jointly. Simply modifying (2) as

\[ J = \mathbb{E} \left[ \| \mathbf{x}_{\text{rem}} - \mathbf{W}^H \mathbf{y} \|^2 \right] \]  

(15)

and using \( \mathbf{y} \) from (9) leads to

\[ \mathbf{w}_{m,i} = \mathbf{p}_{m,i} (\mathbf{S} \mathbf{P}^H + \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nce}})^{-1} \mathbf{s}_m, \]  

(16)
such that the recursive estimation of the filterbank and moving targets naturally excludes the clutter that is already accounted for within the filter structure.

An MVDR form of (16) like that in (8) could likewise be obtained, though it would tend to preserve the clutter instead of canceling it. To enable cancellation we will insert the clutter covariance only into the numerator of (8) as

\[ \mathbf{w}_{m,i} = \frac{\mathbf{p}_{m,i} (\mathbf{S} \mathbf{P}^H + \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nce}})^{-1} \mathbf{s}_m}{\mathbf{s}_m^H (\mathbf{S} \mathbf{P}^H + \mathbf{R}_{\text{nce}})^{-1} \mathbf{s}_m}. \]  

(17)

Consequently, while initial estimation via (6) and (7) does include the clutter component, it will disappear from the moving target estimate as iteration continues. The filter structures of (16) and (17) are collectively denoted as background supplemental loading (BaSL). As will be shown using measured data, this soft cancellation approach provides greater visibility of slow-moving targets that would otherwise be suppressed when performing hard cancellation.

VI. SUPPLEMENTAL COVARIANCE ESTIMATION
The supplementary covariance matrix

\[ \mathbf{R}_{\text{sup}} = \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nce}} \]  

(18)

used in BaSC via (11) and appearing in (16) and (17) for BaSL could be obtained in different ways. The most direct approach is by computing the sample covariance

\[ \mathbf{R}_{\text{sup}} \approx \frac{1}{L} \sum_{\ell=1}^{L} \mathbf{y}_\ell \mathbf{y}_\ell^H, \]  

(19)

where \( \mathbf{y}_\ell \) for \( \ell = 1, 2, \cdots, L \) are snapshots collected over intervals where moving targets do not reside. Of course, it is necessary that these estimates are identically distributed (or at least sufficiently similar) to the clutter and noise within the moving target interval. In other words, we are making precisely the same assumption as is made for standard adaptive clutter cancellation.

Alternatively, a structured supplemental covariance could be formed by leveraging the model from (9) and using the initial estimates from (6) for the \( i = 0 \) filterbank in (7). Denoting these estimates as \( \hat{\mathbf{x}}_\ell \) for \( \ell = 1, 2, \cdots, L \) snapshots, the structured supplementary matrix is obtained via

\[ \mathbf{R}_{\text{sup}} \approx \mathbf{S} \hat{\mathbf{P}}_{\text{clut}} \mathbf{S}^H + \sigma_v^2 \mathbf{I} \]  

(20)

where

\[ \hat{\mathbf{P}}_{\text{clut}} = \left[ \frac{1}{L} \sum_{\ell=1}^{L} \hat{\mathbf{x}}_\ell \hat{\mathbf{x}}_\ell^H \right] \odot \mathbf{I}_{M \times M} \]  

(21)

in the same form as (5). While it appears a bit more cumbersome than (19), this structured approach has the benefit of permitting easy removal of non-clutter components, which could otherwise contaminate the training data (see [3]), by simply zeroing the necessary diagonal elements in (21) that fall outside the expected clutter response. In the following, we shall examine the use of both supplementary matrices.

VII. EXPERIMENTAL VALIDATION
Measured data from two completely separate open-air tests is used to assess the efficacy of the BaSc and BaSL forms of RISR for moving target estimation in clutter. In all cases 5 iterations of the given approach is employed. The first data set involves the use of 150 random FM waveforms [33] having a 3-dB bandwidth of 67 MHz, pulselength of 4.5 Useconds, and pulse repetition frequency (PRF) of 20 kHz that were implemented on a Tektronix arbitrary waveform generator at an S-band center frequency of 3.55 GHz. These were emitted in the direction of a traffic intersection in Lawrence, KS, and the resulting echoes collected using a real-time spectrum analyzer.

Figure 1 shows standard FFT Doppler processing with Taylor windowing and no clutter cancellation. Some moving targets appear to be visible but the large clutter response makes them hard to distinguish. Applying projection-based clutter cancellation (since the platform is stationary) to this data yields the result in Fig. 2, with the moving targets now clearly visible, though the Doppler resolution is rather coarse.
Applying RISR from (8) to this same data set (see Fig. 3) provides an obviously significant sharpening in Doppler, with the moving targets becoming quite clear, though the clutter is still present and may mask some targets. When BaSC is likewise employed via (14) as shown Fig. 4, the clutter is mostly suppressed, though some leakage is still present due to large clutter discretes. Finally, when BaSL from (17) is applied it is observed in Fig. 5 that the clutter is almost completely removed and the Doppler-sharpened moving targets are plainly visible.

To serve as a sort of ground truth for comparison, 1000 random FM waveforms were used to illuminate the same scene. After FFT-based Doppler processing and projection-based clutter cancellation the result in Fig. 6 shows the same set of enhanced targets observed in Figs. 3-5. Because this latter result enjoys nearly 7 times the number of unique pulses, the associated SNR and sidelobe decoherence benefits that accompany it (due to incoherent sidelobe combing for random FM waveforms [33]) are easy to see. More importantly, however, is the very good agreement in moving targets between Figs. 5 and 6, many of which are not discernible in Fig. 2.

The second set of data was collected from a W-band FMCW system developed to capture fast-moving objects [34]. The system generates 500 µs chirps that span a bandwidth of 600 MHz at a center frequency of 108 GHz. The operating mode involved up/down chirp cycles, but we only consider the down chirp portions, resulting in an effective PRF of 1 ms. The received echoes from each sweep are dechirped and sampled, followed by standard stretch processing involving an FFT. Here the FFT is also replaced with the RISR/BaSC or RISR/BaSL methods for each sweep. This manner of fast-time adaptive
processing is particularly well-suited to this arrangement because the fast-moving object requires that spectral estimation be performed separately for each sweep.

For this data set, reusable paintballs were fired away from the receiver and the resulting data is oriented in terms of frequency offset (corresponding to range) on the horizontal axis and (slow) time in milliseconds on the vertical axis (increasing downward). Both forms of the supplementary matrix are considered, with the background data collected when the radar is operating prior to the paintball being fired.

Figure 7 shows the time-history across a set of FMCW sweeps after each sweep has been dechirped followed by an FFT (i.e. standard stretch processing). Many of the strong vertical echoes are background clutter, though the response at 65+ ms at ~2.92 MHz is the vibration of a rubber sheet caused by the paintball impact. Noting that frequency corresponds to range for FMCW, the diagonal paintball trace (more visible in Figs. 8-11) is actually shifted in frequency due to Doppler, with the near-horizontal response at 61 ms arising from rapid deceleration when the paintball strikes the rubber sheet.

In contrast, Figs. 8 and 9 illustrate the impact of BaSC and BaSL when the structured supplementary matrix of (20) is employed to suppress the background clutter. The fast-moving paintball is now clearly visible, especially in the BaSL response, due to this fast-time suppression of background clutter.

Unlike the previous open-air range-Doppler experiment, this data was collected inside an auditorium which presents significant multipath and some modulated clutter effects believed to be caused by ventilation fans. Consequently, the structured supplementary matrix may not adequately capture all of the ambient background clutter. Figures 10 and 11 thus
provide an alternative perspective on BasC (from Fig. 8) and BaSL (from Fig. 9) when the sample covariance matrix from (19) is used. While the sheet vibration effect for 65+ ms is now less clear, the paintball trajectory is more visible (subjectively speaking). Moreover, the impact at 61 ms is also now stronger for both BaSC (by about 6 dB) and BaSL (by about 3 dB).

**VIII. CONCLUSIONS**

The RMMSE-based approach denoted as RISR, developed for DOA estimation, is likewise applicable to spectral estimation, though it does not inherently address clutter cancellation. Here this capability is incorporated via hard and soft implementations. It is shown using measured S-band and W-band data that the corresponding adaptive methods provide significant enhancement for the detection and discrimination of moving targets. Moreover, because it is performed on a per-snapshot basis, this formulation open the door to new applications of interference cancellation.

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