

Challenges and Prospective Solutions for Non-Uniform Radar Waveforms in a Shared Spectrum

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Abstract—The purpose of this paper is to provide an overview of challenges and methods to design and process a non-uniform radar waveform in a shared spectrum with computationally efficient algorithms to mitigate co-channel interference and enable standard constant false alarm rate detection and tracking approaches. Computationally efficient processing approaches for non-uniform radar waveforms paired with emerging computing architectures are paving the way for real-time implementation. However, these non-uniform processing schemes need to be paired with interference informed radar waveform design to support interference mitigation processing of OFDM signals to achieve acceptable performance in real world environments. Current challenges associated with optimizing a non-linear radar processing chain for a non-uniform waveform discussed in this paper include processing and signal environment informed waveform design, OFDM interference rejection, convergence speed of reiterative minimum mean square error (RMMSE) based range Doppler map (RDM) formation, and preservation of Gaussian noise statistics for down-stream processing. Potential solutions discussed to these problems include a waveform optimization based on a restricted isometry property (RIP) constraint, re-mod/demod interference rejection, standard RMMSE, and reduced dimension RMMSE RDM processing.

Index Terms—Spectrum Sharing, Non-Uniform Waveforms, RMMSE, Interference Mitigation

I. INTRODUCTION

Traditional radar systems employ N coherent processing intervals (CPI) utilizing different fixed PRIs paired with M of N non-coherent processing to enable disambiguation in Doppler and mitigation of range blind zones present for each unique PRI. Each CPI can be processed in a computational efficient manner by using a traditional match filter for range compression and fast Fourier Transform (FFT) based processing for Doppler due to the full and uniform sample spacing in the range and Doppler dimensions. However, there is performance regret associated with this scheme due to (a)

non-coherent integration across N CPIs over the time span t_{CPI} rather than coherent integration across t_{CPI} , (b) undesirable sidelobe structure and levels (c) infinite loss at some ranges for each PRI, and (d) difficulty in tracking dynamic targets due to either scan revisit rate or interleaved CPIs. On-going research [1] [2] [3] into alternative waveform and non-linear detection level processing has focused on improving radar performance relative to the first three items listed.

In parallel to advancing radar technologies, the commercial demand for increased spectrum [4] to support the proliferation of high-bandwidth wireless communication devices has resulted in spectrum co-use amongst radar and traditional communication users that will become increasingly commonplace in the future. For ground based radars, medium to high power base stations, such as those associated with 4G and 5G signals, can be dense spatially, have higher interference to signal ratio (ISR), and have higher temporal and spectral overlap due to simultaneous transmit and receive capabilities. Historically, co-channel interference that was temporally aperiodic, low ISR, and sparse spatially meant that spatial nulling and traditional tracking approaches could be used with reasonable success. However, overall radar system performance will degrade beyond acceptable performance bounds when using these approaches while faced with medium to high power interference in the mainlobe. The mitigation of mainlobe communication interference then needs to rely on signal separation of the radar returns and the unknown communication interference. For a ground based radar, OFDM signals used ubiquitously in the communication industry can be mitigated via a combination of radar modulation on pulse to improve separability, pulse repetition interval to minimize cross correlation, and non-linear coherent subtraction of the estimated OFDM signal.

The convergence of more capable radar processing with a

more challenging signal environment provides an opportunity to revisit optimal radar waveform design from a perspective that captures target dynamics, co-channel interference, and processing constraints rather than focusing on addressing range blind zones and Doppler ambiguities.

II. CHALLENGE AREAS

The development of a sensing strategy based on non-uniform waveforms in a shared spectrum requires addressing multiple challenges including but not limited to optimal waveform design, utilizing combinations of non-linear processing techniques, and a computational realizable solution on available compute resources. Later sections discuss potential approaches to address aspects of these challenges, developing an end to end solution from waveform creation through tracking across multiple beams is an on-going area of research within the radar community.

A. Waveform Optimization

The transition from a simple dwell definition that consists of N CPIs of P pulses that can each be described by a single PRI, center frequency, bandwidth, and pulse width, and modulation on pulse to a waveform that has on the order NP unique pulses requires a new mechanism to optimizing the waveform rather focusing only on satisfying range and Doppler disambiguation criteria. Thus, the waveform optimization procedure can be tailored to balance the following factors:

- *Co-channel Interference Structure*: Improving signal separation via adjusting modulation on pulse [14] can improve coherent cancellation of interference and increasing and randomizing the PRI could reduce the impacts of slow-time coherent gain for cyclic interference signals.
- *Variable Slow Time Support Per Range Bin*: Traditional M of N dwell structures had full support, no loss, for some ranges while blind zones have infinite loss. With variable PRIs using coherent processing across across pulses assuming a fixed dwell time, the waveform designer can optimize the structure of unobserved slow-time events for a given range bin. Example optimization approaches could be to prioritize receive scheduling to improve observability at long ranges, for known targets, or provide structure that aids processing algorithms.
- *Hardware Complexity*: The ability to command pulse to pulse unique transmit and receive events can challenge traditional architectures built upon simple waveform definitions which may be a constraint in the optimization.
- *Processing Approaches*: Many approaches to interference cancellation and non-uniform processing rely on non-linear processing strategies that can be biased towards better performance, faster convergence, and/or computational short-cuts when the waveform contains structure.

Additionally, the waveform optimization process may need to run in real time to improve run-time optimization and environmental tailoring which means that the waveform optimization algorithms themselves may need to be computational efficient.

B. Radar Detection Processing

The migration to non-uniform waveforms in a shared spectrum necessitates addressing the following:

- *Doppler Processing and Expansion*: Non-uniform sample spacing in slow-time due to non-observability and variable PRI require a method to control Doppler sidelobes and address a large Doppler space to support tracking.
- *Range Compression and Partial Target Returns*: Fixed PRI waveforms have partial target returns at the edges the receive windows but consistent in Doppler. Non-uniform PRIs can result in partial pulse returns for all targets at a subset of pulses which can degrade performance.
- *Residual Interference*: The residual co-channel interference post mitigation will impact the "noise" regions that are utilized in traditional CFAR based detectors. Measuring and minimizing this residual will impact detection and false alarm rates.
- *Non-Traditional Detection Statistics*: Many approaches to addressing the non-uniform sample support involve non-linear approaches that can change detection statistics that impact target detection and tracking algorithms. Minimizing or measuring these changes is important for performance.
- *Computational Complexity*: A constraint on the entire detection process is the assumption that the algorithms must be real-time realizable.

III. WAVEFORM OPTIMIZATION VIA THE RIP

Reconstructing an accurate representation of a range-Doppler space with a number of resolvable cells that exceeds the time-bandwidth product of the waveform is inherently compressive. Furthermore, as time-bandwidth product of the waveform is increased, resolution improves, so even though the number of fast-time/slow-time samples may increase, the maximum range-Doppler extent does not. Any attempt to reconstruct a larger range-Doppler space, therefore, necessarily involves sensing artifacts such as high sidelobes and ambiguities. True ambiguities can be avoided by staggering pulse repetition times, but signal processing must attempt to handle the resulting high sidelobes. These measurement and processing strategies are consistent with sampling and reconstruction strategies from compressive sensing. In compressive sensing (CS), the quality of a compressive measurement scheme can be quantified by the restricted isometry property (RIP), which quantifies the fraction of distance lost between two unique signals during the compression process, given by

$$(1 - \delta) \|\mathbf{x}_1 - \mathbf{x}_2\|^2 \leq \|\mathbf{A}\mathbf{x}_1 - \mathbf{A}\mathbf{x}_2\|^2 = d_c^2, \quad (1)$$

where d_c^2 is the distance between the any two compressed signals, \mathbf{x}_1 and \mathbf{x}_2 , and \mathbf{A} is the signal compression operator [6]. The RIP property is defined as the minimum value of δ for which the inequality holds over all pairs of signal realizations. This property is, therefore, useful in quantifying the performance of a particular sampling schemes such as non-uniform PRI staggering [2]. Figure 1 shows the results

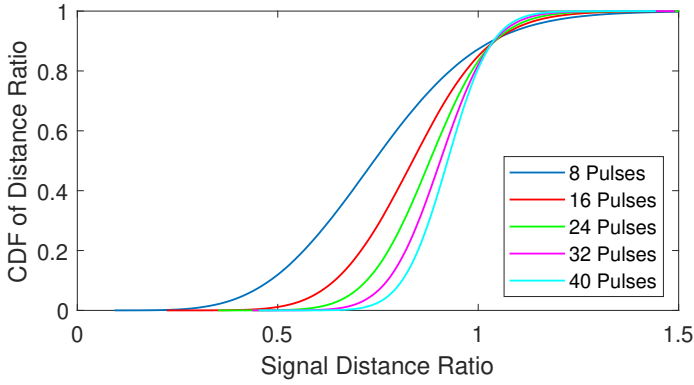


Fig. 1. CDF of distance ratio for different pulsing schemes found from Monte Carlo trials.

of calculating a RIP-style metric for 1D (slow time) pulsing schemes using a Monte Carlo simulation. In the simulation, the Doppler bandwidth of signals was limited such that 96 pulses, constrained to a fixed CPI duration, resulted in an uncompressed and unambiguous scenario. From there, using fewer pulses resulted in a decrease of the average PRF, such that the situation became compressive. Random PRI intervals were generated during the Monte Carlo simulation. Pulse widths were normalized to maintain a fixed amount of energy in the pulse train. For a measurement scheme to behave well, the distance between two unique signals should be preserved, with a ratio of 1 having the same distance between the two signals before and after compression. It's clear from the figure that increasing the number of samples, holding constant the total transmitted energy, results in fewer values below 1, i.e. the measurement scheme is better able to reconstruct the original signal.

The use of the RIP property may be able to help inform the choice of measurement scheme, which is vital when trying to detect targets when non-uniform PRI staggering is used. Extension of the RIP to 2D range-Doppler including receiver blanking and pulse diversity is ongoing.

IV. COMMUNICATION INTERFERENCE MITIGATION VIA REMOD/DEMOM

Communications are a growing source of in-band and out-of-band interference, especially as more spectrum is allocated for communications systems [4]. Here we consider actively cancelling in-band interference from communication signals by leveraging knowledge of the signal structure. Specifically, we attempt to demodulate the communications signal in order to remodulate and create a perfect synthetic copy of the signal. This technique is well-known in the passive radar community [10], [16]. Passive radar techniques use signals of opportunity to illuminate the scene and use two antennas: One pointed toward the scene and one pointed at the emitter. The direct path signal from the emitter can be demodulated to extract the data bits and then subsequently remodulated to create a reference signal [11]. Recent passive radar work has shown that the network and modulation parameters required to decode 5G signals can be extracted to demodulate the reference signal [10].

For an active radar system operating in a simultaneous transmit and receive (STAR) mode, the demodulation and remodulation technique can be used to reconstruct the interfering communications signal and subtract it from the data [13]. However, high power, monostatic, pulsed radar systems generally blank the receiver when transmitting [15]. The receiver blanking results in lost information in the continuous wave communications signal. Figure 2 demonstrates the loss of information for each symbol that results from the receiver blanking with a staggered PRI.

To demodulate the OFDM signals, zeros are inserted into the symbols. To understand the performance degradation caused by the receiver blanking, a Monte Carlo simulation is carried out. OFDM symbols are generated according to the parameters in Table IV and white Gaussian noise is added for an INR of 25 dB. A vector of 32 pulses is created with a mean PRF and those times are used to null the the OFDM symbol vectors.

TABLE I
OFDM PARAMETERS

Subcarrier Spacing	15 kHz
Subcarriers	1200
Sample time	32.55 ns
Symbol Duration	66.67 μ s
Cyclic Prefix Duration	4.7 μ s
3 dB bandwidth	18 MHz
Modulation	16-QAM
Samples/Symbol	2048
Samples/Cyclic Prefix	144

Pulse duration is also going to have an impact on the demodulation performance, as a longer pulse represents more missed symbol data. The pulse duration was tested as a percentage of the mean PRI, here called the duty cycle. Figure 3 shows the reduction in the interference plus noise power using the demodulation/remodulation technique when there are no radar returns present.

The best results occur with a longer mean PRI and a shorter pulse. Using shorter pulses and reducing the frequency of pulses reduces the amount of data loss and the frequency of data loss resulting in the most accurate reconstruction of the interfering communication signal.

Three distinct cases occur when attempting to reconstruct the interference signal. First, when the receiver null occurs during the cyclic prefix of the symbol, the cyclic prefix acts as a buffer between symbols and is stripped off before demodulation. Therefore, the reconstruction of the signal is unaffected by a loss of cyclic prefix information, assuming the receiver is already time-aligned to the channel.

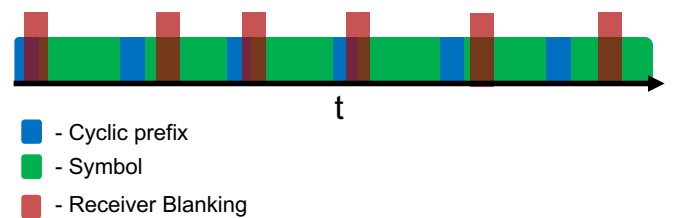


Fig. 2. OFDM symbols with receiver blanking.

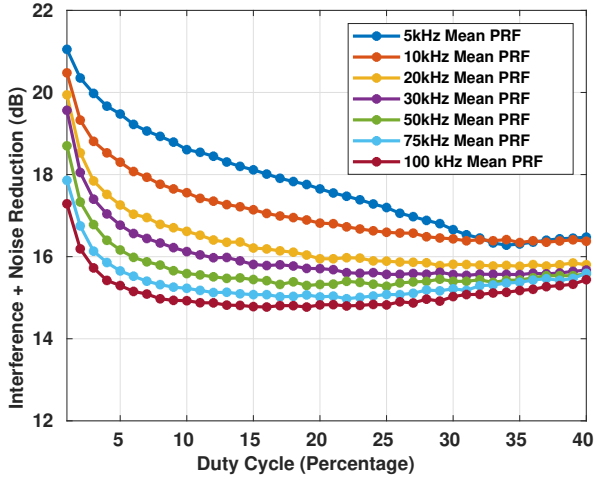


Fig. 3. Interference plus noise power reduction for staggered PRIs with mean PRFs and various duty cycles.

Second, when the receiver is blanked in the middle of a symbol, the reconstruction has reduced power for the time duration of the pulse. The reconstruction of the signal also has this lower power area that rolls to the correct power before and after it in the symbol, which results in a slight increase in interference at the maximum range of the previous pulse and the very close range of the current pulse.

Finally, when the receiver is blanked at the end of the OFDM symbol the data loss heavily impacts the performance of the interference removal. The end of the symbol is used to generate the cyclic prefix and therefore losing the end of the symbol results in a remodulated symbol with a very low power cyclic prefix and minimal interference removal in the time duration of the cyclic prefix. This third case can be mitigated by detecting the loss of the end of the cyclic prefix and using the received cyclic prefix to fill in the end of the OFDM symbol. This allows for better reconstruction of the cyclic prefix and mitigates the performance loss of the interference subtraction that would otherwise be encountered.

Checking for receiver blanking at the end of the symbol and replacing it with the cyclic prefix results in an average performance increase of 1 dB. More importantly, carrying out this check makes the output of the technique more consistent across time and therefore range.

V. RMMSE PROCESSING FOR RDM FORMATION

Pulse repetition interval (PRI) staggering enables expansion of both Doppler and range ambiguity regions but at the cost of higher Doppler sidelobes, with standard Doppler tapering [2]. In [8] it was shown that structure-based adaptive processing based on reiterative minimum mean-square error (RMMSE) estimation in the form of the reiterative super-resolution (RISR) algorithm [5] can suppress these higher Doppler sidelobes down to the noise floor, thereby compensating for one of the main difficulties that arises from staggering. Additionally, by incorporating the benefits of adaptive pulse compression (APC) [1] into a joint 2D range/Doppler RMMSE framework

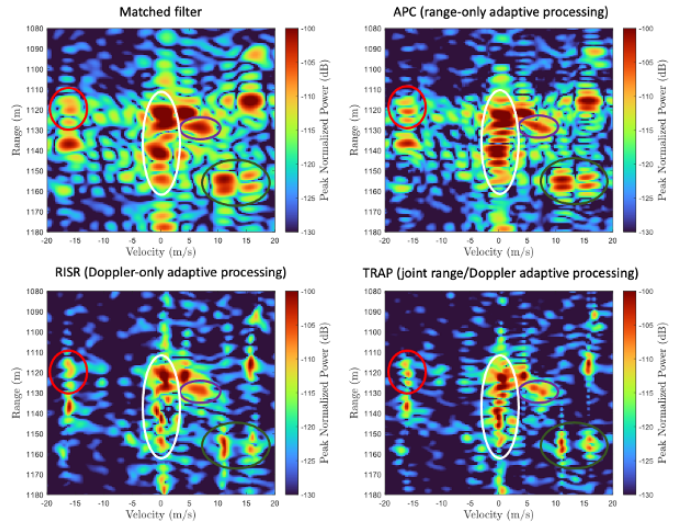


Fig. 4. Open-air range-Doppler response for LFM waveform and uniform PRI: (Upper Left) standard pulse compression and Taylor-windowed Doppler processing, (Upper Right) adaptive pulse compression and Taylor-windowed Doppler processing, (Lower Left) standard pulse compression and RISR Doppler processing, (Lower Right) Time range adaptive Processing (TRAP)

further suppression of sidelobes is achieved as described in [9] but with significant computational burden.

Figures 4 further illustrates the benefits of the performance of various adaptive receive processing techniques using open-air data involving the simple arrangement of repeated LFM waveforms having uniform PRI (i.e. not waveform-diverse). The point of this comparison between standard Doppler processing (using a Taylor taper) and RISR is to illustrate how sidelobe suppression can be achieved without the attendant tapering loss that is otherwise incurred (0.7 dB). A modest degree of Doppler super-resolution is also achieved, though care must be taken because excessive super-resolution can lead to severe mismatch loss. The benefit of improving Doppler sidelobes is seen in the center of the white circle where a large mass in standard processing resolves to separate movers (adjacent blue circle) that are slightly offset in Doppler. Furthermore, by extending to a 2D TRAP formulation, improved separability in range (red circle) is achieved in comparison to RISR that uses standard range compression.

While RMMSE processing can address Doppler sidelobes induced by staggering [8], further complications are introduced when one considers the expansion of the range ambiguity region via the same approach. Specifically, as discussed in [2], relative to a uniform PRI case (and for a constant overall coherent processing interval (CPI) extent) the use of staggering actually reduces the farthest range for which the entire set of PRIs are accessible. Consequently, it becomes necessary to consider both multiple-time-around (MTA) scattering from farther range intervals and the impact of blanked range intervals for particular PRIs that are simply unavailable due to their coinciding with the transmission of a later pulse (i.e. a per-PRI form of blind ranges). These effects can be incorporated into the scattering model so that the usually Vandermonde form of the Doppler steering vector for uniform PRI now becomes

$$v(f_D, l) = \begin{bmatrix} \alpha_1(l) \\ \alpha_2(l)e^{j2\pi f_d T_{acc}(2)} \\ \dots \\ \alpha_M(l)e^{j2\pi f_d T_{acc}(M)} \end{bmatrix}. \quad (2)$$

This generalized steering vector is clearly dependent on Doppler frequency f_d , with the impact of staggering incorporated via the accumulated slow-time [2]

$$T_{acc}(m) = \sum_{q=0}^{m-1} T_q \quad (3)$$

for $m = 1, 2, \dots, M$, with initial condition $T_0 = 0$ and thus $T_{acc}(1) = 0$ for the first pulse. We have further denoted a dependence on range index l by introducing the PRI-specific scaling term $\alpha_m(l)$. At a particular range index for which a given PRI is effectively blanked, $\alpha_m(l) = 0$ would occur. While one might otherwise expect $\alpha_m(l) = 1$ for all other cases, the unity condition actually assumes that no pulse eclipsing occurs, which in general may not be true. Consequently, for each PRI and each range index this scaling value can be $0 \leq \alpha_m(l) \leq 1$ depending on the degree of eclipsing (the $= 0$ blanking case is essentially fully eclipsed). It is likewise interesting to note that, should variable pulsewidth also be permitted during the CPI, this same steering vector model remains applicable, with the $\alpha_m(l) = 1$ condition now associated with longest pulse width and no eclipsing.

To improve separability between different MTA range intervals one could introduce slow-time coding, which as discussed in [2] effectively yields an affine transformation of the Doppler manifold when combined with staggering. Even greater separability could be achieved by allowing for non-repeating waveforms (but with same spectral support) across the CPI. Of course, the latter also imposes a coupling between slow-time and fast-time that requires compensation to address the attendant range sidelobe modulation (RSM). Overall, this increasingly complex emission/scattering framework possesses inherent non-stationary attributes that can greatly benefit from non-uniform waveform receive methods.

VI. REDUCED-DIMENSION RMMSE

Adaptive pulse compression (APC) via RMMSE estimation has been shown to be an effective technique for mitigating range sidelobes and uncovering masked targets, which makes it extremely useful when trying to detect targets masked by the high sidelobe structure of non-uniform PRI staggering [1]. However, APC has a computational complexity of $\mathcal{O}(N^3)$ for each stage of processing, making it difficult to implement in real-time for high- or multi-dimensional problems. Significant reductions are required as the total degrees of freedom in the full-dimension of the problem can be quite high. When non-uniform data collection schemes are introduced to expand the achievable range-Doppler space, the traditional range-Doppler ambiguity structure no longer applies. When sufficient random staggering is used, the range-Doppler space essentially becomes infinite; therefore, limitations are tied to detectability (in

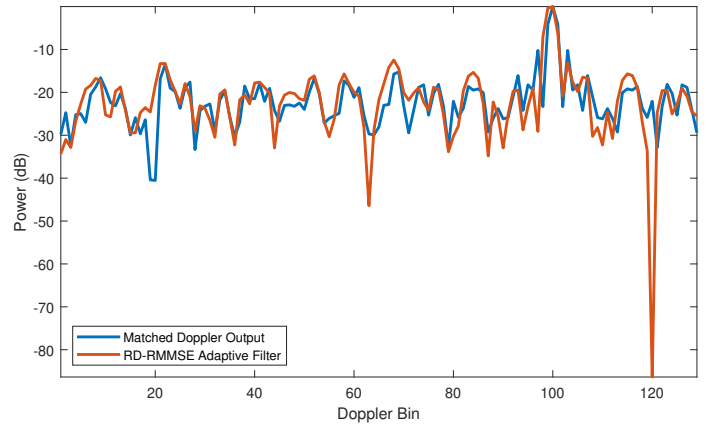


Fig. 5. Matched versus adaptive filter patterns for 64 pulses with randomly staggered PRI.

presence of high sidelobes) and processing constraints. There may be performance benefits using joint RMMSE processing in range and Doppler; however, the large range-Doppler space and number of fast-time/slow-time measurements result in a large number of degrees of freedom that makes real-time implementation difficult. Dimensionality reduction techniques are, therefore, helpful for lessening this computational burden and enabling implementation in real systems. Fast APC is one such technique that has been used to reduce computational complexity by nearly an order of magnitude [3]. The technique proposed here is inspired by post-Doppler STAP techniques that leverage Doppler processing and data formatting to achieve a reduction in computational complexity without sacrificing performance.

Space-Time Adaptive Processing (STAP) is another adaptive filtering technique that provides interference cancellation and improved detection of targets when performing ground-moving target indication [17] but suffers from a similar computational burden. Computational and training data limitations have motivated several reduced-dimension techniques, improving STAP performance and realizability [12]. For example, post-Doppler and beamspace STAP are popular choices of suboptimal approaches that rely on a static processing stage followed by adaptive processing on reduced degrees of freedom [7]. These techniques make use of efficient Doppler processing methods and a reduction in degrees of freedom to achieve reduction of computational complexity up to N^2 from a baseline of N^3 .

Motivated by reduced-dimension STAP, reduced-dimension RMMSE techniques use similar strategies to significantly reduce computational load. This partially adaptive technique will make use of data formatting to statically process portions of the data that can then be adaptively combined. The static portion of the processing can be performed by traditional matched filtering, highly efficient for which implementations exist, while the following adaptive approach uses an RMMSE approach to form adaptive filters and produce updated estimates. A one-dimensional post-Doppler RMMSE approach proposed here first splits a single CPI consisting of N pulses into K sub-CPIs, each containing M_k pulses. For formation

of L Doppler bins, the $N \times L$ slow-time manifold \mathbf{V} is reformatted into a series of K size $M_k \times L$ Doppler manifolds - one for each of the K sub-CPIs. If the slow-time PRI structure is identical for each sub-CPI, then the sub-CPI manifolds are identical, but this structure is not required and the PRI structures within each sub-CPI can be arbitrary. The sub-CPI manifolds are applied to each of the sub-CPIs to produce K coarse-resolution Doppler profiles. Being the result of only a few staggered-PRI pulses in each sub-CPI, these coarse-resolution profiles have high sidelobes and, potentially, Doppler ambiguities that must be resolved by the second, adaptive stage of processing.

The coarse-resolution profiles serve as the input to the adaptive stage of the RD-RMMSE processing. The outputs corresponding to the same l th Doppler bin from each of the K sub-CPIs are adaptively weighted to obtain an estimate of that Doppler bin. The adaptive weights are computed using the RMMSE architecture, which accounts for the strengths of other Doppler bins in the current iteration as well as the correlation between phase histories of different Doppler bins in the second-stage processing. This requirement results in a need to maintain a library of second-stage (length- K) steering vectors for every Doppler bin at the output of every Doppler bin formed at the coarse stage. However, these manifolds are known up front and only need to be computed once. At each iteration of the adaptive processing stage, the current Doppler bin estimates are used along with a $K \times L$ signal manifold to produce a $K \times K$ covariance matrix (one for each Doppler bin), which is used in the RMMSE formulation to obtain adaptive weights [1]. The reduction from an $N \times N$ matrix estimates and inversion to L different $K \times K$ matrix estimates and inversions is similar to the way that, for example, post-Doppler STAP reduces computational load [7]. An adaptive filter is calculated and applied for each of the L Doppler bins of interest to produce an updated estimate of the Doppler profile for each stage of iteration. The output of each iteration stage is then used as the input for the next iteration stage, and the process repeats. Figure 5 shows an example of the Doppler response of an adaptive Doppler filter that is produced by the partially adaptive approach described here. This adaptive Doppler filter has been built for Doppler bin 100, where the peak of the filter output occurs. The previous iteration indicated a strong signal in the 120th Doppler bin; hence, the adaptive weights place a null 20 bins away to prevent energy from the 120th bin corrupting the estimate of the 100th Doppler bin.

The main computational savings that result from this reduced-dimension technique come from the smaller matrix inversion required for every filter. Rather than inverting a $N \times N$ matrix for each Doppler bin, a $K \times K$ matrix inversion is needed instead.

VII. CONCLUSIONS

Non-uniform radar waveforms can be processed with RMMSE class methods and designed to have near uniform data support in range and Doppler via the RIP constraint. The computational load of RMMSE based methods can be

significant but on-going work into methods such as reduced dimension RMMSE and TRAP offer a path to near term hardware. The successful utilization of this technology will involve further maturing and integration of co-channel interference mitigation techniques, such as demod/remod, to enable robust performance of this technology in real world environments.

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