

Structure-Based Adaptive Radar Processing for Joint Clutter Cancellation and Moving Target Estimation

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- Moving target indication (MTI) radars are used to identify movers in the presence of clutter, typically using Doppler as the discriminant
- The broad class of reiterative minimum mean square error (RMMSE) algorithms have been **experimentally shown** to improve sensing performance by adaptively **reducing sidelobe interference** and **enhancing resolution** without the typical SNR loss
- However, from an MTI perspective the base RMMSE algorithms **do not provide a means to suppress clutter** and, likewise, clutter cancellation does not account for the RMMSE filtering process
- Consequently, we develop two modifications to the RMMSE framework to create adaptive algorithms that either **sequentially or jointly cancel clutter while also enhancing the estimation of moving targets**

- Consider a single snapshot $\mathbf{y}$ comprised of N time samples for which we wish to estimate the spectral content
- The generic receive model for spectral estimation can be represented as

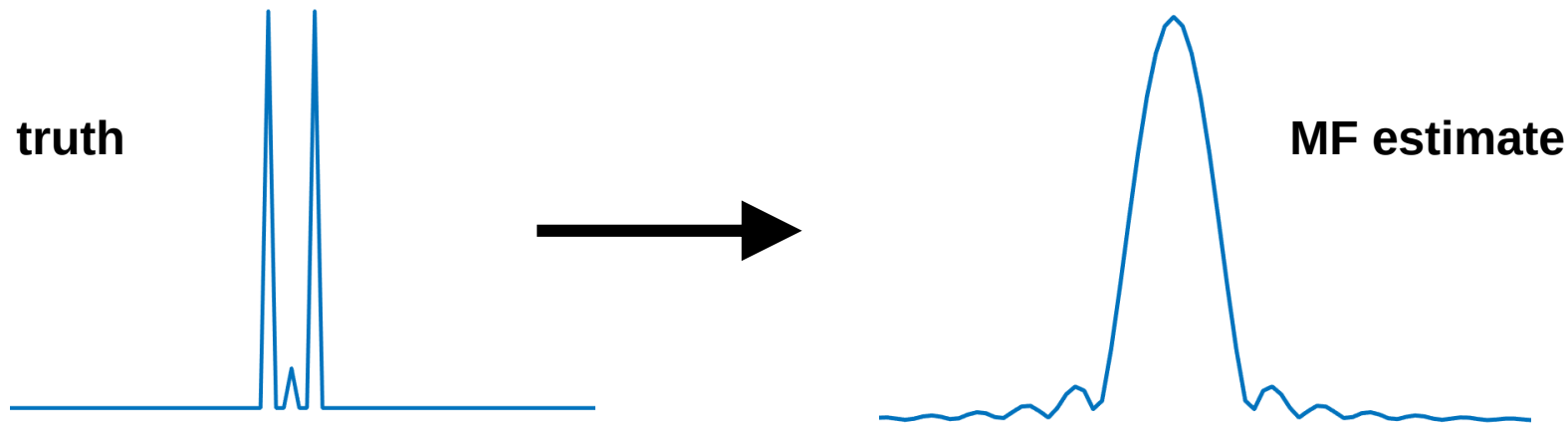
$$\mathbf{y} = \mathbf{S} \mathbf{x} + \mathbf{v}$$

where $\mathbf{x}$ is an $M \times 1$ vector comprised of $M \gg N$ frequency-dependent complex amplitudes, $\mathbf{S}$ is an $N \times M$ bank of frequency steering vectors, and $\mathbf{v}$ is additive noise of arbitrary distribution

- The maximum SNR estimate of $\mathbf{x}$ can be achieved via matched filtering as

$$\hat{\mathbf{x}} = \frac{1}{N} \mathbf{S}^H \mathbf{y}$$

but because $M \gg N$ the mapping between $\mathbf{x}$ and $\mathbf{y}$ is not 1-to-1, resulting in a coupling of the estimates in $\mathbf{x}$ and a sinc-like smearing that can obfuscate the truth



- The Reiterative Super Resolution (RISR) algorithm [1] can be used to adaptively compensate for spectral smearing by iteratively solving the Mean-Square Error (MSE) objective function

$$J = \mathbb{E} \left[\left\| \mathbf{x} - \mathbf{W}^H \mathbf{y} \right\|^2 \right]$$

for the $N \times M$ adaptive filter bank $\mathbf{W}$

- This objective function has the well known MMSE solution of

$$\mathbf{W} = \left(\mathbb{E} \left[\mathbf{y} \mathbf{y}^H \right] \right)^{-1} \mathbb{E} \left[\mathbf{y} \mathbf{x}^H \right]$$

[1] S.D. Blunt, T. Chan, K. Gerlach, "Robust DOA estimation: the re-iterative super resolution (RISR) algorithm," *IEEE Trans. Aerospace & Electronic Systems*, vol. 47, no. 1, pp. 332-346, Jan. 2011.

- RISR differs from traditional minimum variance spectral estimation in that it requires only a **single snapshot**.
- After obtaining $\hat{\mathbf{x}}_0$ from the MF estimate, the m th column of $\mathbf{W}$ at the i th iteration is

$$\mathbf{w}_{m,i} = p_{m,i-1} (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m$$

where $\mathbf{s}_m$ is the m th column of $\mathbf{S}$ and $\mathbf{R}_{\text{nse}}$ is the $N \times N$ noise covariance matrix.

- With the M values of $p_{m,i-1}$ along the diagonal, the $M \times M$ diagonal matrix

$$\mathbf{P}_{i-1} = \begin{bmatrix} \hat{\mathbf{x}}_{i-1} & \hat{\mathbf{x}}_{i-1}^H \end{bmatrix} \odot \mathbf{I}_{M \times M}$$

is the previous power spectrum estimate.

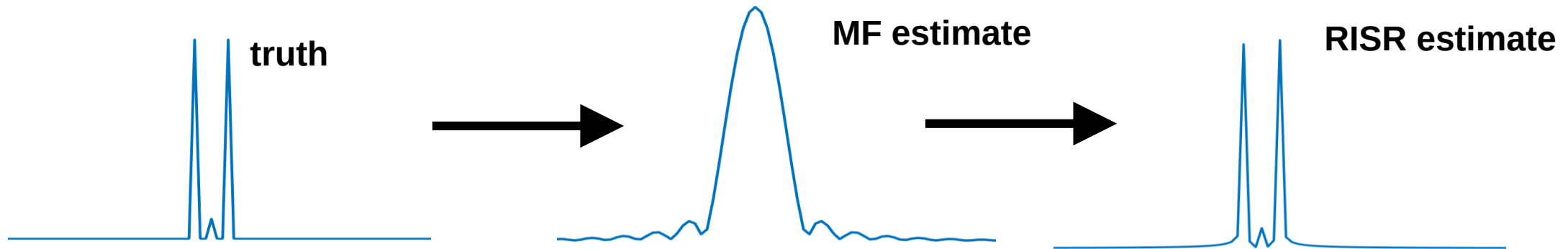
- The i th estimate of $\mathbf{x}$ is then obtained via

$$\hat{\mathbf{x}}_i = \mathbf{W}_i^H \mathbf{y}$$

- RISR was later modified in [2] to incorporate a gain constraint on the individual filters as

$$\mathbf{w}_{m,i} = \frac{(\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}{\mathbf{s}_m^H (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}$$

which has the well-known MVDR form and has been observed to be **more robust towards mismatch effects**, albeit without the same degree of resolution enhancement as unconstrained RISR



[2] E. Hornberger, S.D. Blunt, T. Higgins, "Partially constrained adaptive beamforming for super-resolution at low SNR," *IEEE CAMSAP Workshop*, Cancun, Mexico, Dec. 2015.

- A sequential or “**hard**” cancellation RMMSE technique was developed by separating the original linear model into 1) clutter parameters we wish to cancel and 2) the remaining parameters we wish to estimate as

$$\begin{aligned}\mathbf{y} &= \mathbf{S} \mathbf{x} + \mathbf{v} \\ &= \mathbf{S} \mathbf{x}_{\text{clut}} + \mathbf{S} \mathbf{x}_{\text{rem}} + \mathbf{v} \\ &= \mathbf{y}_{\text{clut}} + \mathbf{y}_{\text{rem}} + \mathbf{v}\end{aligned}$$

Note: here we denote the undesired component as clutter, but can be extended to any known/expected interference

- Ideally, $\mathbf{y}_{\text{clut}}$ and $\mathbf{y}_{\text{rem}}$ would lie in orthogonal subspaces, and thus given an estimate of the clutter (or “**background interference**”) and noise we can construct the normalized cancellation matrix

$$\mathbf{R}_{\text{clut}} = \mathbb{E}[\mathbf{y}_{\text{clut}} \mathbf{y}_{\text{clut}}^H]$$



$$\begin{aligned}\mathbf{R}_{\text{canc}} &= (\mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nse}}) / \sigma_v^2 \\ &= (\mathbf{R}_{\text{clut}} + \sigma_v^2 \mathbf{I}) / \sigma_v^2\end{aligned}$$

- Applying this cancellation matrix to the full covariance then projects onto the orthogonal complement of the clutter as

$$\mathbf{R}_{\text{canc}}^{-1} \left(\mathbb{E}[\mathbf{y} \mathbf{y}^H] \right) = \mathbb{E}[\mathbf{y}_{\text{rem}} \mathbf{y}_{\text{rem}}^H] + \mathbb{E}[\mathbf{v} \mathbf{v}^H] \quad \xrightarrow{\text{implying}} \quad \mathbf{R}_{\text{canc}}^{-1} \mathbf{y} = \tilde{\mathbf{y}} = \mathbf{y}_{\text{rem}} + \mathbf{v}$$

- Incorporation into RISR therefore yields

$$\hat{\mathbf{x}}_{\text{rem},i} = \mathbf{W}_i^H \mathbf{R}_{\text{canc}}^{-1} \mathbf{y} = \mathbf{W}_i^H \tilde{\mathbf{y}}$$

and the corresponding BaSC filters are:

unconstrained

$$\mathbf{w}_{m,i} = p_{m,i-1} \mathbf{R}_{\text{canc}}^{-1} (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m$$

gain-constrained

$$\mathbf{w}_{m,i} = \frac{\mathbf{R}_{\text{canc}}^{-1} (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}{\mathbf{s}_m^H (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}$$

- While BaSC employs a sequential cancel-then-estimate approach, it is also worth considering how the two operations can be performed jointly.
- Now modify the previous objective function to exclude the clutter term as

$$J = \mathbb{E} \left[\left\| \mathbf{x}_{\text{rem}} - \mathbf{W}^H \mathbf{y} \right\|^2 \right]$$

which leads to the set of unconstrained filters

$$\mathbf{w}_{m,i} = p_{m,i-1} (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m$$

- Here the (unnormalized) cancellation matrix is incorporated inside the adaptive framework, yielding Background Supplementary Loading (BaSL).

- A direct MVDR-like extension of the BaSL filters would actually prevent clutter cancellation
- Therefore, by **excluding the clutter loading from the denominator** we can pose a filter that is effectively gain-constrained relative to the absence of clutter (on a per-frequency-bin basis) via

$$\mathbf{w}_{m,i} = \frac{(\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}{\mathbf{s}_m^H (\mathbf{S} \mathbf{P}_{i-1} \mathbf{S}^H + \mathbf{R}_{\text{nse}})^{-1} \mathbf{s}_m}$$

- Note that the initial MF estimate will include the clutter component in $\mathbf{P}_0$ but it will be iteratively suppressed from the MTI estimate.

- The supplementary covariance matrix

$$\mathbf{R}_{\text{sup}} = \mathbf{R}_{\text{clut}} + \mathbf{R}_{\text{nse}}$$

used in BaSC and BaSL can be obtained in various ways

- The most direct approach is via the sample covariance matrix

$$\mathbf{R}_{\text{sup}} \approx \frac{1}{L} \sum_{\ell=1}^L \mathbf{y}_{\ell} \mathbf{y}_{\ell}^H$$

though doing so may involve “data contamination” where slow-moving targets get cancelled along with the clutter.

- Alternatively, a structured supplementary matrix can be formed by leveraging the RISR model and using the MF estimates of the clutter as

$$\mathbf{R}_{\text{sup}} \approx \mathbf{S} \hat{\mathbf{P}}_{\text{clut}} \mathbf{S}^H + \sigma_v^2 \mathbf{I}$$

where

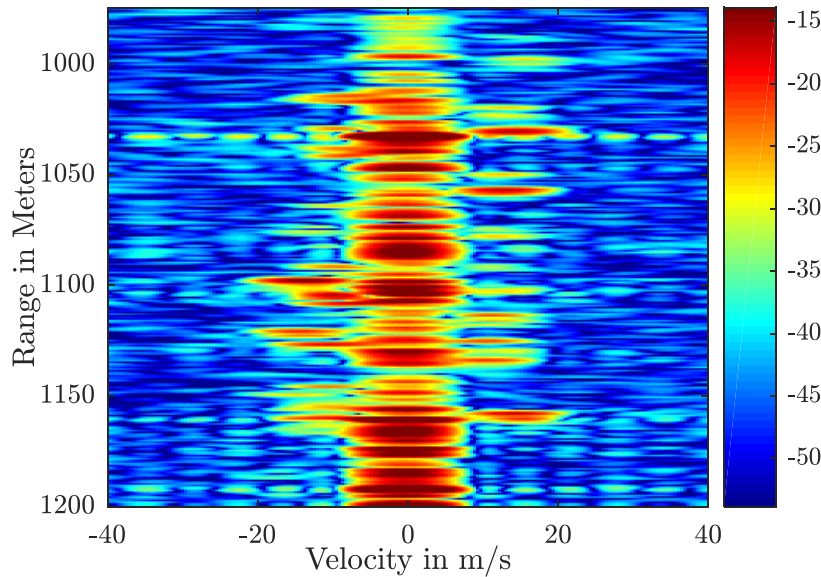
$$\hat{\mathbf{P}}_{\text{clut}} = \left[\frac{1}{L} \sum_{\ell=1}^L \hat{\mathbf{x}}_{\ell} \hat{\mathbf{x}}_{\ell}^H \right] \odot \mathbf{I}_{M \times M}$$

- This form can greatly reduce the required samples to form the supplementary matrix and can be constructed to exclude any non-clutter components that may contaminate the training data

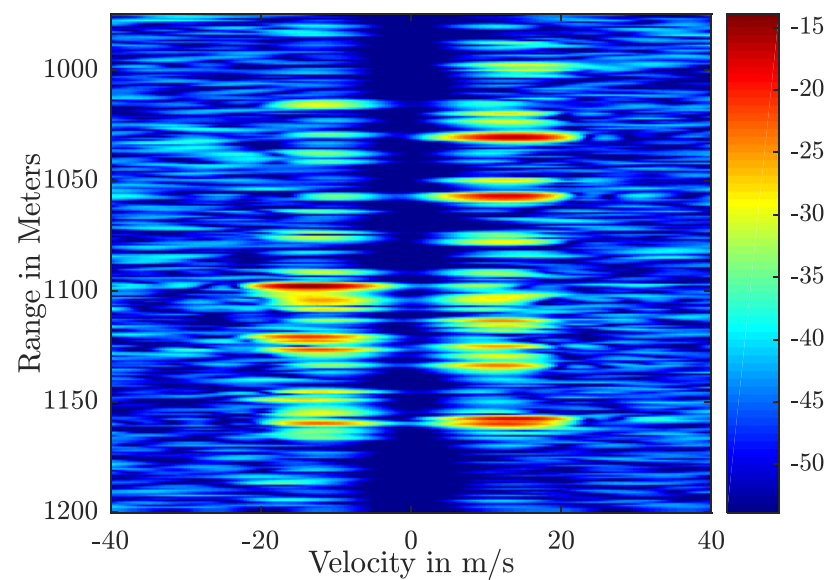
- First experimental assessment examines the impact upon data collected with an S-band radar testbed.
 - 3.55 GHz center frequency, 67 MHz 3-dB bandwidth, 4.5 μ s pulses, 30 kHz PRF
- 1000 pulsed, random FM waveforms (do not repeat during the CPI) illuminated a traffic intersection in Lawrence, KS (from roof of Nicholas Hall on KU campus).
- To demonstrate enhancement using adaptive estimation/cancellation only 150 of the pulses were used, compared with standard processing (FFT + clutter cancellation) using either 150 or all 1000.

Standard FFT processing and projection-based clutter cancellation (Baseline Case)

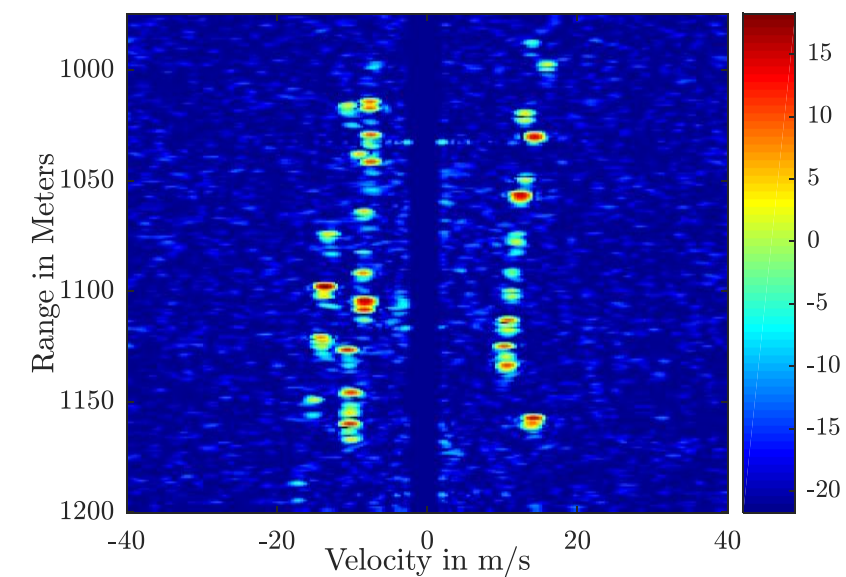
150 pulses, no clutter cancelation



150 pulses, with clutter cancelation



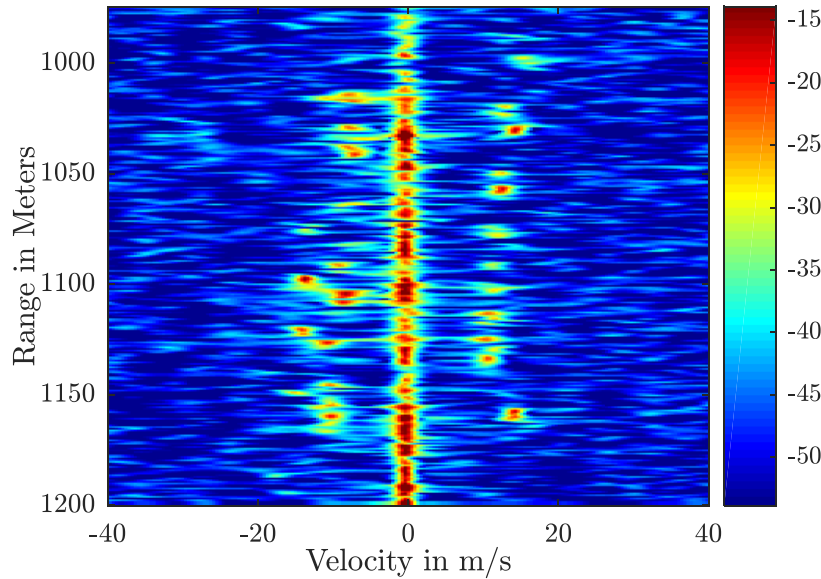
1000 pulses, with clutter cancelation



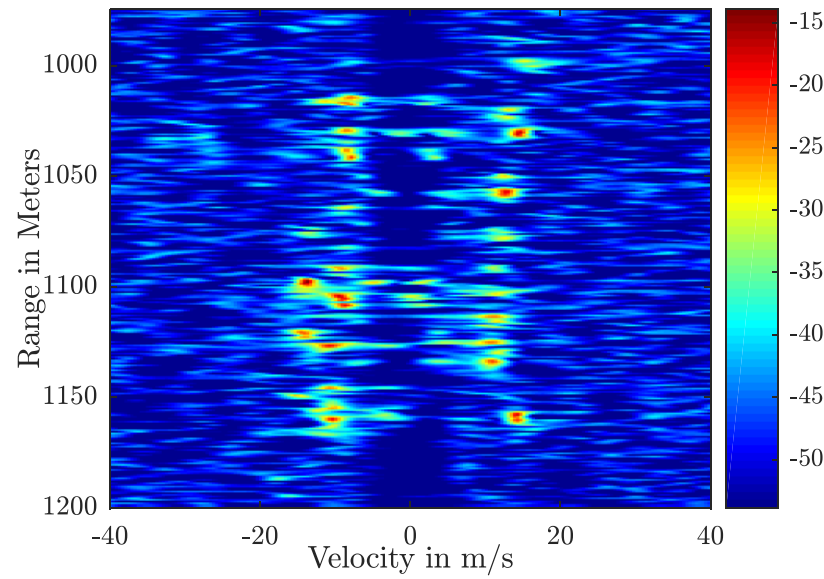
RMMSE without/with clutter cancellation (Structured $\mathbf{R}_{\text{sup}}$)

=> all 150 pulses

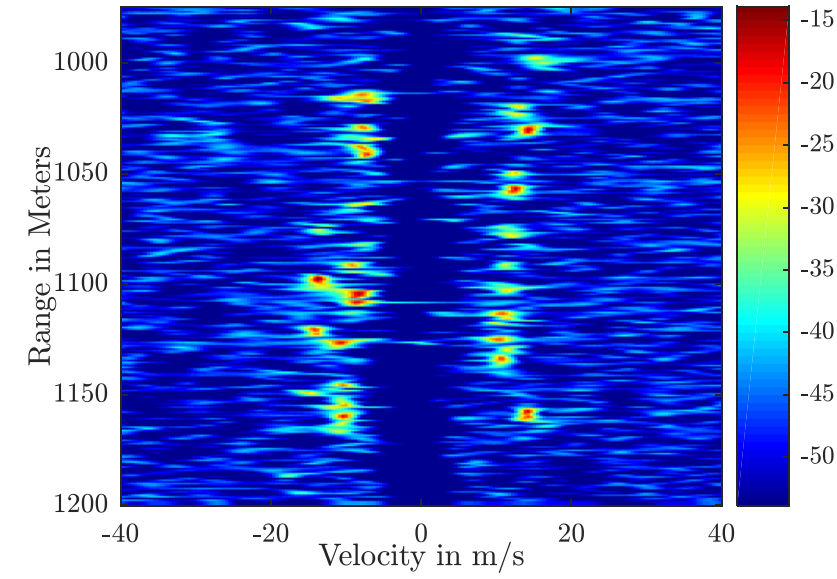
RISR (no cancellation)



BaSC (sequential cancellation)



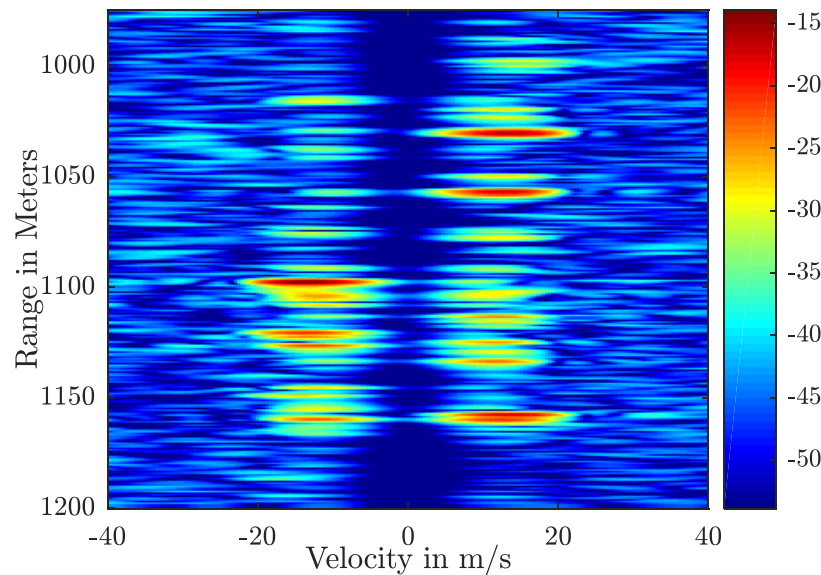
BaSL (joint cancellation)



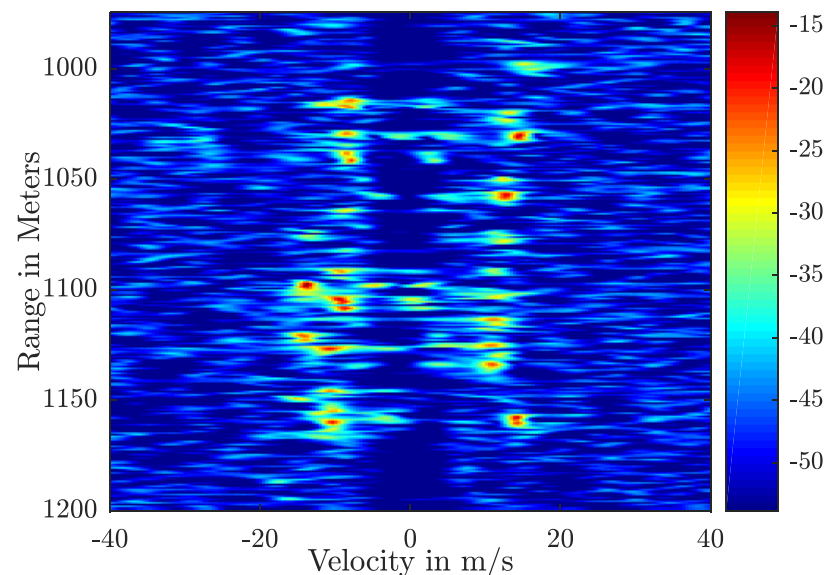
BaSC and BaSL both provide clutter cancellation, with the latter realizing less residue

Compare BaSC/BaSL with standard FFT processing and clutter cancellation (also 150 pulses)

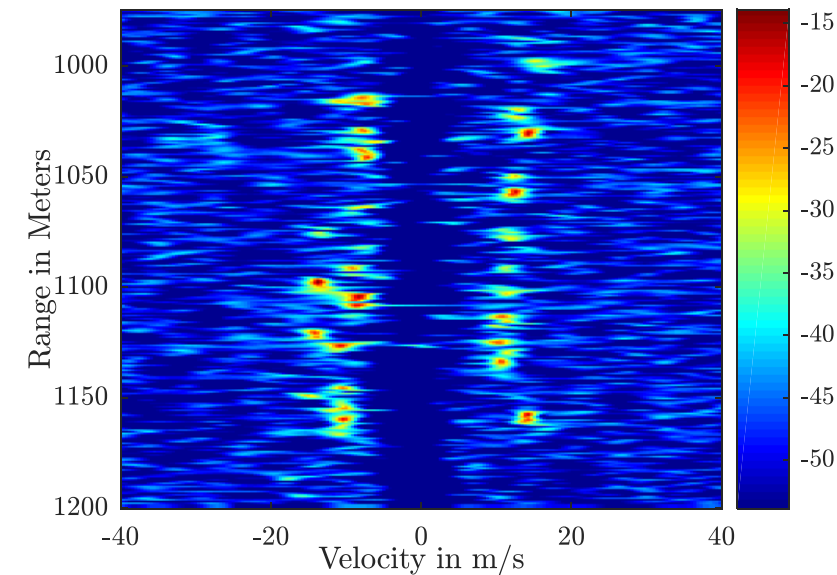
FFT with clutter cancellation



BaSC (sequential cancellation)



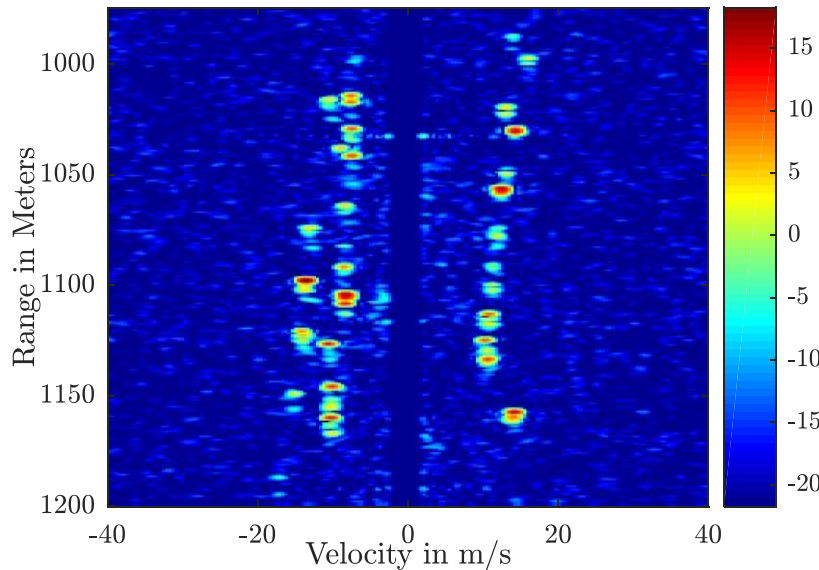
BaSL (joint cancellation)



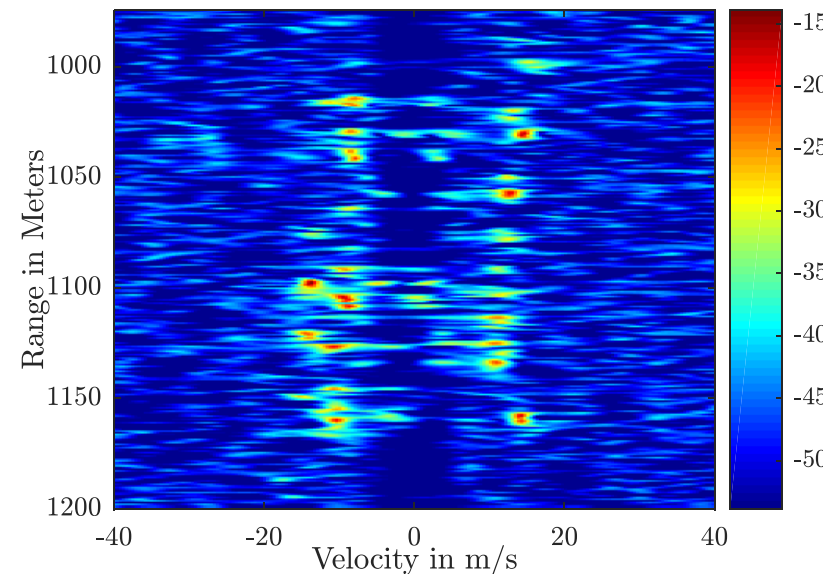
The RMMSE-based estimation of BaSC/BaSL clearly provides enhanced Doppler resolution

Now compare BaSC/BaSL (using 150 pulses) with standard FFT and clutter cancellation based on 1000 pulses

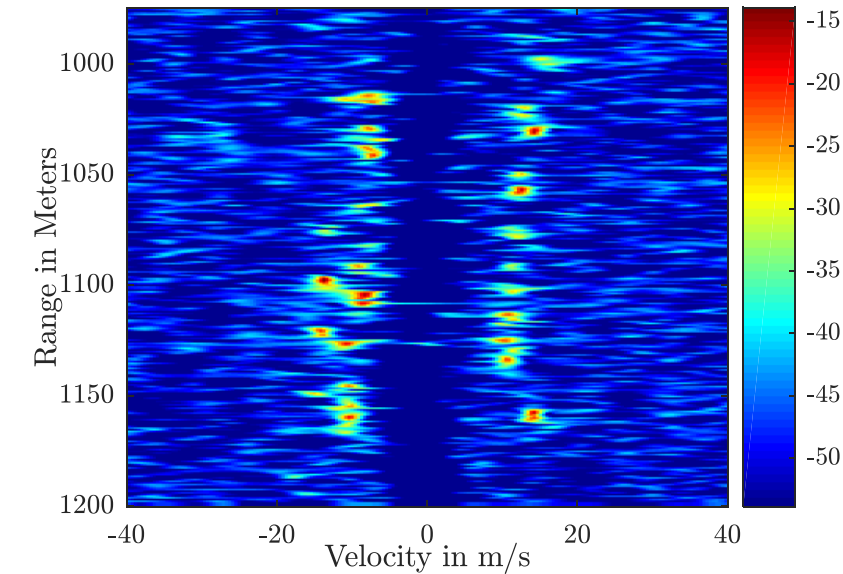
FFT & cancellation (1000 pulses)



BaSC (150 pulses)



BaSL (150 pulses)



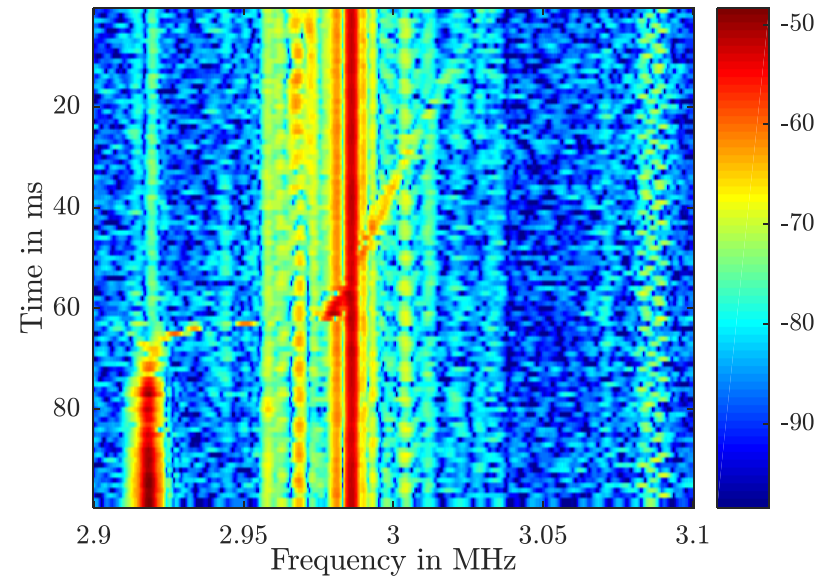
Enhanced Doppler resolution by BaSC/BaSL is on-par with nearly 7x increase in CPI, though without the commensurate suppression in sidelobe modulation

- Now consider data collected from a W-band radar testbed that operates in an FMCW mode being used to capture fast-moving objects [3].
 - 108 GHz center frequency, 600 MHz bandwidth, 500 μ s up/down chirp sweeps
 - Only down-chirp portions considered here
- Data collected in Apollo Auditorium of Nichols Hall, with a reusable paintball (or re-ball) fired away from the radar at 90 m/s.
- Significant multipath and clutter modulation effects (from ventilation fans in the ductwork) are clearly visible. We wish to perform clutter cancellation on a per-sweep basis while the re-ball moves through the scene.

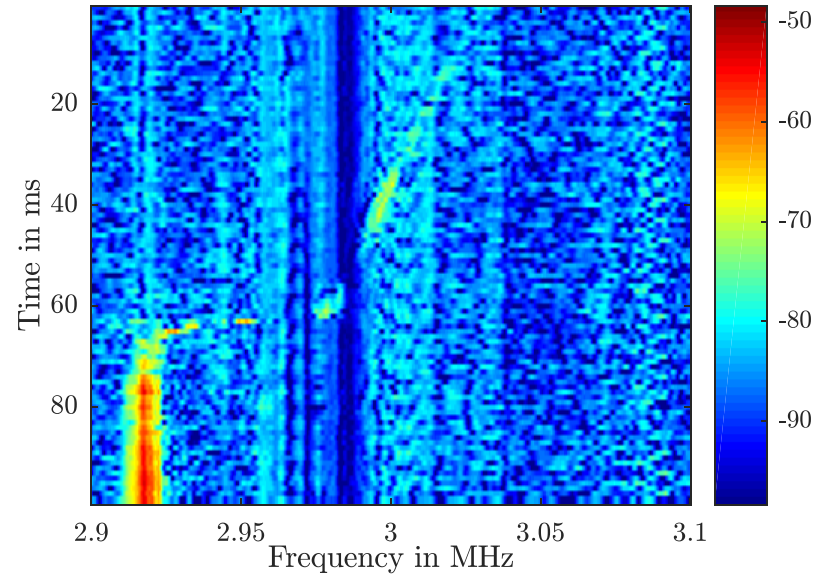
[3] C. Allen, L. Goodman, S.D. Blunt, D. Wikner, “Fast-time clutter suppression in mm-wave FMCW low-IF radar for fast-moving objects,” *2020 IEEE Intl. Radar Conf.*, Washington, DC, Apr. 2020.

- W-band FMCW & stretch processing vs. BaSC/BaSL
(Structured $\mathbf{R}_{\text{sup}}$)

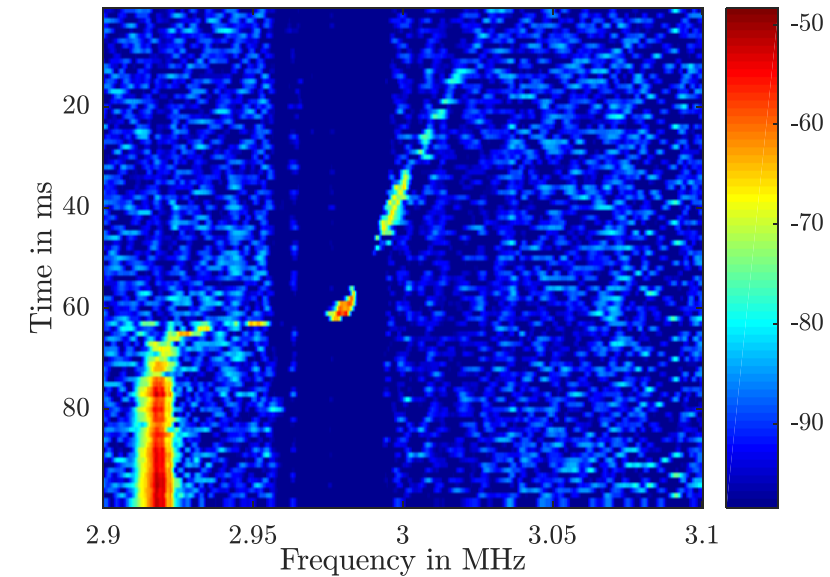
Standard stretch processing



BaSC applied on each sweep



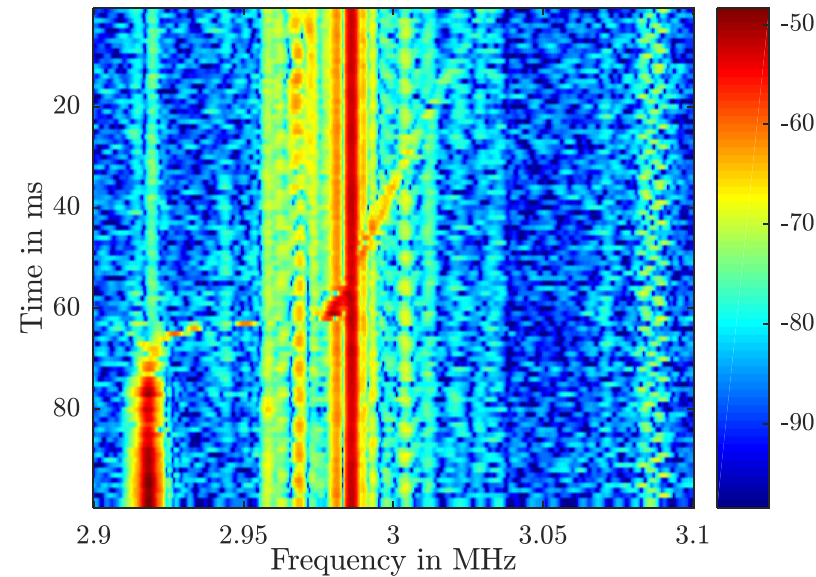
BaSL applied on each sweep



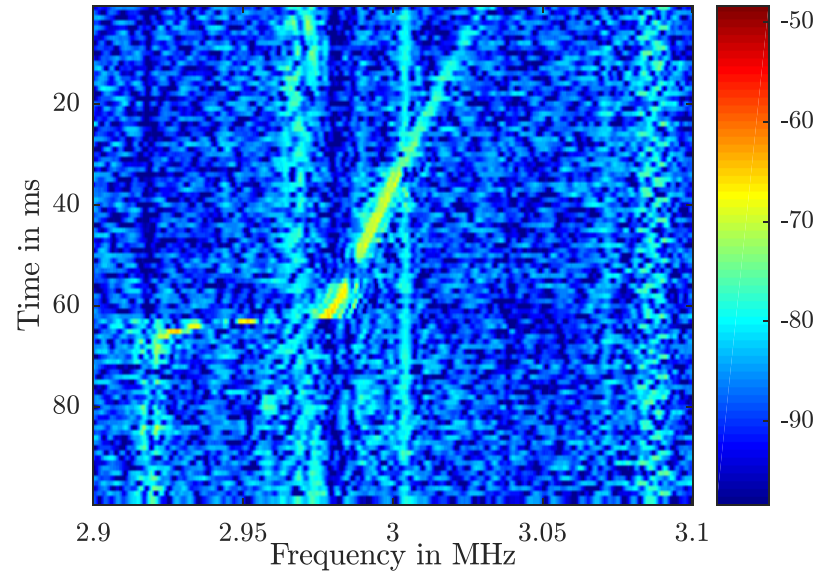
BaSC and BaSL achieve different degrees of clutter cancellation on a per-sweep basis

- W-band FMCW & stretch processing vs. BaSC/BaSL
(Sample $\mathbf{R}_{\text{sup}}$)

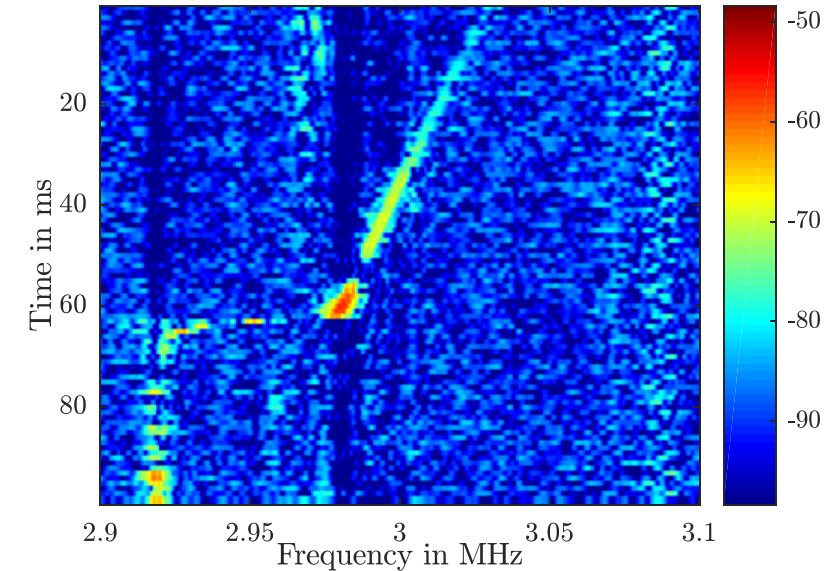
Standard stretch processing



BaSC applied on each sweep

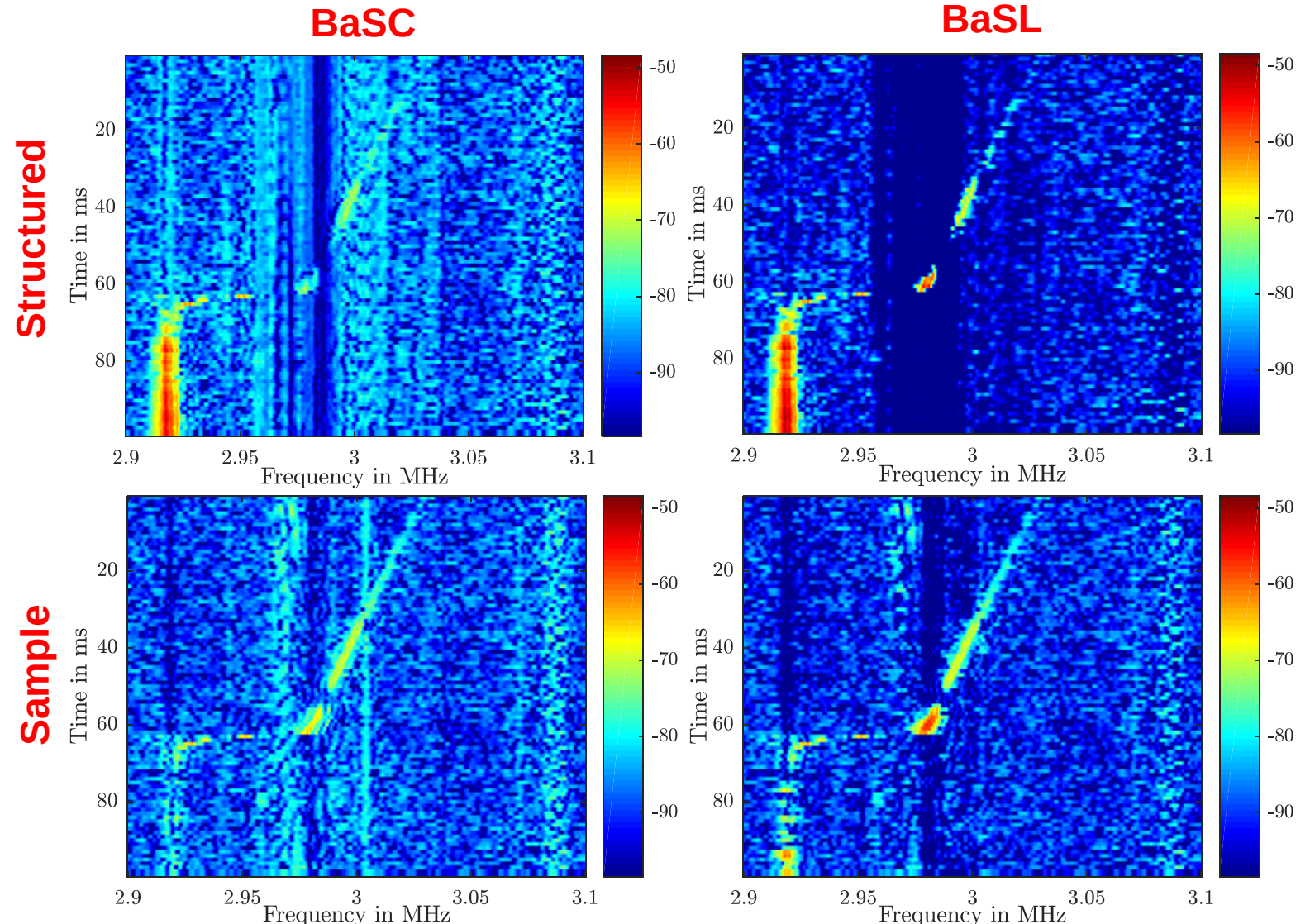


BaSL applied on each sweep

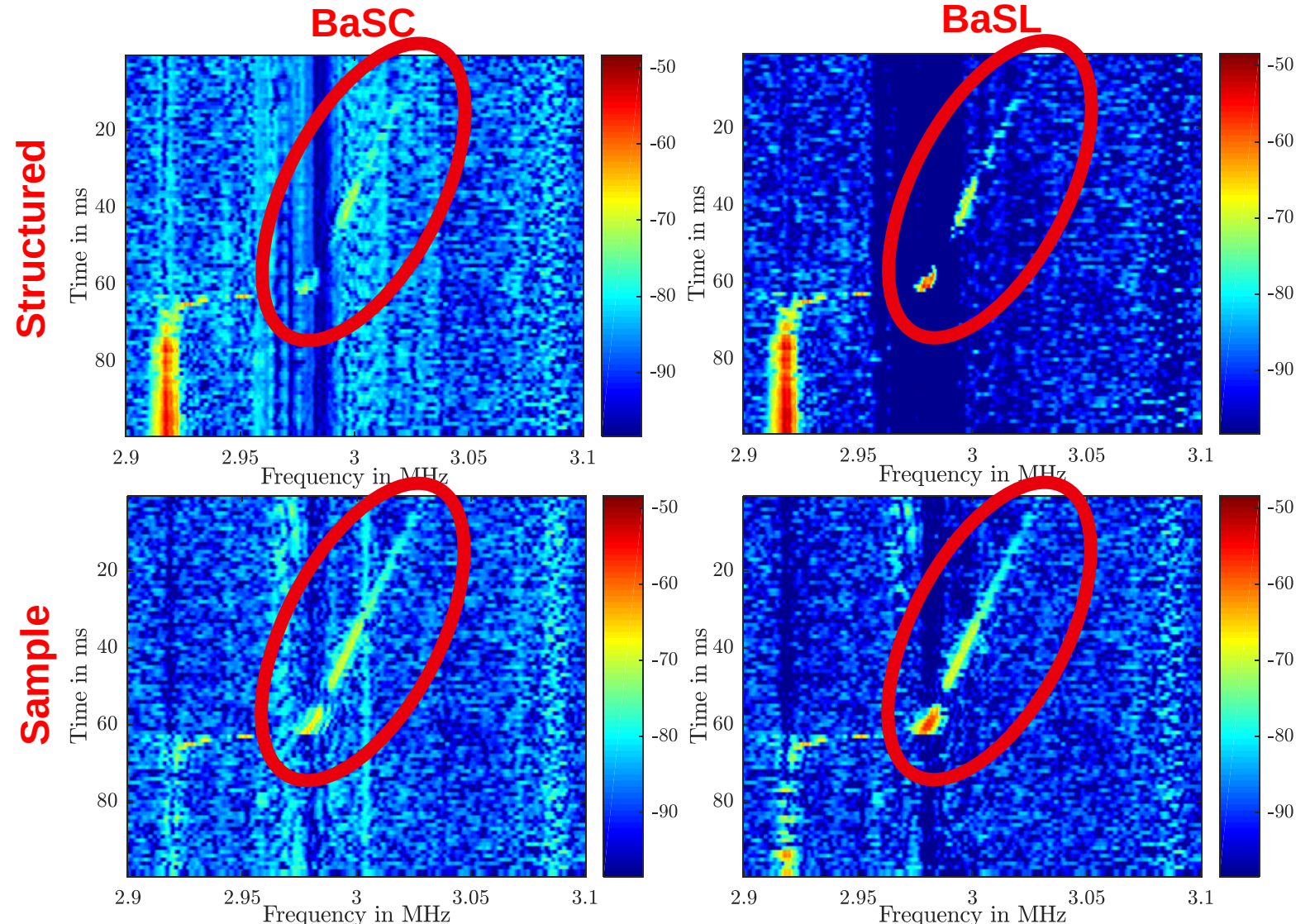


Different clutter attributes are suppressed depending on how supplementary matrix is formed.

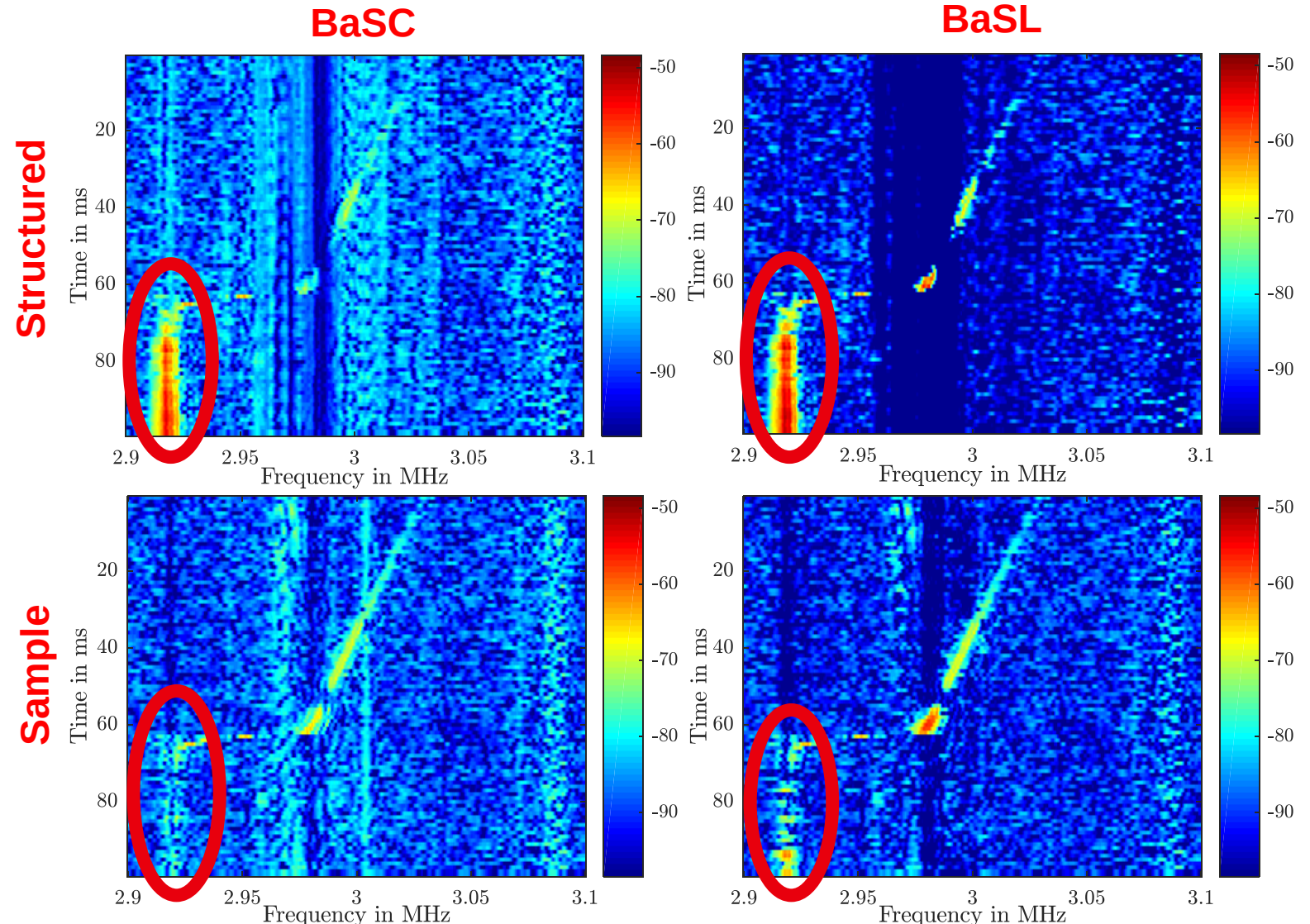
- BaSL achieves better clutter suppression with less target loss
- Structured and sample covariance matrices emphasize different phenomenology



- Re-ball traversing the auditorium until impacting a rubber sheet
- Re-ball velocity induces a Doppler shift (apparent range-shift due to chirp)
- BaSL using the sample clutter covariance is qualitatively the best



- Abrupt horizontal translation due to rapid deceleration upon impact, followed by vibration of sheet
- Structured covariance preserves this behavior, which is cancelled when using sample covariance



- RMMSE-based estimation useful for suppressing sidelobes and enhancing resolution
- When RMMSE-based estimation is combined with appropriate clutter cancellation, the benefits of both can be achieved
- Resulting sequential (BaSC) and joint (BaSL) adaptive estimation/cancellation methods demonstrated to be effective on two different measured data sets
 - May facilitate new sensing capabilities to capture short-time/high-speed dynamics